OPTIMAL CONTROL FOR A DISTRIBUTED PARAMETER SYSTEM WITH DELAYED-TIME, NON-LINEAR

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ABSTRACT

In this paper, to verify the accuracy of the mathematical model describing the object, the physical parameters of the object such as: delayed time (τ), time constant (T) of the furnace, heat-transfer factor (α) from the furnace space to the object, heat-conducting factor (λ), temperature-conducting factor (a), as well as some simulation results. We conducted the experiments on two object samples as a flat-slab of Samot and a flat-slab of Diatomite. Experimental results show that the temperature distribution $q(x,t_f)$ at the layers in the heating object at the end of the heating process is close to the desired value of temperature q, that is satisfy the requirements of the most accurate burning problem. In addition, the experimental results reflect the correctness of the algorithms and the optimal program has been calculated.*

Keywords: *Optimal control, Distributed parameter systems, Delay, Non-linear, Padé approximation.*

1. INTRODUCTION

The paper has continued to develop in some studies as in [2,5,6,9]. Optimal control for distributed parameter system is applied in many fields such as heat treatments, composting the magnetic materials, steel rolling, etc. The optimal control problem for the "most accurate burning process" is considered in [2,5,6,9]. Namely, control object is the resistor furnace and heating which is the optimal control problem for the object with distributed, delayed, nonlinear parameters. Identification of resistor furnace, the solution of optimal control problem, calculating programs as well as simulation results. The paper only conducted experiments so as to verify the correctness of the algorithms and the optimal program has been calculated as well as previous simulation results.

2. THE PROBLEM OF OPTIMAL CONTROL

2.1. The object model

As a typical distributed parameter system, a one-dimensional heat conduction system is considered. The process of one-sided heating of object, which shaped like a retangular box in a furnace is described by the parabolic-type partial differential equation, as follows in [2], [5], [6], [9]:

$$a\frac{\partial^2 q(x,t)}{\partial x^2} = \frac{\partial q(x,t)}{\partial t} \tag{1}$$

where $q(x,t)$, the temperature distribution in the object, is the output needing to be controlled, depending on the spatial coordinate x with $0 \le x \le L$ and the time t with $0 \le t \le t_f$, a is the temperature-conducting factor (m^2/s), L is the thickness of object (m), t_f is the allowed burning time (s)

The initial and boundary conditions are given in [2], [5], [6], [9]:

$$q(x,0) = q_0(x) = \text{const} \tag{2}$$

$$\lambda \left.\frac{\partial q(x,t)}{\partial x}\right|_{x=0} = \alpha\left[q(0,t) - v(t)\right] \tag{3}$$

$$\left.\frac{\partial q(x,t)}{\partial x}\right|_{x=L} = 0 \quad (4)$$

with α as the heat-transfer coefficient between the furnace space and the object (W/m^2.^{0}C), λ as the heat-conducting coefficient of material (W/m.^{0}C), and *v(t)* as the temperature of the furnace respectively (^{0}C).
The temperature *v(t)* of the furnace is controlled by voltage *u(t)*, the temperature distribution *q(x,t)* in the object is controlled by means of the fuel flow *v(t)*, this temperature is controlled by voltage *u(t)*. Therefore, the temperature distribution *q(x,t)* will depend on voltage *u(t)*.

The relationship between the provided voltage for the furnace *u(t)* and the temperature of the furnace *v(t)* is ussually the first order inertia system with time delay as the following equation in [9].

$$T.\dot{v}(t) + v(t) = k\,.u(t-\tau) \quad (5)$$

where T is the time constant, τ is the time delay; k is the static transfer coefficient; *v(t)* is the temperature of the furnace and *u(t)* is the provided voltage for the furnace (controlled function of the system).
However, in expression (5), k is the changing coefficient depending on the temperature in the furnace, or k is a function of temperature v. So, the static transfer coefficient can be expressed by the equation: $\bar{k} = k(v)$, so $\bar{k}$ is a nonlinear coefficient. Actually, by identifying a resistor furnace [10], we saw that k varies considerably, for example in a resistor furnace with a temperature range of 0-500^{0}C. Thus, the expression (5) can be expressed by the equation:

$$T.\dot{v}(t) + v(t) = \bar{k}\,.u(t-\tau) \quad (6)$$

Furthermore, when the coefficient $\bar{k}$ is nonlinear, it is difficult to find a solution and can not to apply the Laplace transform. Therefore, the paper will perform the linearization of coefficients $\bar{k}$ into N values: $\bar{k}_1, \bar{k}_2, \bar{k}_3, ..., \bar{k}_N$. In which, we assume coefficients $\bar{k}_1, \bar{k}_2, \bar{k}_3, ..., \bar{k}_N$ are constants.

