One Numerical method to solve the Time Fractional BBM-Burger

Rasolomampiandry Gilbert –Randimbindrainibe Falimanana

Ecole Supérieure Polytechnique Antananarivo (ESPA) - Université d'Antananarivo
BP 1500, Ankatso – Antananarivo 101 – Madagascar

Abstract.

In **[1]** the residual power series method (RPSM), is used for finding the series solution of the time fractional Benjamin-Bona-Mahony-Burger (BBM-Burger) equation and
the numerical solution of the BBM-Burger equation is calculated by Maple. In this paper, we use MATLAB 2013 to find the numerical solution of this equation

Keys words :
Caputo fractional derivative ;Burger equation ;Power Series

1Introduction

Today, fractional differential equations are more and more important in many fields, such as mathematics and dynamic systems [2,3]The persons who firstly proposed fractional differential equations were Leibniz and L'Hopital in 1695.Lakshmikantham and Vatsala [4] discussed the basic theory for the initial value problem involving Riemann-Liouville differential operators by fractional differential equations. Diethelm and Ford [5] proposed the analytical questions of existence and uniqueness of solutions by fractional differential equations. And many other academics studied different theories in fractional differential equations
The BBM Burger equation can be written in time fractional operator form as [6]

$$D_t^{\alpha} u - \frac{\partial^3 u}{\partial x^2 \partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) = 0, \quad t > 0, x \in I \subset \mathbb{R}, \alpha \in]0,1] \quad (1)$$

where α is a parameter, which is the order of the time fractional derivative and is located in the range of (0,1]. The initial condition is

$$u(x,0) = \sec h^2\left(\frac{x}{4}\right)$$

If $\alpha = 1$, the exact solution [7] is

$$u(x,t) = \sec h^2\left(\frac{x}{4} - \frac{t}{4}\right)$$

Using residual power series method (RPSM) [8-24]**, [1]** gives the analytic solution of (1). By exploiting this result, we will give a numerical solution of (1)calculated with MATLAB

The rest of the paper is as follows. In Section 2, some basic definitions and theorems about the Caputo and modified residual power series method mentionned in [1] are introduced. In Section 3, We will recall the results obtained by [1] in the resolution of the time fractional BBM- Burger equation with residual power series method. Numerical results in Section 4. At last, the conclusion was drawn in Section 5.

2 Basic definitions and theorems

Definition 1 (see [1],[25])

Let $f(t) : [0, \infty[\to \mathbb{R}$ be a function and $n \in \mathbb{N}^*, \alpha > 0$ .The Caputo fractional derivative is defined by

$$D^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^x (t-\tau)^{n-\alpha-1} \frac{d^n f(\tau)}{d\tau^n} d\tau, n-1 < \alpha < n$$

$$D^{\alpha} f(t)=\frac{d^{n} f(x)}{dx^{n}}, \alpha=n \in \square^{*}$$

Theorem1 (see [1], [25])

The Caputo fractional derivative of thr power function satisfies

$$D^{\alpha} x^{q}=\frac{\Gamma(q+1)}{\Gamma(q+1-\alpha} x^{q-\alpha}, \alpha \leq q$$

$$D^{\alpha} x^{q}=0, \alpha>q$$

Definition 2 (see [1],[8],[9])

A power series expansion of the form $\sum_{m=0}^{\infty} c_m (t-t_0)^{m\alpha}$ for $0 \leq n-1<\alpha \leq n$ and $t \geq t_0$, is called fractional power series about $t=t_0$, where t is a variable and c_m are constants called the coefficients of the series

Theorem 2 (see [1], [8])

Suppose that f has a fractional power series representation at $t=t_0$ of the form

$$f(t)=\sum_{n=0}^{\infty} c_m (t-t_0)^{m\alpha}, 0 \leq n-1<\alpha \leq n, t_0 \leq t<t_0+R$$

If $D^{m\alpha} f(t) \in (t_0, t_0+R), m=0,1,2,.....,$ then the coefficients c_m are given by the formula

$c_m=\frac{D^{m\alpha} f(t_0)}{\Gamma(m\alpha+1)}, m=0,1,2,.....,$ where $D^{m\alpha}=D^{\alpha}.D^{\alpha}....D^{\alpha}$ (m-times) and R is the radius of convergence

Definition 3 (see [1], [9])

A power series of the form

, for $0 \leq n-1<\alpha \leq n$ and $t \geq t_0$, is called multiple power seies about $t=t_0$ where t is a variable and f_m are fonctions of χ called the coefficient of series.

