MICRO LOAN PORTFOLIO EQULIBRUIM MIX: A MARKOV CHAIN APPROACH

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ABSTRACT

Credit risk (loan) management has been the priority of almost all Financial Institutions in recent years. The interest lies as to whether the financial institution will be able to meet the demands of their potential clients whereas clients are expected to meet their short term or long term loan obligation. In view of this an optimal loan allocation mix policy from the steady State distribution of loan disbursement process is presented in this study.
The objectives of the study are to (i) obtain an optimal loan allocation mix policy (ii) to estimatethe transition matrix using time series data on loans .iii) To find out whether risk can change or is likely to change due to portfolio changes.[1]
Monthly data on actual loan Disbursement of four loan types for a period of twenty-four monthsis analyzed. An estimated Transition probability matrix is analyzed using the Markov chain approach. It is from this that the transition probability matrix and the steady state distribution of loan disbursement process are obtained. The loan types disbursed by the financial institution include Agricultural, Susu, Small and Medium Enterprise (SME) and Salary loans.
The estimated transition matrix showed that the probability of loan switching from Agricultural loan to Small and Medium Enterprise (SME) loan is the highest (0.54) while loan switching from Salary loan to Agricultural loan is the lowest (0.034).
Probability of no loan switching for Susu loan is (0.380), Probability of no loan switching for SME loan is (0.52), and whiles that of Salary is (0.044).
From the estimated probability transition matrix, the steady state distribution indicated that in the long run, SME loan constitutes 52.36% of the total funds allocated for loans. by Agricultural loan 38.17%, salary loan 4.95% and Susu loan 3.76%, of the total loan amount.

Keyword: - *Stochastic process, Markov chain, Stationarity, Portfolio, State space, transition matrix, Periodicity*

1. INTRODUCTION

Managing credit risk has been the priority of almost all Financial Institutions in recent years. The interest lies as to whether the Financial Institutions would be able to meet the demand of their potential clients whereas clients are to meet their short term or long term loan obligation. There is therefore the need for a solution that will maximize the profit of the Financial Institution whiles reducing the probability of default to the barest minimum. In view of this an optimal loan allocation mix policy from the steady state distribution of loan disbursement process is presented in this research.

1.1 BACKGROUND OF THE STUDY

Accessing a loam from financial institutions is on the increase due to the establishment of Small and Medium Enterprises (SMEs).In the quest of making loans accessible to all, Financial Institutions encounter series of financial crisis that create excessive loss in the banking industry.

Some financial analysts attribute this to the World Wide credit crunch where as others link the issue to poor management practices by some Financial Institutions. Banks are encouraged to manage their finances efficiently in times like this. The Bank of Ghana in view of this initiated the Computerization and Networking of Bank Branches as well as the Operations and product service development of Banks. [2]

The product service development of Banks can be achieved when banks are able to operate efficiently and effectively in their product offering and as well maintain an appropriate fund portfolio for their survival and growth [3]

1.2 SIGNIFICANCE OF THE STUDY.

The findings would

i) Enable management of banking Institutions come out with pragmatic policies for loan portfolio management.

ii) Remind credit staff about the implications of their credit duties in creating quality portfolio for their banks.

iii) Help in the development of the banking industry thereby improving the economic status of the nation.

iv) Be a contribution to existing works on loan portfolio management.

2. LITERATURE REVIEW

THE NEED FOR LOAN

According to Burton (2002), cited in Offei (2011) engaging in loan gives a greater amount of money to fulfill ones project. [4] Loan repayment is difficult for most people but financial constraint compel individuals to resort to loans. Most people apply for loans because they want to; start or develop an existing business, pay off an existing loan ,own a property, for educational purposes and others, cater for unexpected emerge and finally to cater for recurring everyday expense such as rent, food, utilities etc.[5]

BENEFITS OF LOAN PORTFOLIO

Light et al., (2005) indicated that loan portfolio management is one of the responsibilities critical to the success of an institution.[5]

Loan portfolio encompasses all systems and processes used by management to adequately plan, direct, control and monitor the institution lending operation portfolio risk

Cyert et al., (1962) developed a finite stationary Markov chain model to predict uncollectible amounts (receivables) in each of the past due category. The states of the chain were defined as Normal payment, past due, and bad-debt states. [6]

Grinold, (1983) used a finite Markov chain model to analyze a firm's market value. [7]