2.2. The objective function

The problem is set out as follows: we have to determine a control function *u(t)* with ($0 \le t \le t_f$) in order to minimize the temperature difference between the distribution of desired temperature $q^*(x)$ and real temperature of the object $q(x,t_f)$ at time $t = t_f$. It means at the end of the heating process to ensure temperature uniformity throughout the whole material:

$$J_c = \int_0^L \left[q^*(x) - q(x,t_f) \right]^2 dx \to \min \quad (7)$$

The constrained conditions of the control function is:

$$U_1 \le u(\text{t}) \le U_2 \quad (8)$$

U_1, U_2 are the lower and upper limit of the supply voltage respectively (V). This problem is called the most accurate burning problem.

3. EXPERIMENT SYSTEM MODEL

Conducting experiments aims to check the correctness of algorithms and optimal programs that have been calculated by simulation. In addition, the real experiment is also to check the accuracy of the mathematical model of the object as well as the physical parameters of the resistor furnace and the heating object. The experiment was carried out at Thai Nguyen University of Technology.

3.1 Introduction of experimental system model

Figure 3.1: *Experimental system model*

The experimental system model includes the following blocks:

- Resistor furnace and heating object.
- Temperature sensor, thermocouple amplifier
- Power circuit (single phase AC/AC voltage converter)
- Pulse phase control circuit, card NI USB 6008, computer.

The block diagram of the experimental system is shown in Figure 3.2..