Theorem 3(see,[8],[9])

Suppose that $u(t,x)$ has a multiple power series representation at $t=t_0$ of the form

$$u(t,x)=\sum_{m=0}^{\infty} f_m(x)(t-t_0)^{m\alpha}, 0 \leq n-1<\alpha \leq n, x \in I, t_0 \leq t<t_0+R$$

If $D_t^{m\alpha} u(x,t)$ are continous on $I \times (t_0, t_0+R), m=0,1,2,.....,$ then the coefficients $f_m(x)=\frac{D_t^{m\alpha} u(t,x)}{\Gamma(m\alpha+1)}, m=0,1,2,....$ when $D_t^{m\alpha}=\frac{\partial^{m\alpha}}{\partial t^{m\alpha}}=\frac{\partial^{\alpha}}{\partial t^{\alpha}}.\frac{\partial^{\alpha}}{\partial t^{\alpha}}.....\frac{\partial^{\alpha}}{\partial t^{\alpha}}(m-times)$ and

$R=\min_{C \in I} R_C$, R_C is the radius of convregence of fractional power series $\sum_{m=0}^{\infty} f_m(c)(t-t_0)^{m\alpha}$

If $t_0=0$ then $u(t,x)=\sum_{m=0}^{\infty} f_m(x) \frac{t^{m\alpha}}{\Gamma(m\alpha+1)}$. The truncatured series of $u_i(t,x)$ is defined by

$$u_i(t,x)=\sum_{m=0}^{i} f_m(x) \frac{t^{m\alpha}}{\Gamma(m\alpha+1)}$$

If $t=0, u(x,0)=f_0(x)$. We define the i th residual fonction as follows

$$\operatorname{Re}s_i(x,t) = D_t^{\alpha} u_i - u_{i,xxt} + u_{i,x} + \left(\frac{u_i^2}{2}\right)_x$$

In order to get $f_n(x), n \in \mathbb{N}^*$, we look for the solution of $D_t^{(n-1)\alpha} \operatorname{Re}s_n(x,0) = 0$

3 Solution of the Time Fractional BBM-Burger Equation by Residual Power Series Method

In **[1]** the initial condition the Time Fractional BBM-Burger Equation is $u(x,0) = \sec h^2(\frac{x}{2})$

And the exact solution is $u(x,t) = \sec h^2(\frac{x}{4} - \frac{t}{4})$ .The solution the Time Fractional BBM-Burger Equation is $u(t,x) = \sum_{m=0}^{\infty} f_m(x) \frac{t^{m\alpha}}{\Gamma(m\alpha+1)}$ The truncatured series of $u_i(t,x)$ is defined by $u_i(t,x) = \sum_{m=0}^{i} f_m(x) \frac{t^{m\alpha}}{\Gamma(m\alpha+1)}$ If i=4 $u_4(t,x) = \sum_{m=0}^{4} f_m(x) \frac{t^{m\alpha}}{\Gamma(m\alpha+1)}$

With $f_0(x) = u(x,0) = \sec h^2(\frac{x}{4})$; $f_1(x) = \frac{1}{2}\sec h^2(\frac{x}{4})\tanh(\frac{x}{4}) + \frac{1}{2}\sec h^4(\frac{x}{4})\tanh(\frac{x}{4})$

$$f_2(x) = \frac{7}{8}\sec h^6(\frac{x}{4})\tanh^2(\frac{x}{4}) - \frac{1}{8}\sec h^6(\frac{x}{4}) + \frac{5}{4}\sec h^4(\frac{x}{4})\tanh^2(\frac{x}{4}) - \frac{1}{4}\sec h^4(\frac{x}{4}) +$$
$$\frac{3}{8}\sec h^2(\frac{x}{4})\tanh^2(\frac{x}{4}) - \frac{1}{8}\sec h^2(\frac{x}{4})$$

$$f_3(x) = \frac{35}{16}\sec h^8(\frac{x}{4})\tanh^3(\frac{x}{4}) - \frac{11}{16}\sec h^8(\frac{x}{4})\tanh(\frac{x}{4}) + \frac{17}{4}\sec h^8(\frac{x}{4})\tanh^3(\frac{x}{4}) -$$
$$\frac{19}{16}\sec h^8(\frac{x}{4})\tanh(\frac{x}{4}) + \frac{3}{8}\sec h^2(\frac{x}{4})\tanh^3(\frac{x}{4}) - \frac{1}{4}\sec h^2(\frac{x}{4})\tanh(\frac{x}{4})$$

