

OPTIMAL WAVELENGTHS FOR MULTI-SPECTRAL PYROMETER USED FOR HEAT TREATMENTS OF STEELS

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ABSTRACT

Optimum wavelengths for a pyrometer that can be used during the heat treatment of steels will be selected at the end of this article. Thanks to one of the physical properties of hot steels which has the possibility to radiate, it is possible to remotely measure its temperature on the surface. This temperature is proportional to the infrared or visible radiation emitted. The system is equipped with optical filters to control electromagnetic radiation and converge them to the appropriate detector. These optical filters are characterized according to the wavelengths used. The pyrometer we talked about will be a multi spectral pyrometer that is to say a pyrometer that uses several wavelengths. The number of optical path depends on the number of the wavelength used. Steel is one of the nonlinear emissivity metals, so the measurement of temperature requires a technique capable of overcoming this nonlinearity. A model called TNL.Tabc or Temperature by Non Linear model with T, a, b, and c the parameters to estimate will be used to select the optimal wavelengths. It will focus on minimizing a cost function by the least squares method. With this model we will sequentially choose the optimal wavelengths one after the other by inverse method. This method consists in fixing the temperature and finding the wavelength corresponding to the temperature set. The first wavelength obtained will be used to calculate the second. And this principle will be applied to find the third as well as the fourth. Optimum wavelengths will be obtained from five (5) selected temperatures in the temperature range of heat treatment of steels. Each of those wavelengths must pass the test of the various criteria to minimize the errors of measurements on the temperature. The power of the light energy obtained depends strongly on these optimal wavelengths. The wavelength groups that will meet these criteria will be the optimum wavelengths for a pyrometer for the heat treatment of steels.

Keyword: Multi spectral pyrometer, optimal wavelength, Infrared radiation, Temperature, and Heat treatment of Steel

1. INTRODUCTION

The temperature is the physical quantity very indispensable in the fields of productions. And most of the time, to know it, we use thermometers that are in direct contact with the object whose temperature is measured. But this technique is not applicable at all for moving objects, located in a hazardous area, for objects with poor thermal conductivity, deformable surface and especially for very high temperature. The metallurgical industries are the most affected by these problems, such as the heat treatment of metals. It is for this reason that the radiative property of

materials is exploited so that its temperature can be measured remotely, this is the general principle of the pyrometer [1].

There are several techniques used for the realization of a pyrometer. One of these techniques is the multi spectral method that uses several wavelengths. A major problem in the design of such a multi-spectral pyrometer is the choice of wavelengths used because it is indispensable and plays a very important role in the temperature calculation. In addition, metals are characterized by their nonlinear emissivity, low in the range of visible and near infrared waves. This non-linearity of the emissivity makes it difficult to measure the temperature and causes serious errors. The choice of wavelengths is very important in minimizing temperature errors and relative errors due to the spectral emissivity of metals.

In this article, we will try to find the optimal wavelengths used for a four-band multi spectral pyrometer for the heat treatment of steels. The goal is to have the four wavelengths whose error on the temperature of the fluxes obtained and the error relating to the spectral emissivity of the steels are minimal.

2. LOI DE RAYONNEMENT ELECTROMAGNETIQUE

2.1. Law of Planck

Let a black body at the temperature T , the energy density of the radiation of this body can be calculated. The calculations are based on the assumption that the electromagnetic field in the limited cavity of the black body is equivalent to a set of independent harmonic oscillators in thermodynamic equilibrium at temperature T and obeying the Boltzmann statistic. It is shown that the luminance $L_{\lambda}^0(T)$ of the black body is equal to the energy density of the radiation multiplied by $\frac{4\pi}{c}$, where the luminance is the ratio of the luminous intensity or energy density of the radiation to the emission surface [2].

$$L_{\lambda}^0(T) = \frac{2hc^2\lambda^{-5}}{\exp\left(\frac{hc}{k\lambda T}\right) - 1}$$

Where $h = 6.6255 \times 10^{-34}$ Js Planck constant, $k = 1.38 \times 10^{-23}$ JK⁻¹ Boltzmann constant, $c = 2.996 \times 10^8$ ms⁻¹ speed of electromagnetic waves in vacuum.