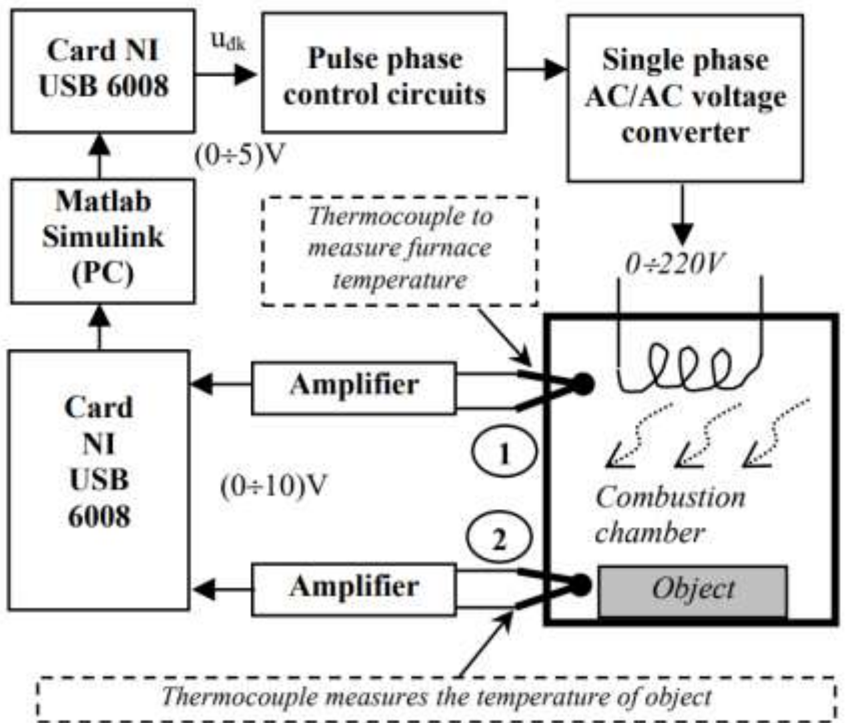

Figure 3.2. *Block diagram of the experimental system*

The control voltage signal (u_{dk}) outputting from the computer (control diagram on Matlab-simulink software) through the analog output of the NI USB-6008 Card is applied to the pulse phase control circuit. The pulse phase control circuit outputs two control pulses with a difference of 180^0 to put to a single phase AC/AC converter consisting of two thyristors connected in reverse parallel in the dynamic circuit. The output voltage of the converter changes from 0-220 (V) in proportion to the control signal (u_{dk}) that changes from 0-5(V) output from the computer to supply the resistor furnace. The set temperature q* is given in the optimal problem solving program. The furnace temperature varies from ambient temperature to about 500^0C. To measure the temperature of resistor furnace and heating object, we use 04 K-type thermocouples with almost linear characteristics. The furnace temperature through the first thermocouple **(1)**, the thermocouple **(2)** includes three thermocouples, to measure the temperature of the surface layer, the center layer and the bottom layer of the heating object, the output voltage of the thermocouples is very small value from 0-50.6(mV). Because the analog input signal AI0, AI2, AI4, AI5 of the USB-6008 Card is a normalized signal from -10V to +10V, we must use an amplifier to amplify the output voltage from the thermocouple. The amplifier has a maximum gain of 250 times .

a. Resistor furnace and heating object.

Resistor furnace with chamber furnace structure, with furnace chamber size: height is 19 cm; width is 20cm and depth is 50cm. The heating objects used for the experiment are two samples of Samot and Diatomite refractory bricks as shown in Figure 3.3.

Figure 3.3. *Two samples of Samot and Diatomite refractory bricks*

b. Temperature sensor

The furnace temperature and the temperature at the layers in the heating objects are measured by a Chromel/Alumel type K thermocouple commonly used in industry with the temperature measurement range from 0 to 500^0C, the output voltage from 0 to 50mV. Average sensitivity ΔE=0.04 mV/^{0}C. The measurement error of thermocouple in the experimental temperature range from 0 to 500^0C is ±0.16mV (about 4^0C). The temperature sensor is shown in Figure 3.4.

Figure 3.4. *Temperature sensor (type K thermocouple)*

c. Amplifiers

The output voltage of the thermocouple is very small, usually in the range of 0÷50 mV. These voltages will be amplified about 250 times to have an output voltage that varies from 0 to 10V to put into the Card NI USB 6008.

Figure 3.5. *Thermocouple amplifier circuit*

d. Card NI USB 6008

In the control system, the interface makes it possible for the computer to collect data from the feedback measurement block, store and process the received signal according to a predefined program, and send a control signal (u_{dk}) to the pulse phase control circuit for two thyristors via the Card NI USB 6008.

The Card NI USB 6008 is an analog-digital I/O card. The card reads 8 analog input channels from AI0 to AI7 (14-bit resolution, 48 kS/s), outputs 2 analog channels from A00÷A01 (12-bit, 150 S/s); 12 digital input/output channels (digital I/O); The 32-bit counter connects to the USB of a desktop or laptop computer. Use Matlab software. Card NI USB 6008 as shown in Figure 3.6

Figure 3.6. *Card NI USB 6008*

e. Pulse phase control circuit

The pulse phase control circuit (Figure 3.7) is responsible for generating two ranges of control pulses (u_{dk1}, u_{dk2}) with a difference of 180^0, synchronized with the voltage applied to the thyristors by adjusting the moment of pulse appearance (angle α) sent to the thyristors to change the effective value of the output voltage of the AC/AC converter placed on the burning wire in the resistor furnace.

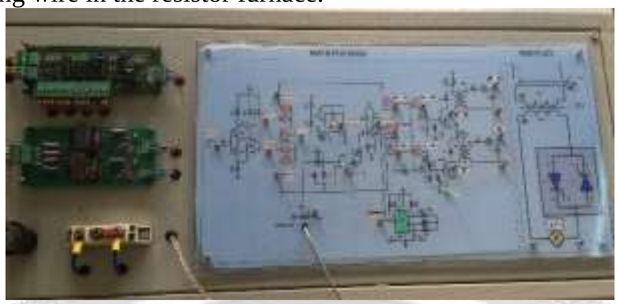

Figure 3.7. *Pulse phase control circuit*

3.2. Real experimental process

With the resistor furnace to conduct the experiment, through the re-identification of the resistor furnace as described in [10], it shows that the furnace is the first order inertia system with delayed time, with the coefficients determined: $\overline{k}_1 \approx 1.8;\ \overline{k}_2 \approx 3.3;\ \overline{k}_3 \approx 5$ are the static transfer coefficients of the resistor furnace for three temperature ranges $\Delta v_1;\ \Delta v_2;\ \Delta v_3$, respectively.

T≈1200(s) is the time constant of the furnace.

τ≈130 (s) is the delayed time of the furnace.