$$f_4(x) = \frac{385}{64}\sec h^8(\frac{x}{4})\tanh^4(\frac{x}{4}) - \frac{35}{16}\sec h(\frac{x}{4})(\frac{-1}{2}\sec h^2(\frac{x}{4})\tanh(\frac{x}{4}))^8\tanh(\frac{x}{4})$$
$$\times(\frac{-1}{2}\sec h^2(\frac{x}{4})\tanh(\frac{x}{4}))^3 - \frac{51}{16}\sec h^8(\frac{x}{4})\tanh^2(\frac{x}{4}) + \frac{153}{16}\sec h^6(\frac{x}{4})\tanh^4(\frac{x}{4}) +$$
$$\frac{11}{16}\sec h(\frac{x}{4})(\frac{-1}{2}\sec h^2(\frac{x}{4})\tanh(\frac{x}{4}))^8\tanh(\frac{x}{4}) - \frac{17}{4}\sec h(\frac{x}{4})(\frac{-1}{2}\sec h^2(\frac{x}{4})\tanh(\frac{x}{4}))^9$$
$$\times\tanh(\frac{x}{4}) + \frac{11}{64}\sec h^8(\frac{x}{4}) - \frac{193}{32}\sec h^6(\frac{x}{4})\tanh^2(\frac{x}{4}) + \frac{273}{64}\sec h^4(\frac{x}{4})\tanh^4(\frac{x}{4}) +$$
$$\frac{13}{8}\sec h(\frac{x}{4})(-\frac{1}{2}\sec h^2(\frac{x}{4}))^6\tanh(\frac{x}{4})(\frac{-1}{2}\sec h^2(\frac{x}{4})\tanh(\frac{x}{4})) - \frac{39}{16}\sec h(\frac{x}{4})$$
$$\times\tanh(\frac{x}{4})(\frac{-1}{2}\sec h^2(\frac{x}{4})\tanh(\frac{x}{4}))^7 + \frac{13}{32}\sec h^6(\frac{x}{4}) - \frac{53}{16}\sec h^4(\frac{x}{4})\tanh^2(\frac{x}{4}) +$$

$$\frac{15}{32}\operatorname{sech}^2(\frac{x}{4})\tanh^4(\frac{x}{4})+\frac{19}{16}\operatorname{sech}(\frac{x}{4})(-\frac{1}{2}\operatorname{sech}^2(\frac{x}{4})\tanh(\frac{x}{4}))^5\tanh(\frac{x}{4})-\frac{3}{8}\operatorname{sech}(\frac{x}{4})$$
$$\times(-\frac{1}{2}\operatorname{sech}^2(\frac{x}{4})\tanh(\frac{x}{4}))^5\tanh(\frac{x}{4})+\frac{19}{64}\operatorname{sech}^4(\frac{x}{4})-\frac{15}{32}\operatorname{sech}^2(\frac{x}{4})\tanh^2(\frac{x}{4})+\frac{1}{4}\operatorname{sech}(\frac{x}{4})$$
$$\times(-\frac{1}{2}\operatorname{sech}^2(\frac{x}{4})\tanh(\frac{x}{4}))^3\tanh(\frac{x}{4})-\operatorname{sech}(\frac{x}{4})-\frac{35}{8}\operatorname{sech}^8(\frac{x}{4})\tanh^4(\frac{x}{4})+\frac{105}{16}\operatorname{sech}^8(\frac{x}{4})\tanh^2(\frac{x}{4})$$
$$\times(\frac{1}{4}-\frac{1}{4}\tanh^2(\frac{x}{4}))+\frac{11}{8}\operatorname{sech}^8(\frac{x}{4})\tanh^2(\frac{x}{4})-\frac{11}{16}\operatorname{sech}^8(\frac{x}{4})(\frac{1}{4}-\frac{1}{4}\tanh^2(\frac{x}{4}))-\frac{51}{8}\operatorname{sech}^6(\frac{x}{4})\tanh^4(\frac{x}{4})$$
$$+\frac{51}{4}\operatorname{sech}^6(\frac{x}{4})\tanh^2(\frac{x}{4})(\frac{1}{4}-\frac{1}{4}\tanh^2(\frac{x}{4}))+\frac{39}{16}\operatorname{sech}^6(\frac{x}{4})\tanh^2(\frac{x}{4})-\frac{13}{8}\operatorname{sech}^6(\frac{x}{4})(\frac{1}{4}-\frac{1}{4}\tanh^2(\frac{x}{4}))$$
$$\times\frac{39}{16}\operatorname{sech}^4(\frac{x}{4})\tanh^4(\frac{x}{4})+\frac{117}{16}\operatorname{sech}^4(\frac{x}{4})\tanh^2(\frac{x}{4})(\frac{1}{4}-\frac{1}{4}\tanh^2(\frac{x}{4}))+\frac{19}{16}\operatorname{sech}^4(\frac{x}{4})\tanh^2(\frac{x}{4})-\frac{19}{16}\operatorname{sech}^4(\frac{x}{4})$$
$$\times(\frac{1}{4}-\frac{1}{4}\tanh^2(\frac{x}{4}))-\frac{3}{16}\operatorname{sech}^2(\frac{x}{4})\tanh^4(\frac{x}{4})+\frac{9}{8}\operatorname{sech}^2(\frac{x}{4})\tanh^2(\frac{x}{4})(\frac{1}{4}-\frac{1}{4}\tanh^2(\frac{x}{4}))+\frac{1}{8}\operatorname{sech}^2(\frac{x}{4})\tanh^2(\frac{x}{4})$$
$$-\frac{1}{4}\operatorname{sech}^2(\frac{x}{4})(\frac{1}{4}-\frac{1}{4}\tanh^2(\frac{x}{4}))^2+\frac{1}{16}\operatorname{sech}^2(\frac{x}{4})$$

