# MATHEMATICAL ANALYSIS OF MAGNETOHYDRODYNAMICS FLUIDS (MHD)

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## Abstract

Fluid dynamics, which is specifically liquids and gases, has a wide range of applications in physics, engineering, meteorology, oceanography and several other fields. Fluid dynamics demands higher mathematics to be fully demonstrated. For the mathematical formulation of the problem both magnetization and electrical conductivity of magneto hydrodynamics (MHD) are taken into account to characterize the electrical and magnetic properties of Magneto hydrodynamics (MHD). The aim of this is to mathematical explanation of Magneto hydrodynamics (MHD) of the study of the magnetic properties and behaviour of electrically conducting fluids.

***Key words***: Magneto hydrodynamics, Magnetic properties, Electrical properties.

## 1. INTRODUCTION

The mathematical models represent clear relationships and interrelationships among the variables and other factors considered important in solving problems. In Electrical engineering, mathematical modeling and simulations are important during the designing stage and the operational stage of electrical power systems. Mathematical models are also used when launching the stability of electrical circuits, when investigating microchips and during the enhancement of power supply networks. Mathematical models and equations leading mechanics and fluid flow have also been used to explain the flow of fluid when subjected to various stress configurations. In mechanical engineering, mathematical modeling is of prominence in crash simulations [1] and in structural optimization of motor vehicle skeleton under different amount of load / stress. In the last ten years, one of the significant examination centers worldwide has been around the investigation of fluid materials [2]. The formulation of electrically conducting fluids is made by adopting the principles of the well-known magneto hydrodynamics (MHD) which is incorporated with electrical and magnetic behavior [3]. The study of MHD flow over a stretching sheet still constitutes a topic of current ongoing research. It was also assumed that the magnetization of the fluid varied with the magnetic field strength H and the temperature T [4]. The administering field conditions for the progression of incompressible Newtonian liquids are the incompressible coherence condition and the Navier–Feeds conditions. Different examinations have been made to contemplate stream issues of Newtonian and non-Newtonian liquids with Navier slip limit condition [5].

In this paper, fluid flow of the magneto hydrodynamics (MHD) is studied and also electrical and magnetic properties of magnetohydrodynamics (MHD) are analyzed. For the mathematical formulation of the problem both magnetization and electrical conductivity of magneto hydrodynamics (MHD) are taken into account and consequently principle of magneto hydrodynamics (MHD) is adopted.

## 2. MATHEMATICAL FORMULATION

Let us consider the viscous, steady, two-dimensional, laminar flow of an incompressible and electrically conducting biomagnetic fluid past a flat elastic sheet which is stretched with a velocity proportional to distance, i.e., $u = cx$, where $c$ is a dimensional constant. The temperature of the stretched sheet $T_w$ is kept fixed, and the temperature of the fluid far away from the sheet is $T_c$, where $T_c > T_w$. The fluid is confined to the half space above the sheet, and magnetic dipole is located at distance $d$ below the sheet, giving rise to a magnetic field of sufficient strength to saturate the biomagnetic fluid. The flow configuration is shown schematically at Fig. 1.