During the experiment, we measure the temperature in the furnace space and at three points of the heating object: the first point is the surface layer of the object ($q_m(t)$); the second point is the layer at the center of the object ($q_t(t)$); the third point is the bottom layer of the object ($q_c(t)$). All four measurement signals are passed through the amplifier to give the output voltage from 0 to 10V, and then give to the computer via the Card NI USB 6008.

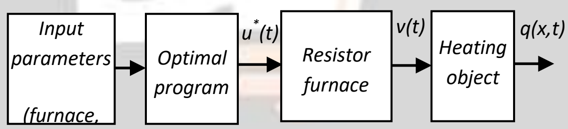

Figure 3.8. *Optimal control experimental circuit block diagram*

In the block diagram of Figure 3.8, the parameters of the furnace and the object are known, and the optimal program has been calculated from the solution of the optimal problem, which generates the optimal voltage applied to the furnace. The optimal voltage line *u*(t)* has the form of a square pulse, followed by a resistor furnace (object with delayed time, nonlinear), and finally a heating object (object with distributed parameters).

From the block diagram of Figure 3.8, we have a real experimental circuit diagram as shown in Figure 3.9

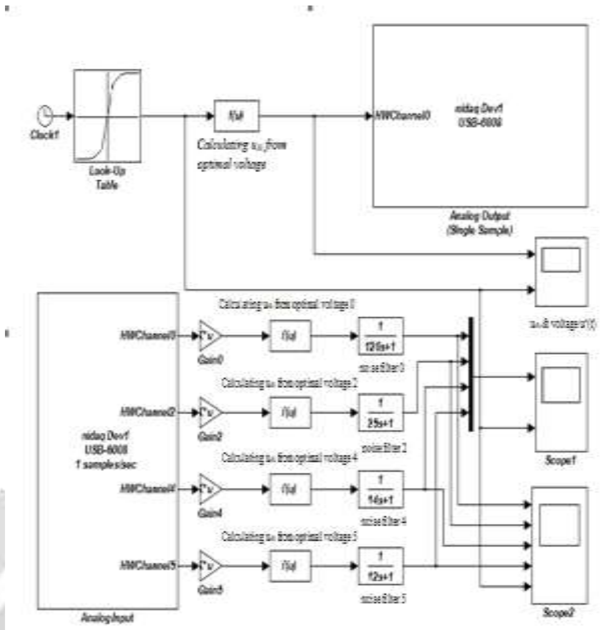

Figure 3.9. *Real experimental circuit diagram*

4. SOME EXPERIMENTAL RESULTS

4.1. Experiment with Samot sample

a. Experiment 1

First, we conduct an experiment with a Samot sample with the following parameters [9], [10], [11]:

- The physical parameters of the object
  - The thickness of object: $L = 0.03\ (m)$
  - The heat-transfer factor: $\alpha = 60\ (W/m^2.{}^0C)$
  - The heat-conducting factor: $\lambda = 0.955 (W/m.{}^0C)$
  - The temperature-conducting factor:

  $a = 4.8 * e^{-7}\ (m^2/s)$
- The parameters of the furnace
  - The time constant of the furnace: $T = 1200\ (s)$
  - The delayed time of the furnace: $\tau = 130\ (s)$
  - The desired temperature distribution: $q^* = 300^0C$
  - The period of heating time: $t_f = 4200\ (s)$
  - Limit the temperature of furnace: $v(t) \leq 500^0C$
  - Limit the temperature of flat-slab surface:

  $q(0,t) \leq 350\ {}^0C$
  - Limit under voltage: $U_1 = 125$ (V)
  - Limit upper voltage: $U_2 = 205$ (V)

With these parameters, the coefficient Bi is calculated as follows:

$$Bi = \alpha.L / \lambda = 60.0,03 / 0.955 \approx 1.88$$

Hence, the flat-slab of Samot is a thick object because the coefficient $Bi > 0.5$.