4 Numerical results

4.1 Programming

The programming under MATLAB of results of the dicretizations above is :

```
alpha=input('simulation time alpha=')
[x,t] = meshgrid(-4:.1:4, 0.1:.2:0.4);
exact=(sech(x/4-t/4)).^2
f0=(sech(x/4)).^2
f1=1/2.*(sech(x/4)).^2.*tanh(x/4)+1/2.*(sech(x/4)).^4.*tanh(x/4)
f2=7/8.*(sech(x/4)).^6.*(tanh(x/4)).^2-1/8.*(sech(x/4)).^6
+5/4.*(sech(x/4)).^4.*(tanh(x/4)).^2-1/4.*(sech(x/4)).^4
+3/8.*(sech(x/4)).^2.*(tanh(x/4)).^2-1/8.*(sech(x/4)).^2
f3=35/16.*(sech(x/4)).^8.*(tanh(x/4)).^3-11/6.*(sech(x/4)).^8.*tanh(x/4)
+17/4.*(sech(x/4)).^6.*(tanh(x/4)).^3-13/8.*((sech(x/4))).^6.*tanh(x/4)
+39/16.*(sech(x/4)).^4.*(tanh(x/4)).^3-19/16.*(sech(x/4)).^4.*tanh(x/4)
+3/8.*(sech(x/4)).^2.*((tanh(x/4)).^3)-1/4.*(sech(x/4)).^2.*tanh(x/4)
f4=385/64.*(sech(x/4)).^8.*(tanh(x/4)).^4-35/16.*sech(x/4).*tanh(x/4)-51/16
.*(sech(x/4)).^8.*(tanh(x/4)).^2+153/16.*(sech(x/4)).^6.*(tanh(x/4)).^4
+11/16.*sech(x/4).*(-1/2.*(sech(x/4)).^2.*tanh(x/4)).^8.*tanh(x/4)
.*(-1/2.*(sech(x/4)).^2.*tanh(x/4))-17/4.*sech(x/4).*(-1/2.*(sech(x/4)).^2
.*tanh(x/4)).^6.*tanh(x/4).*(-1/2.*(sech(x/4)).^2.*tanh(x/4)).^3+11/64
.*(sech(x/4)).^8-193/32.*(sech(x/4)).^6.*(tanh(x/4)).^2+273/64
.*(sech(x/4)).^4.*(tanh(x/4)).^4+13/8.*sech(x/4).*(-1/2*(sech(x/4)).^2).^6
.*tanh(x/4).*(-1/2.*(sech(x/4)).^2.*tanh(x/4))-39/16.*sech(x/4)
.*(-1/2.*(sech(x/4)).^2.*tanh(x/4)).^7.*tanh(x/4)+13/32.*(sech(x/4)).^6
-53/16.*(sech(x/4)).^4.*(tanh(x/4)).^2+15/32.*(sech(x/4)).^2
.*(tanh(x/4)).^4+19/16.*sech(x/4).*(-1/2.*(sech(x/4)).^2.*tanh(x/4)).^5
.*tanh(x/4)-3/8.*sech(x/4).*(-1/2.*(sech(x/4)).^2.*tanh(x/4)).^5.*tanh(x/4)
+19/64.*(sech(x/4)).^4-15/32.*(sech(x/4)).^2.*(tanh(x/4)).^2+1/4.*sech(x/4)
.*tanh(x/4).*(-1/2.*(sech(x/4)).^2.*tanh(x/4)).^3-sech(x/4)-35/8
.*(sech(x/4)).^8.*(tanh(x/4)).^4+105/16.*(sech(x/4)).^8.*(tanh(x/4)).^2
.*(1/4-1/4.*(tanh(x/4)).^2)+11/8.*(sech(x/4)).^8.*(tanh(x/4)).^4-11/16
.*(sech(x/4)).^.8.*(1/4-1/4.*(tanh(x/4)).^2)-51/.8.*(sech(x/4)).^.6
.*(tanh(x/4)).^4+51/4.*(sech(x/4)).^6.*(tanh(x/4)).^2
.*(1/4-1/4.*(tanh(x/4)).^2)+39/16.*(sech(x/4)).^6.*(tanh(x/4)).^2 -13/8
```

```
.*(sech(x/4)).^6.*(1/4-1/4.*(tanh(x/4)).^2)-39/16.*(sech(x/4)).^4
.*(tanh(x/4)).^4+117/16.*(sech(x/4)).^4.*(tanh(x/4)).^2
.*(1/4-1/4.*(tanh(x/4)).^2)+19/16.*(sech(x/4)).^4.*(tanh(x/4)).^2-19/16
.*(sech(x/4)).^4.*(1/4-1/4.*(tanh(x/4)).^2)-3/16.*(sech(x/4)).^2
.*(tanh(x/4)).^4+9/8.*(sech(x/4)).^2.*(tanh(x/4)).^2
.*(1/4-1/4.*(tanh(x/4)).^2)+1/8.*(sech(x/4)).^2.*(tanh(x/4)).^2-1/4
.*(sech(x/4)).^2.*(1/4-1/4.*(tanh(x/4)).^2).^2+1/16.*(sech(x/4)).^2
u4=f0+f1.*((t.^alpha)/gamma(1+alpha))+f2.*((t.^(2.*alpha))/gamma(1+2.*alpha
))+f3.*((t.^(3.*alpha))/gamma(1+3.*alpha))+f4.*((t.^(4.*alpha))/gamma(1+4.*
alpha))
Z1=uexact
 mesh(t,x,Z1)
   Z2=u4
 mesh(t,x,Z2)
```