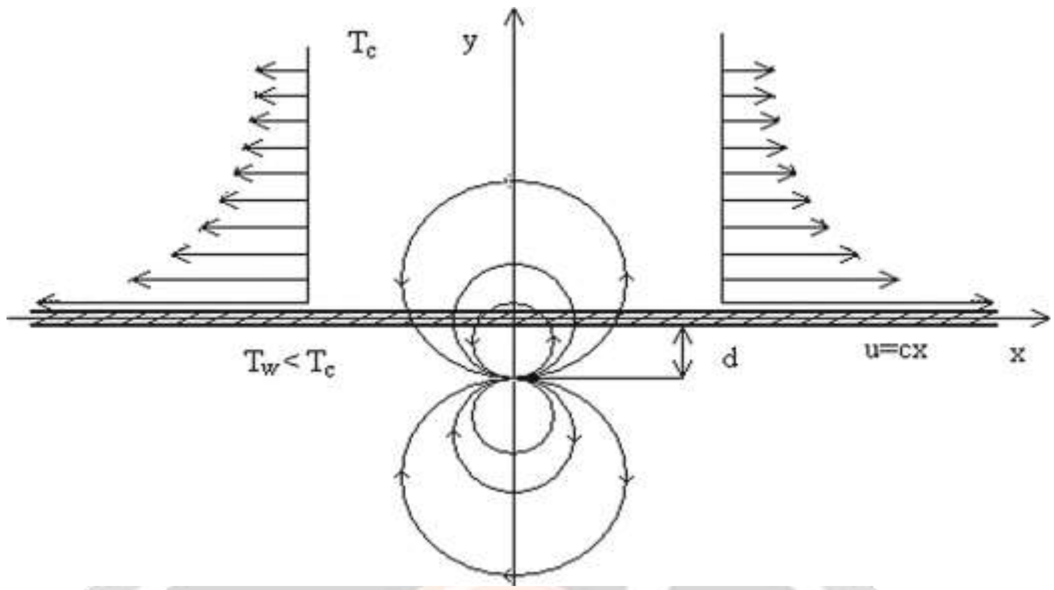


Fig.1. Flow configuration of the flow field

Under the above assumptions the equations governing the flow under consideration are [3, 6]:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 -----------(1)$$

Momentum equations:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu_0 M\frac{\partial H}{\partial x} - \sigma B_y^2 u + \sigma B_x B_y v + u\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) ----(2)$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu_0 M\frac{\partial H}{\partial y} - \sigma B_x^2 u + \sigma B_x B_y u + u\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) ----(3)$$

Energy equation:

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) + \mu_0 T\frac{\partial M}{\partial T}\left(u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y}\right) - \sigma B^2 u$$
$$= k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\left[2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] ----(4)$$

Subject to the boundary conditions:

$$y = 0: \quad u = cx, \quad v = 0, \quad T = T_W -------- (5)$$

$$y = \infty: \quad u = 0, \quad T = T_c, \quad p + {}^{1}\!/_{2\rho q^2} = const -------- (6)$$

In the above equations $\mathbf{q} = (u, v)$ is the dimensional velocity, $p$ is the pressure, $\rho$ is the biomagnetic fluid density, $\sigma$ is the electrical conductivity, $\mu$ is the dynamic viscosity, $C_p$ the specific heat at constant pressure, $k$ the thermal conductivity, $\mu_0$ is the magnetic permeability, $\mathbf{H} = (Hx, Hy)$ is the magnetic field strength, and $B$ is the magnetic induction ($\mathbf{B} = \mu_0\mathbf{H} \Rightarrow (B_x, B_y) = \mu_0 (H_x, Hy)$).

Thus, the magnitude $||H|| = H$ of the magnetic field intensity by

$$H(x, y) = \left(H_x^2 + H_y^2\right)^2 = \frac{\gamma}{2\pi}\,\frac{x}{x^2 + (y + d)^2} ----- (7)$$

The magnetic field intensity $H$ can be expressed by an analogous manner, as

$$H(x, y) \approx \frac{\gamma}{2\pi}\left[\frac{1}{(y + d)^2} - \frac{x^2}{(y + d)^4}\right] ----- (8)$$

The above relations of the magnetic field strength $H$ and its gradients, i.e., (8) is valid close to region where $x = 0$ and are used for the further transformation of the system of the governing equations.

This relation expresses the magnetization as a function of the magnetic field strength intensity $H$ and the temperature of the fluid $T$ [7].

$$M = KH(T_C - T) \text{ --------- } (9)$$

Where $K$ is a constant called pyromagnetic coefficient and $T_c$ is the Curie temperature.

## 3. MAGNETOHYDRODYNAMICS (MHD) FLOW

Magnetohydrodynamics (MHD) was first found by Hannes Alfvén (1908-1995) [8]. Magnetohydrodynamics (MHD; also called magneto-fluid dynamics or hydromagnetics) is the study of the magnetic properties and behaviour of electrically conducting fluids Examples of such magnetofluids include plasmas, liquid metals, salt water, and electrolytes. The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself. The set of equations that describe MHD are a combination of the Navier–Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism [9].