This formula is also used with the so-called Planck constants C_1 and C_2 :

$$L_{\lambda}^0(T) = \frac{C_1\lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right) - 1} \text{ with } C_1 = 2hc^2 \text{ and } C_2 = \frac{hc}{k}$$

2.2. Definition of spectral emissivity

The ratio between the monochromatic luminance of the real source $L_{\lambda}(T)$ and that of the black body $L_{\lambda}^0(T)$, for the same values of the wavelength λ and the temperature T , defines the monochromatic emissivity or spectral emissivity ε_{λ} of the source [3].

$$\varepsilon_{\lambda} = \frac{L_{\lambda}(T)}{L_{\lambda}^0(T)}$$

In the general case, ε_{λ} depends on the source, the wavelength λ and the temperature T and the direction of emission. Whereas the total emissivity is defined in the same way by the following relation:

$$\varepsilon_i = \int_0^{\infty} \varepsilon_{\lambda} d\lambda$$

The luminance of the black body does not depend on the direction of emission, and if it is the same for the real source (source radiating according to Lambert's law), the spectral emissivity does not depend either on the direction of emission.

2.3. Spectral emissivity of metals

The dependence on wavelength emissivity can be expressed in several forms, but we will consider those that can adjust experimental measurements or simplify the analysis. Most surfaces have emissivity that varies with wavelength and temperature. The emissivity of metal surfaces in the wavelengths of the visible and the near infrared often have a polynomial dependence on the wavelength [4].

$$\varepsilon_{\lambda} = c_0 + c_1\lambda + c_2\lambda^2 + \dots + c_n\lambda^n$$

In our case, we use the polynomial model of order 2: $\varepsilon_{\lambda} = a + b\lambda + c\lambda^2$

But in theory, the emissivity depends on the material, the nature of its surface, the temperature, the wavelength and possibly the measurement configuration used. Since metals often reflect radiation, they are generally characterized by a low, non-linear emission level, which is highly dependent on the surface structure and tends towards long wavelengths. This dependence can lead to different and unreliable measurement results [4].

When choosing the appropriate thermal measuring devices, it should be ensured that the infrared radiation is measured with a certain wavelength and a temperature range for which the metals have a relatively high degree of emission.

3. MULTI-SPECTRAL METHOD BASED ON PLANCK LAW

3.1. Presentation of the TNL.Tabc model

The goal is to find the temperature of the steel during the heat treatment with its emissivity simultaneously. The model "TNL.Tabc" means Temperature by Non-Linear model with T, a, b and c, the parameters to be estimated. This model is unbiased and based on the estimation of flux expressed using Planck's law. It will also take into account the polynomial modeling of the emissivity up to the order 2 of the global spectral transfer function of the measurement chain using coefficients (a, b, c). The flux as a function of wavelength and temperature is $L_{\lambda}(T, a, b, c)$ [5].

$$L_{\lambda_i}(T, a, b, c) = (a + b\lambda_i + c\lambda_i^2) \frac{C_1 \lambda_i^{-5}}{\exp\left(\frac{C_2}{\lambda_i T}\right) - 1}$$

With $\varepsilon_{\lambda_i} = a + b\lambda_i + c\lambda_i^2$ spectral emissivity.

The estimation of the parameters (T, a, b, c) will then be carried out by minimizing the function $J(T, a, b, c)$, in which $L_{\lambda_i}^{exp}$ denotes the spectral experimental flux measured at the wavelength λ_i , and $L_{\lambda_i}(T, a, b, c)$ is the theoretical spectral flux at the wavelength λ_i .

$$J(T, a, b, c) = \sum_{i=1}^4 \left(L_{\lambda_i}^{exp} - L_{\lambda_i}(T, a, b, c) \right)^2$$

$$J(T, a, b, c) = \left(L_{\lambda_1}^{exp} - L_{\lambda_1}(T, a, b, c) \right)^2 + \dots + \left(L_{\lambda_4}^{exp} - L_{\lambda_4}(T, a, b, c) \right)^2$$

Note: The index 4 designates the four (04) wavelengths for the estimation of the four parameters $\{T, a, b, c\}$. So we have four (04) equations for the theoretical flows $L_{\lambda_1}(T, a, b, c), L_{\lambda_2}(T, a, b, c), L_{\lambda_3}(T, a, b, c)$ et $L_{\lambda_4}(T, a, b, c)$.