4.2 Graphics representation

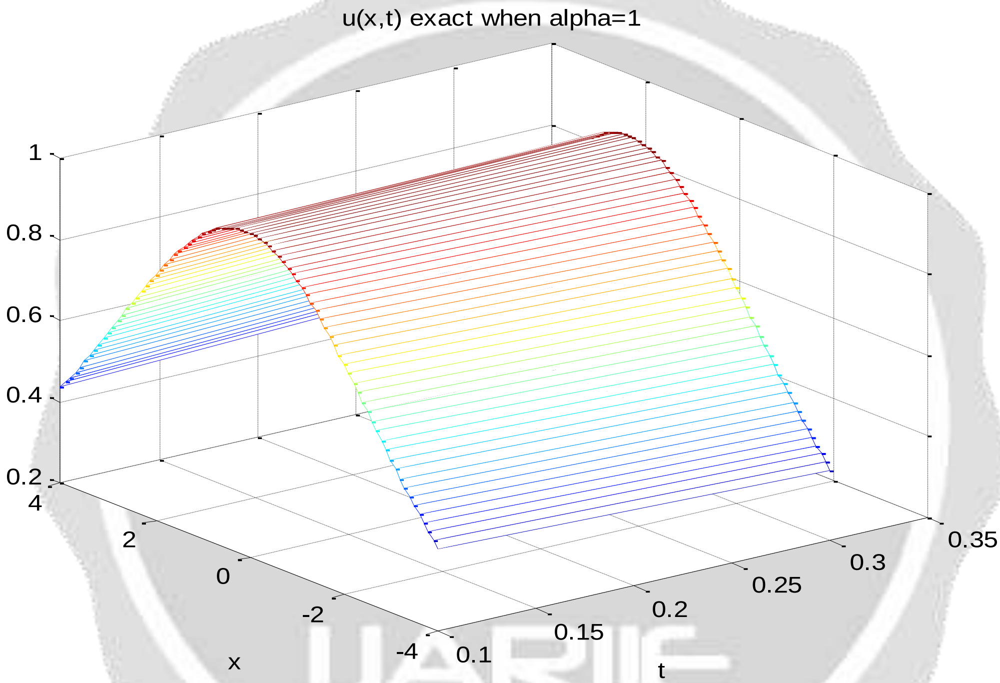

Figure 1 : 3D graphic of the exact solution u(x,t) when α=1

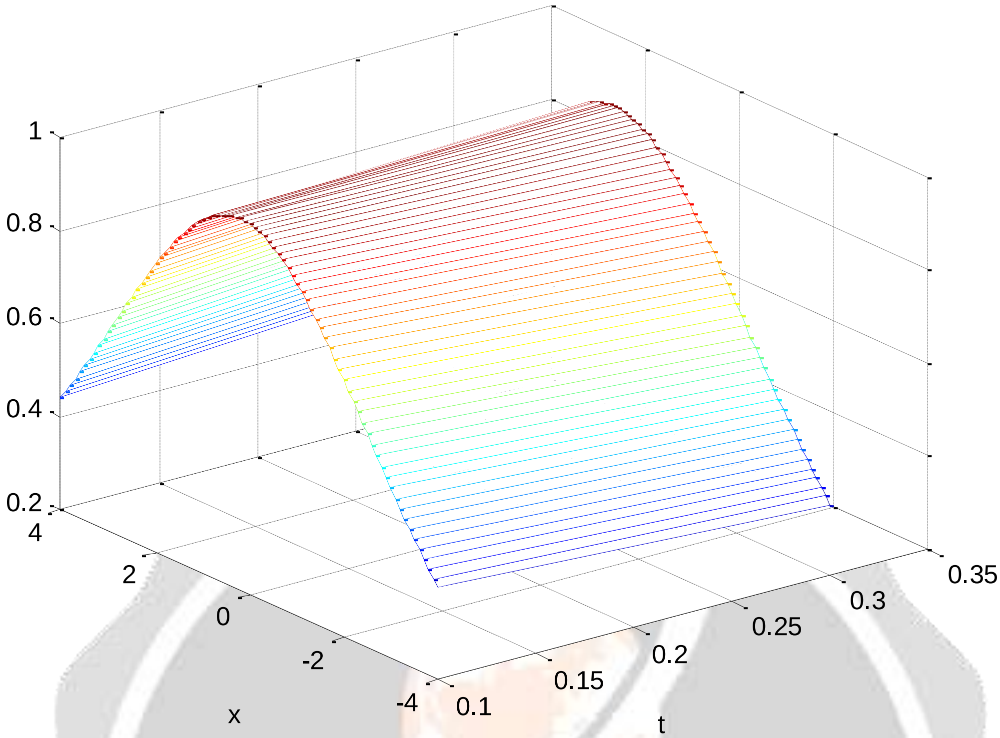

Figure 2 : 3D graphic of approximate solution $u_4(x,t)$ when α=1

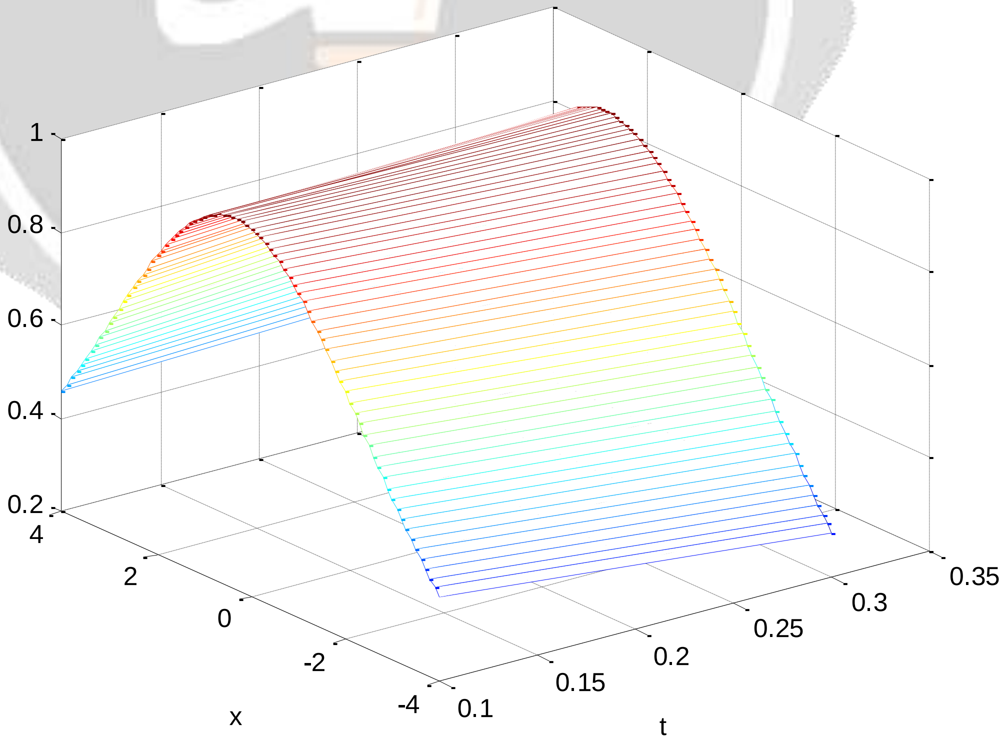

Figure 3 : 3D graphic of approximate solution $u_4(x,t)$ when α=0.8

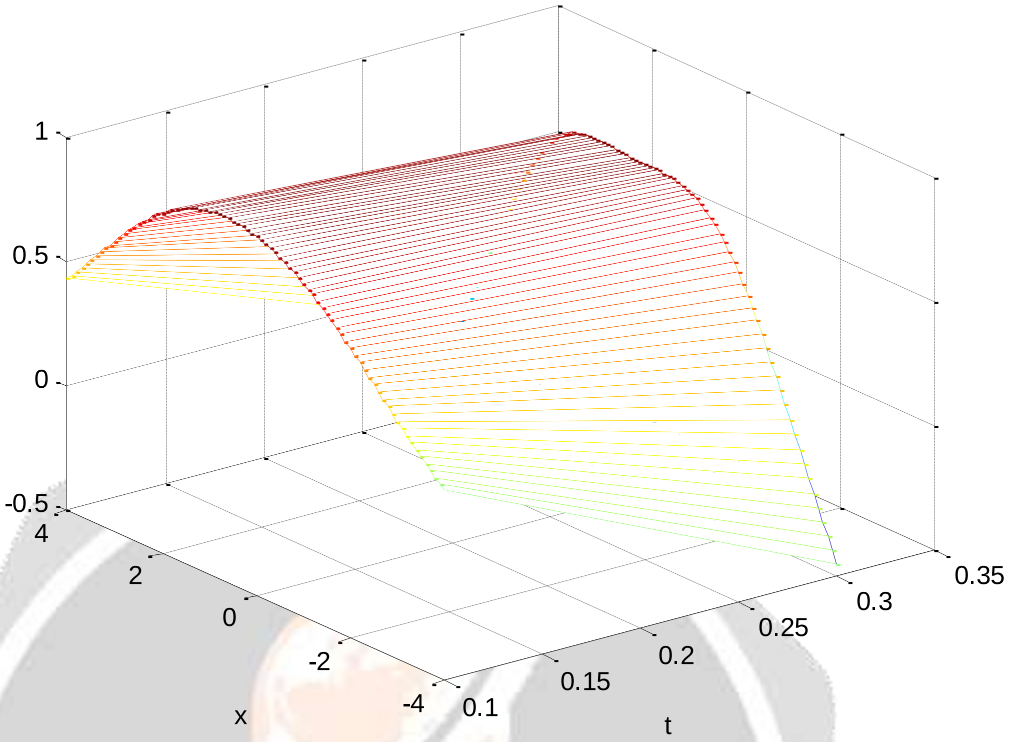

Figure 4 : 3D graphic of approximate solution $u_4(x,t)$ when α=0.5

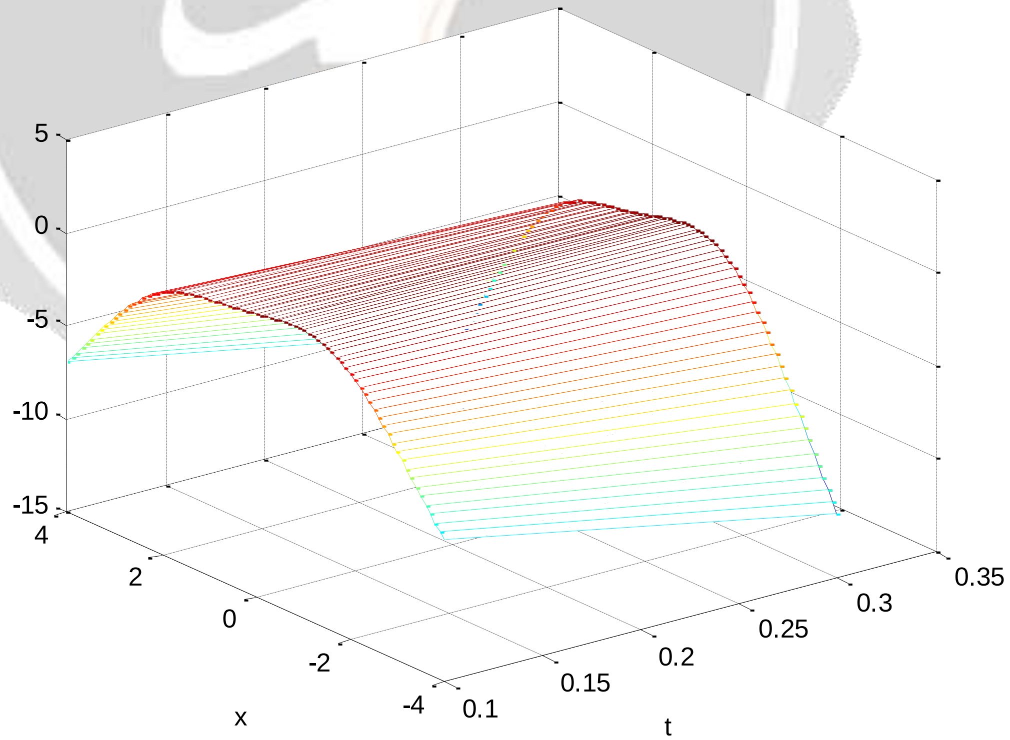

Figure 5 : 3D graphic of approximate solution $u_4(x,t)$ when α=0.1

4.3 Comments of the results.

We can compare the exact solution of the BBM-Burger equation with the analytical approximate solution by graphics . Figure 1 presents the exact solution, Figure 2 shows the approximate solution when $\alpha = 1$. Figure 3, when $\alpha = 0.8$. Figure 