Experimental result as shown in Figure 4.1

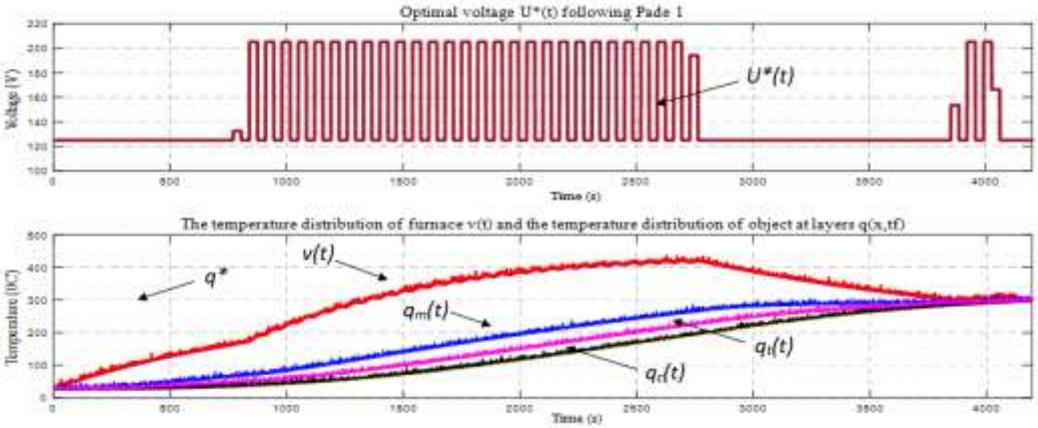

Figure 4.1. *Experimental result with Samot sample ($q^*=300^0C$ and $t_f=4200s$)*

Remark

In Figure 4.1, $U^*(t)$ is the optimal voltage applied to the resistor wire in the resistor furnace, q^* is the set temperature, $v(t)$ is the temperature in furnace space, $q_m(t)$, $q_t(t)$ and $q_c(t)$ are the surface layer temperature, center temperature and bottom layer temperature of the heating object, respectively.

Experimental result in Figure 4.1 show that, at the end of the heating process (at $t_f = 4200s$), the temperature distribution in the surface layer, the center layer and the bottom layer of the object all reaches approximately the set temperature $q^* = 300^0C$.

During the heating process, the maximum value of the furnace temperature $v(t)$ is always less than 500^0C, the maximum value of the surface temperature of the object $q(0,t) \approx 310^0C$.

Experimental result have satisfied the requirements of the most accurate burning problem as well as satisfying the set limit conditions.

b. Experiment 2

We continue to conduct the experiment with the Samot sample above with the parameters of the object and the heating time being kept the same, but here we only change the following parameters, namely: $q^* = 400\ ^0C;\ v(t) \leq 600^0C;\ q(0,t) \leq 450\ ^0C$

Because the heating time t_f = 4200s is kept the same, we increase the heating temperature from 300^0C to 400^0C, so we need to increase the supply voltage to the furnace.

We have conducted many experiments with the aim of selecting the voltage limit so that the temperature at the end of the heating process reaches the required temperature of $q^* = 400^0C$, finally choosing the voltage limit as follows:

- Limit under voltage: $U_1=140$ (V)
- Limit upper voltage: $U_2=220$ (V)

Experimental result as shown in Figure 4.2

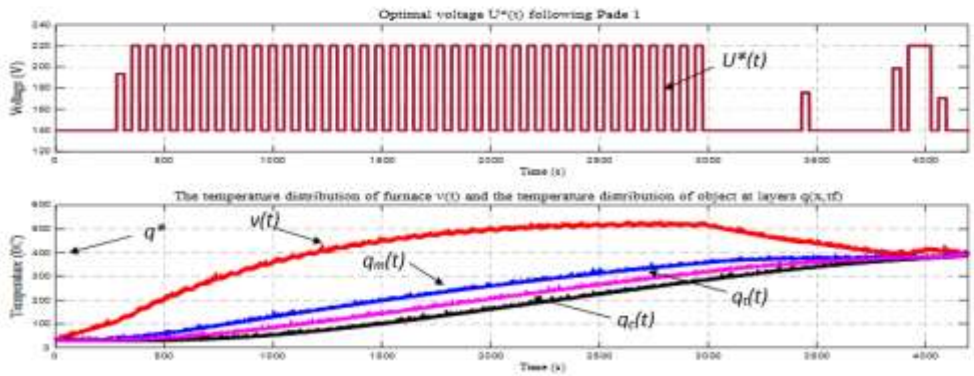

Figure 4.2. *Experimental result with Samot sample ($q^*=400^0C$ and t_f =4200s)*

Remark

In Figure 4.2, *U*(t)* is also the optimal voltage applied to the resistor wire in the resistor furnace, q^* is the set temperature, $v(t)$ is the temperature in furnace space, $q_m(t)$, $q_t(t)$ and $q_c(t)$ are the surface layer temperature, center temperature and bottom layer temperature of the heating object, respectively.