White (1993) presented several models employed in the banking industry. These included discriminant analysis, decision tree, expect system for static decision, dynamic programming, linear programming, and Markov chains for dynamic decision making. Markov chain modeling is a common approach used in the analysis of credit risk. As discussed by White (1993), Markov decision models have been used extensively to analyze real world data in (i) Finance and Investment, (ii) Insurance, (iii) Credit area. [8]

Lee (1997) used an ARMA model to analyze the linkage between time-varying risk premier in the term structure and macroeconomic state variables. [9]

Light et al., (2005) indicated that loan portfolio management is one of the responsibilities critical to the success of an institution. [10]

It is the dynamic process of managing an institution’s primary earning asset to achieve the primary objectives of the board’s strategic business and capital plans.

Mariano et al., (1970) used probability in constructing a monthly model for predicting currency crises in Southeast Asia. The approach was designed to avoid the estimation inconsistency that might arise from Misclassification errors in the construction of crisis dummy.[11]
Demiris, (2006) uses Markov chain in his Bayesian Inference for Stochastic Epidemic model..[12]
Baik et al., (2006) stated that the accurate prediction of the current and future conditions of waste water systems using available assessment data is crucial for developing appropriate proactive maintenance and rehabilitation strategies for an aging waste water collection and conveyance System.[13] Soyer and Feng, (2010) considered reliability models for assessing mortgage default risk. [14]
Gardiner (2010) presented an optimal loan allocation mix policy from the steady state distribution of loan disbursement. Monthly data on actual loan disbursement of four loans types for a period of twenty-four months was considered by using a transition matrix. From the estimated probability transition indicated that in a long run trade loan should constitute 77.3% of the total loan, 10.3%,for service loan ,2.0% for production loan and 10.4%for susu loan.[15] Thyagarajan et al., (2011) analyzed the actual loan sanctions with the non-documented method of loan allocation of the selected retail bank over a period of twenty-four (24) months revealed that there is a scope to their income earning. From the results it was suggested that the proportions of loan disbursed should be as follows: Housing (32.0%), others (28.1%), Business (20.0%) and Education (19.7%). [16]

2. METHODOLOGY

Monthly data on Actual loan disbursement for four loan types for a period of twenty four months is obtained and the estimated transition probability matrix of the movement of one loan type to the other is analyzed. It is from this transition matrix that the steady state distribution of loan disbursement process is obtained. The name of the Financial Institution used for this study will not be mentioned for confidentiality sake. Among the loan types offered are Agricultural loan, susu loan, Small and Medium Enterprise, Salary loan. Several statistical tests will be conducted to investigate the suitability of the data to test the stationarity, homogeneity, periodicity and absorption status of the process. Forecast of loan disbursement for a period of twenty-four (24) months will then be made, in order to forecast the future allocation of each type of loan.

2.1 MARKOV CHAIN.

A Markov chain is a random process usually characterized as memory less, this is due to the fact that the next state depends only on the current state and not on the sequence of events that preceded it.
Let X_n n ϵ $\{0,1,2| \ldots\}$ be a stochastic process with state space which can be finite or countable and is a transition matrix.

$P = [P_{ij}] = [P\{X_{n+1} = j/X_n = i\}]$ for i, j ϵ D…….. (1)

X is a Markov chain if

$P[\{X_{n+1}=j/X_{0,} \ldots . X_n\} = P\{X_{n+1}=j/X_n\}]$…… (2)

X is time homogeneous if P $\{X_{n+1}=j/X_n$ is free of n ϵ D is considered to be discrete (countable number of states).The n step transition probabilities $P_{ij}{}^{n}$=P $\{PX_{m+n}=j/PX_m=i\}$ are the i,j entries of the matrix P^n

$\sum_{j \in D} P\{X_{n+1} = \frac{j}{X_n} = i\} = 1$…… (3)

The possible values of X_i form a countable set S called the state space of the chain.Markov chains are often described by a directed graph, where the edges are labeled by the Probabilities of going from one state to the other states.
The stochastic relation is used to estimate the transition probability matrix.

$Y_i(t) = Y_i(t\text{-}1)\ P_{ij} + U_j(t)$ …….. (4)

The relation is equivalent to $Y_j = X_j P_j + U_j$… …… (5)

For j = 1, Y_j is a 23 component vector of proportion for loan type 1 beginning from t = 2 February 2015 to t = 24 (December 2016. Similarly loan type 2, 3 and 4, are defined accordingly.