4, when $\alpha = 0.5$.Figure 5 when $\alpha = 0.5$. We see that the approximate solution is close to the exact solution, when α approaches 1. So, we can conclude that, as parameter α increases, the graphics get closer and closer to the exact solution of the graphic.

5. Conclusion

In this paper, The time fractional BBM-Burger equation is calculated by MATLAB 2013. We can conculde that we have the similar results with the results in [1]

BILBLIOGRAPHY

[1] Jianke Zhang[1] ;Zhirou Wei[2] ;Longquan Yong[3] ;Yuelei Xiao[4] . Analytical Solution for the Time Fractional BBM-Burger Equation by Using Modified Residual Power Series Method
[1]School of Science,Xi'an University of Posts Telecommunications,Xi'an 710121,China
[2]Shaanxi Key Laboratory of Network Data Analysis and Intelligent Processing, Xi'an University of Posts and Telecommunications,Xi'an, Shaanxi 710121, China
[3]School of Mathematics and Computer Science, Shaanxi University of Technology, Hanzhong 723000, China
[4]Institute of IOT and IT-based Industrialization, Xi'an University of Posts and Telecommunications, Xi'an 710061, China
October 2018

[2] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, "Theory and applications of fractional differential equations," North-Holland Mathematics Studies, vol. 204, no. 49–52, pp. 2453–2461, 2006.

[3] A. Arikoglu and I. Ozkol, "Solution of fractional differential equations by using differential transform method," Chaos Solitons and Fractals, vol. 34, no. 5, pp. 1473–1481, 2007.

[4] V. Lakshmikantham and A. S. Vatsala, "Basic theory of fractional differential equations," Nonlinear Analysis Theory
Methods and Applications, vol. 69, no. 8, pp. 2677–2682, 2008.