3.2. Model with the method of sequential selection of wavelengths

The method used to estimate the temperature is based on the minimization of a cost function using ordinary least squares method. With this method we will define optimal wavelengths with the inverse method. This method consists of fixing the temperature and finding the wavelength corresponding to this temperature. These wavelengths minimize the standard deviation on the estimated temperature. The determination of the different optimal wavelengths will be carried out using the cost function associated with the model "TNL.Tabc" because this does not require the approximation of Wien, does not present any systematic bias in the presence of $J(T, a, b, c)$ additive noise to the flux and zero average [4][5][6]. The statistical properties of the parameter estimator associated with the TNL.Tabc model and the parameters provided by the least squares method are given by the Variance-Covariance matrix. The matrix from which one can determine the standard deviations σ_{β_i} of the different parameters, and in particular, that of the temperature σ_T . The TNL.Tabc model is a nonlinear model, we will then use the approximate expression of the Ordinary Least Squares variance-Covariance matrix, which is given for a parameter vector $\beta = (T, a, b, c)$, under assumptions of an additive noise, independent, identically distributed (variance σ_{noise}^2 is constant, and zero mean), by:

$$\text{cov}(\beta) = \begin{bmatrix} \sigma_T^2 & \text{cov}(T, a) & \text{cov}(T, b) & \text{cov}(T, c) \\ \text{cov}(T, a) & \sigma_a^2 & \text{cov}(a, b) & \text{cov}(a, c) \\ \text{cov}(T, b) & \text{cov}(a, b) & \sigma_b^2 & \text{cov}(b, c) \\ \text{cov}(T, c) & \text{cov}(a, c) & \text{cov}(b, c) & \sigma_c^2 \end{bmatrix} = (X^T X)^{-1} \sigma_{noise}^2$$

With X the sensitivity matrix associated with the variance-covariance matrix, defined by:

$$X = \begin{bmatrix} \frac{\partial L_{\lambda_1}(T, a, b, c)}{\partial T} & \frac{\partial L_{\lambda_1}(T, a, b, c)}{\partial a} & \frac{\partial L_{\lambda_1}(T, a, b, c)}{\partial b} & \frac{\partial L_{\lambda_1}(T, a, b, c)}{\partial c} \\ \frac{\partial L_{\lambda_2}(T, a, b, c)}{\partial T} & \frac{\partial L_{\lambda_2}(T, a, b, c)}{\partial a} & \frac{\partial L_{\lambda_2}(T, a, b, c)}{\partial b} & \frac{\partial L_{\lambda_2}(T, a, b, c)}{\partial c} \\ \frac{\partial L_{\lambda_3}(T, a, b, c)}{\partial T} & \frac{\partial L_{\lambda_3}(T, a, b, c)}{\partial a} & \frac{\partial L_{\lambda_3}(T, a, b, c)}{\partial b} & \frac{\partial L_{\lambda_3}(T, a, b, c)}{\partial c} \\ \frac{\partial L_{\lambda_4}(T, a, b, c)}{\partial T} & \frac{\partial L_{\lambda_4}(T, a, b, c)}{\partial a} & \frac{\partial L_{\lambda_4}(T, a, b, c)}{\partial b} & \frac{\partial L_{\lambda_4}(T, a, b, c)}{\partial c} \end{bmatrix}$$

And the standard deviation on the temperature σ_T is given according to the standard deviation on the noise σ_{noise} , by:

$$\sigma_T = \sqrt{(X^T X)^{-1}} \sigma_{noise}$$

We will take as value the standard deviation of the noise, which we have experimentally with the infrared camera, and having for value $\sigma_{noise} \approx 8,97.10^4 Wm^{-2}$, That is to say $7.43 \cdot 10^{-3} \%$ of the maximum of Planck's law [5].

3.3. Pseudo-optimal method for the selection of wavelengths

The pseudo-optimal method consists in sequentially selecting the wavelengths while respecting all the different criteria. These wavelengths are those that minimize the standard deviations on the temperature at a fixed temperature finding the temperature range of the heat treatment of the steels. In our case, we will use a temperature set (T_s) for the calculation at 1073.15K, 1173.15K, 1223.15K, 1273.15K and 1373.15K.