Experimental results in Figure 4.2 show that, at the end of the heating process (at $t_f = 4200s$), the temperature distribution in the surface layer, the center layer and the bottom layer of the object all reaches approximately the set temperature $q^* = 400^0C$.

In addition, during the heating process, the maximum value of the furnace temperature $v(t)$ is always less than limit value 600^0C, the maximum value of the surface temperature of the heating object $q(0,t) \approx 410^0C$.

Experimental result have also satisfied the requirements of the most accurate burning problem and satisfying the set limit conditions.

4.2. Experiment with Diatomite sample

To check the diversity of the optimal program, we continue to conduct the experiment with a Diatomite sample which has a larger thickness than the Samot sample. In this case, we also experiment with the set temperature of 300^0C and 400^0C.

a. Experiment 1

- The physical parameters of the object in [9,10,11]:
  - The thickness of object: $L= 0.04\ (m)$
  - The heat-transfer factor: $\alpha = 60\ (W/m^2.\,^0C)$
  - The heat-conducting factor: $\lambda = 0.2\ (W/m.\,^0C)$
  - The temperature-conducting factor:
  - $a=3.6*e^{-7}\ (m^2/s)$
- The desired temperature distribution: $q^* = 300\ ^0C$
- The period of heating time: $t_f = 4500\ (s)$
- Limit the temperature of furnace: $v(t) \le 500^0C$
- Limit the temperature of flat-slab surface:

$q(0,t) \le 400\ ^0C$

- Limit under voltage: $U_1=125$ (V)
- Limit upper voltage: $U_2=205$ (V)

Experimental results as shown in Figure 4.3

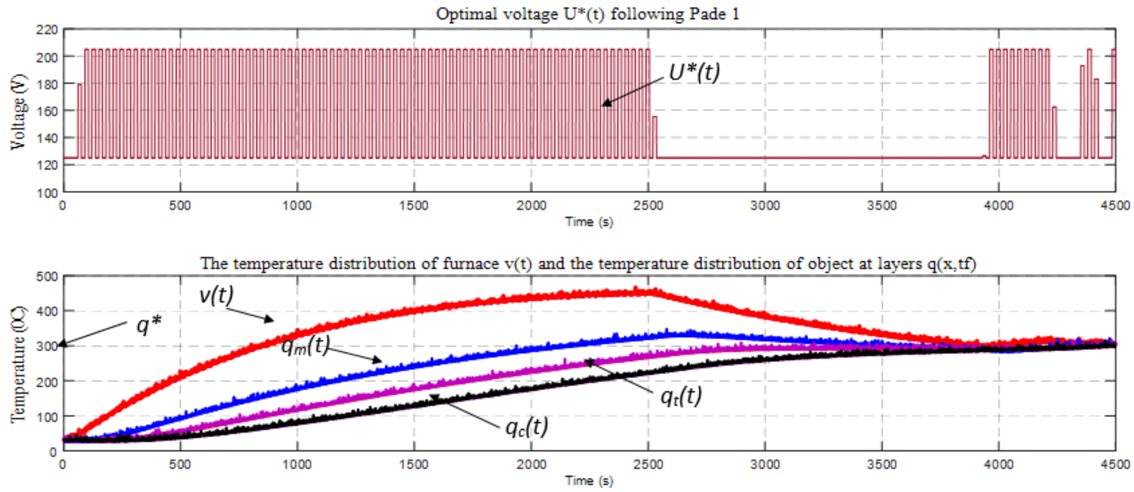

***Figure 4.3**. Experimental result with Diatomite sample ($q^*=300^0C$ and t_f =4500s)*

Remark

In Figure 4.3, *U*(t)* is the optimal voltage applied to the resistor wire in the resistor furnace, *q** is the set temperature, *v(t)* is the temperature in furnace space, $q_m(t)$, $q_t(t)$ and $q_c(t)$ are the surface layer temperature, center temperature and bottom layer temperature of the heating object, respectively. Experimental result in Figure 4.3 show that, at t_f = 4500s, the $q_m(t)$, $q_t(t)$ and $q_c(t)$ are all reaches approximately the set temperature $q^* = 300^0C$.