[5] K. Diethelm and N. J. Ford, "Multi-order fractional differential equations and their numerical solution," Applied Mathematics and Computation, vol. 154, no. 3, pp. 621–640, 2004.

[6] S. Kumar and D. Kumar, "Fractional modelling for BBM Burger equation by using new homotopy analysis transform
method," Journal of the Association of Arab Universities for Basic and Applied Sciences, vol. 16, no. 1, pp. 16–20, 2014.

[7] A. Fakhari, G. Domairry, and Ebrahimpour, "Approximate explicit solutions of nonlinear BBM B equations by homotopy
analysis method and comparison with the exact solution,"Physics Letters A, vol. 368, no. 1-2, pp. 64–68, 2007.

[8] A. El-Ajou, O. A. Arqub, Z. A. Zhour, and S. Momani, "New results on fractional power series: theories and applications,"
Entropy, vol. 15, no. 12, pp. 5305–5323, 2013.

[9] A. El-Ajou, O. A. Arqub, and S. Momani, "Approximate analytical solution of the nonlinear fractional KdV–Burgers equation: a new iterative algorithm," Journal of Computational Physics, vol. 293, pp. 81–95, 2014.

[10] K. Moaddy, M. AL-Smadi, and I. Hashim, "A novel representation of the exact solution for differential algebraic equations system using residual power-series method," Discrete Dynamics in Nature and Society, vol. 2015, Article ID
205207, 12 pages, 2015.

[11] H. M. Jaradat, S. Al-Shar, Q. J. A. Khan, M. Alquran, and K. Al-Khaled, "Analytical solution of time-fractional Drinfeld-Sokolov-Wilson system using residual power series method," IAENG International Journal of Applied Mathematics,vol. 46, no. 1, pp. 64–70, 2016.

[12] L. Wang and X. Chen, "Approximate analytical solutions of time fractional Whitham-Broer-Kaup equations by a residual power series method," Entropy, vol. 17, no. 12, pp. 6519– 6533, 2015.

[13] F. Xu, Y. Gao, X. Yang, and H. Zhang, "Construction of fractional power series solutions to fractional Boussinesq equations using residual power series method," Mathematical Problems in Engineering, vol. 2016, Article ID 5492535, 15 pages, 2016.

[14] A. Kumar, S. Kumar, and S. P. Yan, "Residual power series method for fractional diffusion equations," Fundamenta Informaticae,vol. 151, no. 1–4, pp. 213–230, 2017.

[15] H. Tariq and G. Akram, “Residual power series method for solving time-space-fractional Benney-Lin equation arising in falling film problems,” Journal of Applied Mathematics and Computing, vol. 55, no. 1-2, pp. 683–708, 2017.
[16] O. A. Arqub and H. Rashaideh, “Solution of Lane-Emden equation by residual power series method,” in ICIT 2013 The 6th International Conference on Information Technology, Amman, Jordan, April 2013.
[17] M. I. Syam, “Analytical solution of the fractional Fredholm integrodifferential equation using the fractional residual power series method,” Complexity, vol. 2017, Article ID 4573589, 6 pages, 2017.
[18] W. Li and Y. Pang, “Asymptotic solutions of time-space fractional coupled systems by residual power series method,”Discrete Dynamics in Nature and Society, vol. 2017, Article ID 7695924, 10 pages, 2017.
[19] B. A. Mahmood and M. A. Yousif, “A novel analytical solution for the modified Kawahara equation using the residual power series method,” Nonlinear Dynamics, vol. 89, no. 2, pp. 1233–1238, 2017.
[20] M. Alquran and I. Jaradat, “A novel scheme for solving Caputo time-fractional nonlinear equations: theory and application,” Nonlinear Dynamics, vol. 91, no. 4, pp. 2389–2395, 2018.
[21] M. Alquran, H. M. Jaradat, and M. I. Syam, “Analytical solution of the time-fractional Phi-4 equation by using modified residual power series method,” Nonlinear Dynamics, vol. 90, no. 4, pp. 2525–2529, 2017.
[22] M. Alquran, K. Al-Khaled, and J. Chattopadhyay, “Analytical solutions of fractional population diffusion model: residual power series,” Mathematical Sciences, vol. 8, no. 4, pp. 153– 160, 2015.
[23] M. Alquran, K. Al-Khaled, S. Sivasundaram, and H. M.Jaradat, “Mathematical and numerical study of existence of bifurcations of the generalized fractional Burgers-Huxley equation,” Nonlinear Studies, vol. 24, no. 1, pp. 235–244, 2017.
[24] M. Alquran, “Analytical solution of time-fractional twocomponent evolutionary system of order 2 by residual power series method,” Journal of Applied Analysis and Computation, vol. 5, no. 4, pp. 589–599, 2015.
[25] I. Podlubny, “Fractional differential equations: an introduction to fractional derivatives, fractional differential quations, to methods of their solution and some of their applications,” Mathematics in Science and Engineering, vol. 198,pp. 1–340,1999.