Matrix X_j is a (23 ×4) matrix of loan proportions beginning at t = 1, to t = 23. P_j is a probability vector for all $j \in E$.

Let Q be the fundamental matrix

$$Q = \begin{matrix} 0.035300 & 0.541110 & 0.378352 & 0.045299 \\ 0.035150 & 0.522400 & 0.391430 & 0.051030 \\ 0.035710 & 0.528610 & 0.380405 & 0.055230 \\ 0.034890 & 0.537234 & 0.3831940 & 0.044682 \end{matrix}$$

The time to absorption T is given by

$$T = NC$$

Where C is a column vector with all entries being 1.

$$N = (I - Q)^{-1}$$

The system takes a maximum of four steps to reach the absorbed state. A major useful application of Markov chain is its ability to make forecast of loan proportions. The forecast of loan proportion for the period January 2015 to December, 2016 is made in this study. Table 3.2 shows the monthly forecast proportions of the individual loans disbursed.

The forecast values in the transition matrix gives the policy maker an indication on the average proportion of the different types of loans. In practice forecast have to be updated so long as data is available.

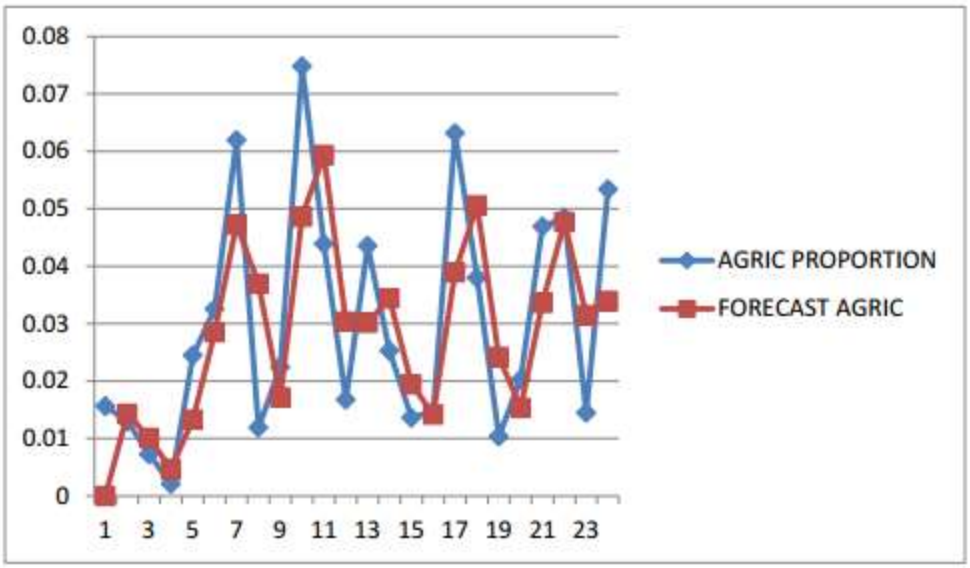

FIG 3.2 ACTUAL AND FORECAST PROPORTION OF AGRIC LOAN.

The actual loan proportion of the Agric loan rises sharply from January to June 2015 until it falls in July,2015.It then rises sharply until it reaches its peak in July,2015.It then rises sharply until it reaches its peak in July,2016 until it finally declines in December,2016.