During the heating process, the maximum value of the furnace temperature *v(t)* is always less than 500^0C, the maximum value of the surface temperature of the object $q(0,t) \approx 330^0C$.

b. Experiment 2

We continue to conduct the experiment with the Diatomite sample with the parameters of the object and the heating time t_f = 4500 (s) being kept the same, but here we increase the heating temperature of the object to 400^0C, so it is also necessary to increase the supply voltage for the resistor furnace, in this case we keep the under limit of the voltage, only increase the upper limit, namely:

- Limit the temperature of furnace: $v(t) \leq 700^0C$
- Limit the temperature of surface: $q(0,t) \leq 500\,^0C$
- Limit under voltage: $U_1=125$ (V)
- Limit upper voltage: $U_2=220$ (V)

Experimental result as shown in Figure 4.4

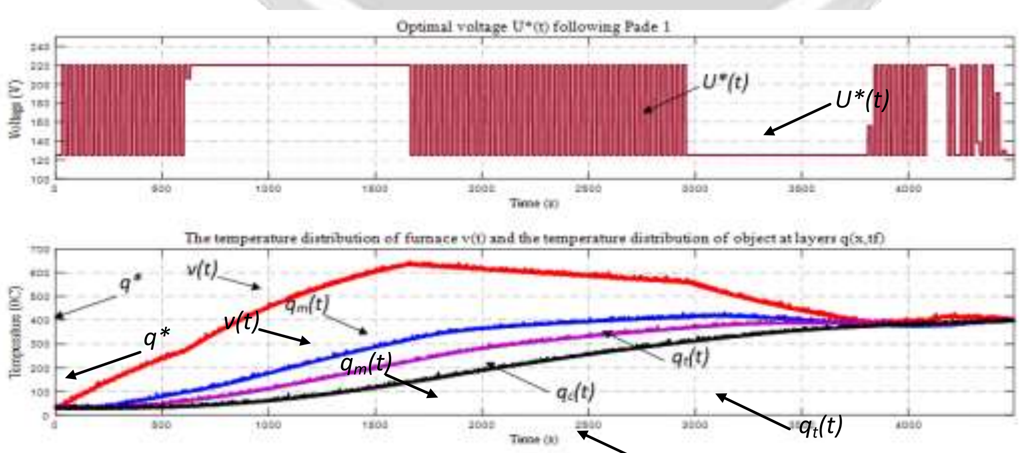

***Figure 4.4**. Experimental result with Diatomite sample ($q^*=400^0C$ and t_f =4500s)*

Remark

Experimental result in Figure 4.4 also show that at the time t_f = 4500s, the temperature distribution in the surface layer $q_m(t)$, the center layer $q_t(t)$ and the bottom layer $q_c(t)$ are approximately the set temperature is 400^0C.

During the heating process, the maximum value of the furnace temperature *v(t)* is always less than the limit temperature value of 700^0C, and the maximum value of the object surface temperature *q(0,t)* is about 420^0C. The experimental result also satisfy the requirements of the most accurate burning problem and satisfy the set limit conditions.

5. CONCLUSIONS

Through the experimental processes to verify some simulation results, we see that the control quality depends on the accuracy of the mathematical model describing the object as well as the assumption that the impact of noise is ignored unwanted entry into the system during control process. Therefore, we have conducted experiments many times to accurately determine the transfer function of the resistor furnace as well as the physical parameters of the heating object such as: delayed time (τ), time constant (T) of the furnace, heat-transfer factor (α) from the furnace space to the object, heat-conducting factor (λ), temperature-conducting factor (a). The more accurate the determination of the above parameters, the more reliable the solution of the optimal problem.

In addition, during the experiment, we calibrated and denoised the measurement from the thermocouple many times to obtain the desired temperature display characteristic curves such as the furnace temperature display curve *v(t)*, the characteristic curves show the heat distribution at layers $q_m(t)$, $q_t(t)$ and $q_c(t)$.

After conducting experiments on two heating object samples as Samot and Diatomite, we have the following conclusions:

- From the experimental results, we see that the temperature distribution $q(x,t_f)$ at the layers in the heating object at the end of the heating process is close to the desired value of temperature q^*, satisfy the requirements of the problem.
- The experimental results reflect the correctness of the algorithms and the optimal program has been calculated. In addition, the experimental results also proved the accuracy and stability of the optimal solution.

6. ACKNOWLEDGEMENT

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