- Selection of the first optimal wavelength

The method of sequential selection of "pseudo-optimal" wavelengths consists of choosing for the first wavelength filter λ_{OP1} , the one which minimizes the standard deviation σ_T on the temperature, assuming that the measurement is mono spectral. The cost function $J(\beta)$ consists of only one parameter: the temperature T.

$$J(T) = (L_{\lambda_1}^{exp} - L_{\lambda_1}(T, a, b, c))^2$$

The temperature T is then the only parameter to estimate. The sensitivity matrix X is composed only of the first column and first row.

$$X = \left[\frac{\partial L_{\lambda_1}(T, a, b, c)}{\partial T} \right]$$

The minimization of the cost function $J(T)$, involves the sensitivity matrix X of the flux at the various parameters to be estimated. The first optimal wavelength λ_{OP1} will minimize this standard deviation.

- Selection of the second optimal wavelengths

The selection of the second filter is performed by setting a = 1, b = 1, and $\lambda_1 = \lambda_{OP1}$. And looking for second wavelength, the shortest that minimizes the local standard deviation of temperature in the TNL.Ta model. The function cost $J(T, a)$ and the sensitivity matrix X are respectively composed as follows:

$$J(T, a) = \sum_{i=1}^2 (L_{\lambda_i}^{exp} - L_{\lambda_i}(T, a, b, c))^2 = (L_{\lambda_1}^{exp} - L_{\lambda_1}(T, a, b, c))^2 + (L_{\lambda_2}^{exp} - L_{\lambda_2}(T, a, b, c))^2$$

$$X = \begin{bmatrix} \frac{\partial L_{\lambda_1}(T, a, b, c)}{\partial T} & \frac{\partial L_{\lambda_1}(T, a, b, c)}{\partial a} \\ \frac{\partial L_{\lambda_2}(T, a, b, c)}{\partial T} & \frac{\partial L_{\lambda_2}(T, a, b, c)}{\partial a} \end{bmatrix}$$

- Selection of the third optimal wavelengths

For the third wavelength, it is obtained by minimizing the cost function $J(T, a, b)$ with the sensitivity matrix X associated with the model TNL.Tab by fixing $\lambda_1 = \lambda_{OP1}$ and $\lambda_2 = \lambda_{OP2}$.

$$J(T, a, b) = \sum_{i=1}^3 (L_{\lambda_i}^{exp} - L_{\lambda_i}(T, a, b, c))^2 = (L_{\lambda_1}^{exp} - L_{\lambda_1}(T, a, b, c))^2 + \dots + (L_{\lambda_3}^{exp} - L_{\lambda_3}(T, a, b, c))^2$$

$$X = \begin{bmatrix} \frac{\partial L_{\lambda_1}(T, a, b, c)}{\partial T} & \frac{\partial L_{\lambda_1}(T, a, b, c)}{\partial a} & \frac{\partial L_{\lambda_1}(T, a, b, c)}{\partial b} \\ \frac{\partial L_{\lambda_2}(T, a, b, c)}{\partial T} & \frac{\partial L_{\lambda_2}(T, a, b, c)}{\partial a} & \frac{\partial L_{\lambda_2}(T, a, b, c)}{\partial b} \\ \frac{\partial L_{\lambda_3}(T, a, b, c)}{\partial T} & \frac{\partial L_{\lambda_3}(T, a, b, c)}{\partial a} & \frac{\partial L_{\lambda_3}(T, a, b, c)}{\partial b} \end{bmatrix}$$

- Selection of the fourth and last optimal wavelengths

The fourth optimal wavelength will be obtained on the same principle as how to obtain the second and the third optimal wavelength by fixing $a = 1, b = 1, c=1, \lambda_1 = \lambda_{OP1}, \lambda_2 = \lambda_{OP2}$ and $\lambda_3 = \lambda_{OP3}$. The cost function $J(T, a, b, c)$ and the sensitivity matrix X associated with the model TNL.Tabc are respectively represented as follows:

$$J(T, a, b, c) = \sum_{i=1}^4 (L_{\lambda_i}^{exp} - L_{\lambda_i}(T, a, b, c))^2 = (L_{\lambda_1}^{exp} - L_{\lambda_1}(T, a, b, c))^2 + \dots + (L_{\lambda_4}^{exp} - L_{\lambda_4}(T, a, b, c))^2$$