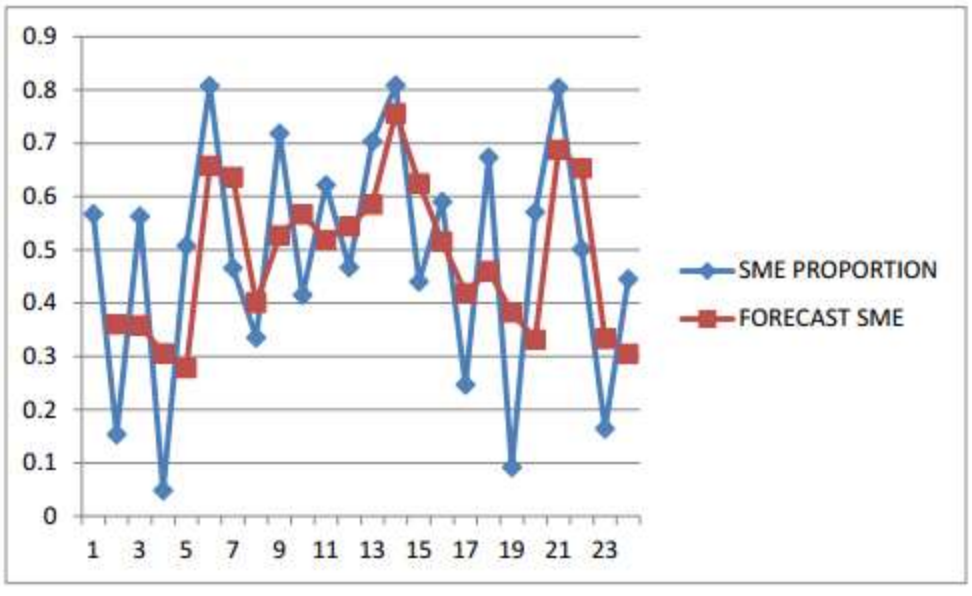

FIG.3.3 ACTUAL AND FORECAST PROPORTIONS OF SME LOANS.

The Actual loan proportions of the SME exhibits inconsistent movement until it reaches its peak in September, 2016.However the trend for the forecast proportion is quite smooth which connotes a stable trend.

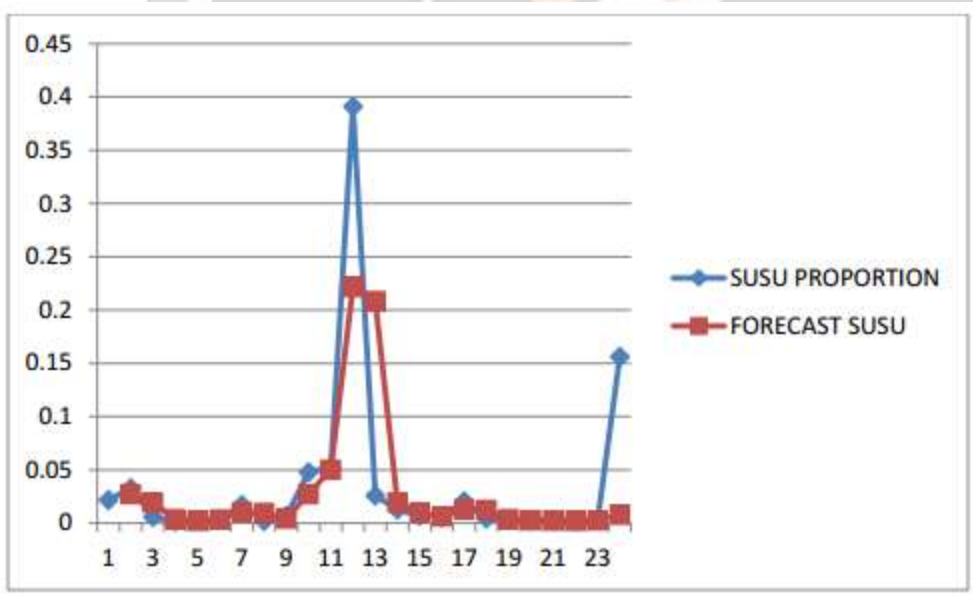

FIG 3.3 ACTUAL AND FORECAST PROPORTIONS OF SUSU LOANS.

The Actual and forecast proportions of the susu loan remained stable from January,2015 to September,2015 and it gradually rose to its peak from November to January,2015 until it finally dropped in February,2016 where it remained stable until December,2016.