The incompressible Navier–Stokes–Maxwell system with Ohm's law in two and three space-dimensions:

$$\partial_t u + u.\nabla u - \mu \Delta u = -\nabla p + j \times B, \quad div\, u = 0 \quad \text{-------}(10)$$

$$\frac{1}{c}\,\partial_t E - \nabla \times B = -j, \; j = \sigma(cE + u \times B), \; \text{-------}(11)$$

$$\frac{1}{c}\,\partial_t B + \nabla \times E = 0, \; div\, B = 0\,, \; \text{-------}(12)$$

Where $c > 0$ denotes the speed of light, $\mu > 0$ is the viscosity of the fluid and $\sigma > 0$ is the electrical conductivity. The t is the time, $u = (u_1, u_2, u_3) = u(t, x)$ stands for the velocity field of the (incompressible) fluid while $E = (E_1, E_2, E_3) = E(t, x)$ and $B = (B_1, B_2, B_3) = B(t, x)$ are the electric and magnetic fields respectively. All are three-component vector fields. Finally, the scalar function $p = p(t, x)$ is the pressure and is also an unknown. Observe, though, that the electric current $j = j(t, x)$ is not an unknown, for it is fully determined by (u, E, B) through Ohm's law.

In MHD, when the magnetic field and the conducting fluid comes into interaction, 4 electric current of density $J$ is induced into the conducting fluid which results in induced magnetic field. The total field $B$ interacts with induced current developing Lorentz force $F = jXB$ MHD has a wide-ranging application and they include: MHD pump, MHD propulsion, MHD generators and MHD flow meters.

The MHD generator has the ability to work under very high temperatures than the traditional electric generators. The thermal or kinetic energy directly is directly converted into electrical energy. As MHD generators have no movable parts, the chances of mechanical failure get reduced. The simplest MHD generator has a gas nozzle that serves as a combustion chamber injecting pulses of the gas into the duct. The walls of the duct act as electrodes. The first MHD generator was developed with copper disks and horse shoe magnet in 1831 by Michael Faraday. The powerful electromagnet in Faraday's generator acts as a source of magnetic field and the current flowing between the two installed electrodes which are perpendicular to magnetic field serves as the main electrical output of the MHD generator. The schematic diagram of MHD generator is shown in Fig.2.

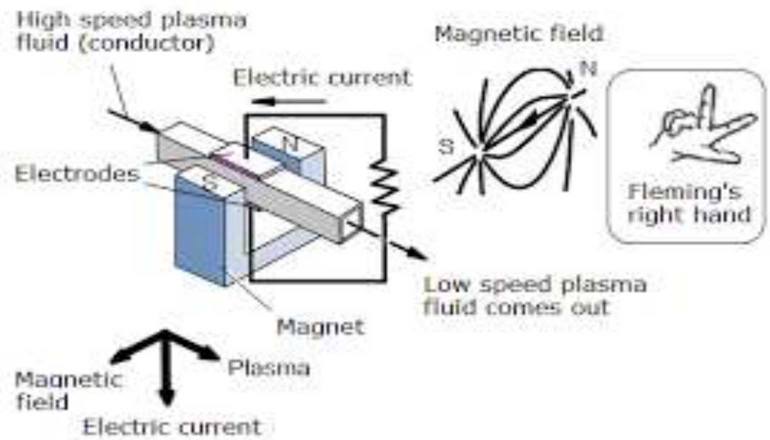


Fig. 2. MHD generator

**CONCLUSION**

Mathematical formulation of the problem both magnetization and electrical conductivity of magneto hydrodynamics (MHD) are utilized to characterize the electrical and magnetic properties of Magneto hydrodynamics (MHD). Magneto hydrodynamics (MHD) of the magnetic properties and behaviour of electrically conducting fluids by Mathematical methods are studied.

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