$$X = \begin{bmatrix} \frac{\partial L_{\lambda_1}(T, a, b, c)}{\partial T} & \frac{\partial L_{\lambda_1}(T, a, b, c)}{\partial a} & \frac{\partial L_{\lambda_1}(T, a, b, c)}{\partial b} & \frac{\partial L_{\lambda_1}(T, a, b, c)}{\partial c} \\ \frac{\partial L_{\lambda_2}(T, a, b, c)}{\partial T} & \frac{\partial L_{\lambda_2}(T, a, b, c)}{\partial a} & \frac{\partial L_{\lambda_2}(T, a, b, c)}{\partial b} & \frac{\partial L_{\lambda_2}(T, a, b, c)}{\partial c} \\ \frac{\partial L_{\lambda_3}(T, a, b, c)}{\partial T} & \frac{\partial L_{\lambda_3}(T, a, b, c)}{\partial a} & \frac{\partial L_{\lambda_3}(T, a, b, c)}{\partial b} & \frac{\partial L_{\lambda_3}(T, a, b, c)}{\partial c} \\ \frac{\partial L_{\lambda_4}(T, a, b, c)}{\partial T} & \frac{\partial L_{\lambda_4}(T, a, b, c)}{\partial a} & \frac{\partial L_{\lambda_4}(T, a, b, c)}{\partial b} & \frac{\partial L_{\lambda_4}(T, a, b, c)}{\partial c} \end{bmatrix}$$

4. CRITERIA FOR THE SELECTION OF OPTIMUM WAVE LENGTHS OBTAINED

Our pyrometer must be very sensitive to the temperature between 975.15K and 1473.15K. This range borders the temperature range of the heat treatment of steels which is between 1000.15K and 1421.15K [8].

So that we can have better optimal wavelengths, we will try to find optimal wavelengths from the temperature set (T_s) at 1073.15K, 1173.15K, 1223.15K, 1273.15K and 1373.15K.

4.1. Criteria on the spectral range of the pyrometer

Our first criterion for the selection of optimal wavelengths is the spectral range of our pyrometer which operates in the band between $0.7\mu\text{m}$ to $3\mu\text{m}$ (Table-1). Measuring the temperature of a metal requires the use of short wavelength to avoid the relative error due to emissivity.

The multi spectral measurement minimizes the error so the choice of short wavelength gives a better precision on the temperature. It is observed that each time a wavelength is added, the standard deviation deteriorates by about a factor of 10.

Table-1: Optimum wave lengths preselected according to the spectral range of the pyrometer

T_s [K]	CHANNEL 1		CHANNEL 2		CHANNEL 3		CHANNEL 4	
	λ_{OP1} [μm]	σ_T [K]	λ_{OP2} [μm]	σ_T [K]	λ_{OP3} [μm]	σ_T [K]	λ_{OP4} [μm]	σ_T [K]
1073.15	2.246	0.000959	1.535	0.005373	1.186	0.043606	0.967	0.461628
							1.392	0.471311
							2.000	0.132117
					1.945	0.038898	1.099	0.128315
							1.719	0.484844
							2.122	0.659303
1173.15	2.054	0.000671	1.403	0.003772	1.085	0.030674	0.884	0.323962
							1.273	0.333100
							1.896	0.090029
					1.778	0.027332	1.005	0.090259
							1.572	0.340205
							1.939	0.465602
1223.15	1.970	0.000568	1.346	0.003185	1.041	0.025831	0.848	0.272897
							1.222	0.279541
							1.821	0.075742
					1.707	0.023056	0.965	0.075973
							1.509	0.287155
							1.857	0.392977
1273.15	1.893	0.000684	1.294	0.002712	1	0.022004	0.815	0.232649
							1.174	0.238138
							1.749	0.064520
					1.640	0.019644	0.900	0.065571
							1.449	0.244430
							1.788	0.334194
1373.15	1.755	0.000357	1.199	0.002005	0.927	0.016302	0.756	0.172321
							1.088	0.176781
							1.622	0.047746
					1.520	0.014494	0.855	0.047922
							1.343	0.180200
							1.658	0.245986