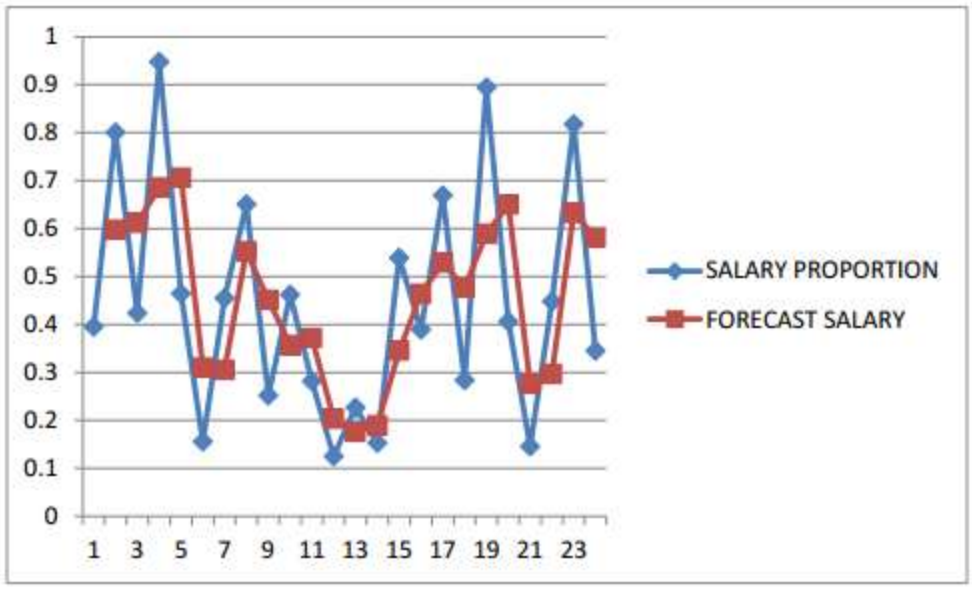

FIG.3.4 ACTUAL AND FORECAST PROPORTIONS OF SALARY LOANS.

The actual loan proportion together with its forecast proportions exhibited inconsistent movement until it reaches its peak in April 2015.The trend continues until it finally declines in December,2016.However, the trend in the forecast proportion is quite smooth which connotes a stable trend.

STATIONARITY AND HOMOGENEITY OF THE PROCESS.

In general it is observed from Table 4.1 that the Agric loan, SME Loan, Susu and Salary loans have an increasing trend. The same phenomena are also observed for the forecast Proportion. Though the trend for the actual and forecast proportion seems to be consistent, the actual proportion has a fluctuating movement.

However for the forecast proportion, the trend is quite smooth which symbolizes a stable trend. Thus one would conclude that the estimated transition matrix produces a stable trajectory which Implies stationarity.

3. RESULTS.

The transition matrix for the problem is

	AGRIC	SME	SUSU	SALARY
AGRIC	0.035300	0.541110	0.378352	0.045299
SME	0.035150	0.522400	0.391430	0.051030
SUSU	0.035710	0.528610	0.380405	0.055230
SALARY	0.034890	0.537234	0.3831940	0.044682

The transition probability matrix shows that the probability of loan switching from AGRIC to SME loan is quite high (0.541) while loan switching from any other type of loan to AGRIC is low (0.035). Loan switching to SME loan is relatively high from other loan types but relatively low from all other types to Agric loan (0.035). One important observation could be highlighted. With nonzero probability loan switching will take place from any other loan to SME loan indicating that SME loan allocation is not fully utilized. The interpretation of this probability values should be made cautiously. The probability value gives us the indication of loan switching. It may actually affect the Switching or it may not. If it affects the switching then the probability value gives the probability of switching to other loan types. If the bank receives a loan application say Agric loan, then if its allocation is still available, then there will be no loan switching. Otherwise loan switching will be made. From the transition matrix,

there is a probability of (0.035) for no switching for Agric, Probability of (0.54) for switching from Agric to SME, probability of (0.378) for switching from
Agric to Susu and probability of (0.04) for switching from Agric to Salary. Other probability values should be interpreted accordingly.