4.2. Criteria on the minimum deviation of the two successive wavelengths

To avoid amplifying the measurement error, while remaining as close as possible in order to minimize the measurement error due to the spectral variation of the emissivity, the minimum difference of the two successive wavelengths $\Delta_{Min} \lambda$ must be respected.

$$\Delta_{Min} \lambda_{ji} = \left| \lambda_j - \lambda_i \right| f \frac{T \lambda_j^2}{C_2} \Bigg|_{\lambda_j f \lambda_i}$$

The minimum difference between the first and the second wavelength will therefore be $\lambda_{OP1} - \lambda_{OP2} \geq \Delta_{Min} \lambda_{1-2}$. So the second maximum wavelength will be selected according to this relation $\lambda_{OP2Max} \leq \lambda_{OP1} - \Delta_{Min} \lambda_{1-2}$. The difference between the second and the third wavelength will be $\lambda_{OP2} - \lambda_{OP3} \geq \Delta_{Min} \lambda_{2-3}$. The maximum value of the third wavelength is then $\lambda_{OP3Max} \leq \lambda_{OP2} - \Delta_{Min} \lambda_{2-3}$. Same principle for the last and fourth optimal wavelength, $\lambda_{OP3} - \lambda_{OP4} \geq \Delta_{Min} \lambda_{3-4}$ then $\lambda_{OP4Max} \leq \lambda_{OP3} - \Delta_{Min} \lambda_{3-4}$. The four selected optimal wavelengths respecting the criterion of the minimum standard deviation on the temperature and the minimum difference between two successive wavelengths, for a temperature of 1073.15K, 1173.15K, 1223.15K, 1273.15K and 1373.15K will be represented in table-2.

In the infrared, the standard deviation can not be exceeded by 5%. The highest values were achieved for the shortest wavelength and wavelengths disturbed by atmospheric absorption. In the other infrared spectral ranges, the standard deviation between the first and the last spectrum was less than 2% [4].

Table-2: Optimum wavelengths obtained from 1073.15K, 1173.15K, 1223.15K, 1273.15K and 1373.15K according to the criterion of minimum deviation of the two successive wavelengths

Ts[K]	λ_{OP} [μm]	σ_T [K]	σ_T [%]	$\Delta \lambda$ [μm]	$\Delta_{min} \lambda$ [μm]
1073.15	$\lambda_{OP1} = 2.246$	0.000959	0.000089	$\lambda_{OP1} - \lambda_{OP2} = 0.711$	$\lambda_{OP1} - \lambda_{OP2} = 0.411$
	$\lambda_{OP2} = 1.535$	0.005373	0.000501	$\lambda_{OP2} - \lambda_{OP3} = 0.349$	$\lambda_{OP2} - \lambda_{OP3} = 0.192$
	$\lambda_{OP3} = 1.186$	0.043606	0.004063	$\lambda_{OP3} - \lambda_{OP4} = 0.219$	$\lambda_{OP3} - \lambda_{OP4} = 0.114$
	$\lambda_{OP4} = 0.967$	0.461628	0.043016		
1173.15	$\lambda_{OP1} = 2.054$	0.000671	0.000057	$\lambda_{OP1} - \lambda_{OP2} = 0.651$	$\lambda_{OP1} - \lambda_{OP2} = 0.344$
	$\lambda_{OP2} = 1.403$	0.003772	0.000321	$\lambda_{OP2} - \lambda_{OP3} = 0.318$	$\lambda_{OP2} - \lambda_{OP3} = 0.160$
	$\lambda_{OP3} = 1.085$	0.030674	0.002614	$\lambda_{OP3} - \lambda_{OP4} = 0.201$	$\lambda_{OP3} - \lambda_{OP4} = 0.095$
	$\lambda_{OP4} = 0.884$	0.323962	0.027614		
1223.15	$\lambda_{OP1} = 1.970$	0.000568	0.000046	$\lambda_{OP1} - \lambda_{OP2} = 0.624$	$\lambda_{OP1} - \lambda_{OP2} = 0.329$
	$\lambda_{OP2} = 1.346$	0.003185	0.000260	$\lambda_{OP2} - \lambda_{OP3} = 0.305$	$\lambda_{OP2} - \lambda_{OP3} = 0.154$
	$\lambda_{OP3} = 1.041$	0.025831	0.002112	$\lambda_{OP3} - \lambda_{OP4} = 0.193$	$\lambda_{OP3} - \lambda_{OP4} = 0.092$
	$\lambda_{OP4} = 0.848$	0.272897	0.022311		
1273.15	$\lambda_{OP1} = 1.893$	0.000684	0.000054	$\lambda_{OP1} - \lambda_{OP2} = 0.599$	$\lambda_{OP1} - \lambda_{OP2} = 0.317$
	$\lambda_{OP2} = 1.294$	0.002712	0.000213	$\lambda_{OP2} - \lambda_{OP3} = 0.294$	$\lambda_{OP2} - \lambda_{OP3} = 0.148$
	$\lambda_{OP3} = 1.000$	0.022004	0.001728	$\lambda_{OP3} - \lambda_{OP4} = 0.185$	$\lambda_{OP3} - \lambda_{OP4} = 0.088$
	$\lambda_{OP4} = 0.815$	0.232649	0.018273		
1373.15	$\lambda_{OP1} = 1.755$	0.000357	0.000026	$\lambda_{OP1} - \lambda_{OP2} = 0.556$	$\lambda_{OP1} - \lambda_{OP2} = 0.293$
	$\lambda_{OP2} = 1.199$	0.002005	0.000146	$\lambda_{OP2} - \lambda_{OP3} = 0.272$	$\lambda_{OP2} - \lambda_{OP3} = 0.119$
	$\lambda_{OP3} = 0.927$	0.016302	0.001187	$\lambda_{OP3} - \lambda_{OP4} = 0.171$	$\lambda_{OP3} - \lambda_{OP4} = 0.082$
	$\lambda_{OP4} = 0.756$	0.172321	0.012549		