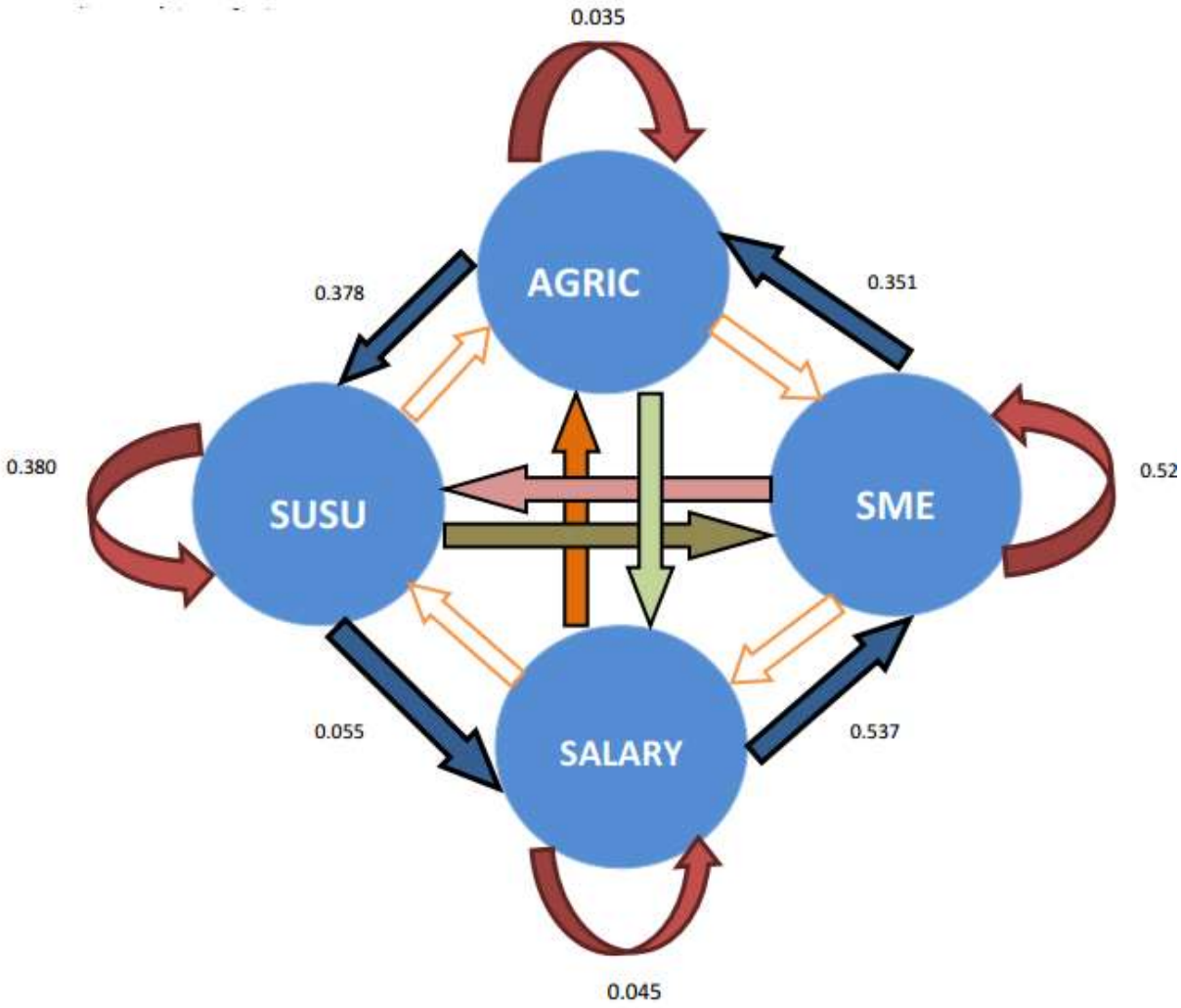

FIG 3.1 PICTORIAL REPRESENTATION OF THE TRANSITION MATRIX

STEADY STATE DISTRIBUTION

The steady state distribution indicates the long term proportion of loan disbursement. It is from this distribution that we obtain the optimal portfolio mix. The estimated steady state distribution for the process is as follows:

	AGRIC	SME	SUSU	SALARY
Π =	0.0376	0.5236	0.3817	0.0495

Thus in the long run, Agric loan should constitute 3.76% of the total loan, 52.36% of SME loan, 38.17% of Susu loan and 4.95% of salary loan.

4. DISCUSSIONS

i) The transition matrix obtained is periodic and that loan switching is possible.
ii) The estimated transition matrix produces a stable trajectory which indicates stationarity.
ii) The steady state distribution of the loan disbursement process shows that the optimal loan Portfolio mix is as follows: SME loan constitutes 52.36% of the total funds allocated for loans.
This is followed by Susu loan 38.17%, Agric loan 3.76% and salary loan 4.95%.
iii) It is observed from the transition matrix that the loan proportions reach the steady state in shorter period. This indicates that the Markov chain model is a short term forecasting model
iv) Agricultural loan should be the least among the loan types disbursed. This is due to an unexpected loss or gain as a result of seasonal changes in the production sector. However a different repayment module can be given to them to prevent them from defaulting.
vii) Financial Institutions should be able to maximize returns by investing in profitable assets.

5. CONCLUSIONS

The transition matrix obtained shows that the process is periodic and stationary. The steady state distribution of the loan disbursement process reveals that the optimal loan Portfolio mix is as follows: SME loan should constitutes 52.36 % of the total funds allocated for loans that is followed by Susu loan 38.17%, Agric loan 3.76% and Salary loan 4.95%.

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