First observation: It is observed that only one group of optimal wavelength which is obtained from each temperature set respects the criterion of minimum distance between two successive optimal wavelengths.

Second observation: In addition, the minimum distance required between two wavelengths is proportionally with the largest wavelength among the two successive ones.

Third observation: We also note that all groups of optimal wavelengths that have the lowest standard deviation do not meet the criterion of minimum distance between two successive wavelengths.

4.3. Standard deviation on the temperature at the temperature range of the fluxes obtained from the optimal wavelengths

Verification of the standard deviation of the optimal wavelengths is almost necessary to know the errors on the temperature throughout the temperature range from 975.15K to 1473.15K of the heat treatment of the steels (Table-3).

First observation: In table-3, we have four (04) optimal lengths that respect the minimum difference between two (2) successive wavelengths for a temperature set. More the temperatures sets increases to 1373.15K, the wavelengths decrease to at least 0.140µm.

Second observation: We also note that the standard deviation on temperature improves as well as the temperature to be measured increases.

Third observation: More the wavelength decreases, the standard deviation deteriorates, that is to say increases ($T_s = 1223.15K$: $\lambda_{OP1} = 1.970\mu m$, $\lambda_{OP2} = 1.346\mu m$, $\lambda_{OP3} = 1.041\mu m$, $\lambda_{OP4} = 0.848$ If we apply these wavelengths at $T = 1073.15K$, we have respectively a standard deviation of 0.001011K, 0.002417K, 0.001023K, 0.052467K).

Fourth observation: the standard deviation deteriorates rapidly if the wavelength exceeds the lower limit of the near-infrared range ($\lambda_{OP4} = 0.756\mu m$ calculated from the temperature set at $T_s = 1373.15K$, the standard deviations on the temperature of this wavelength for the temperature range of our pyrometer are, at $T = 975.15K$ gives $\sigma_T = 0.917752K$ and at $T = 1473.15K$ gives $\sigma_T = 0.002754K$).

Table-3: Standard deviation on the temperature σ_T [K] at the spectral range of the pyrometer of the optimal wavelengths obtained

T_s [K]	λ_{OP} [µm]	975.15K	1073.15K	1173.15K	1223.15K	1273.15K	1373.15K	1473.15K
1073.15	$\lambda_{OP1} = 2.246$	0.001459	0.000959	0.000686	0.000595	0.000524	0.000420	0.000349
	$\lambda_{OP2} = 1.535$	0.003146	0.001559	0.000884	0.000693	0.000555	0.000378	0.000273
	$\lambda_{OP3} = 1.186$	0.011394	0.004336	0.001977	0.001408	0.001033	0.000600	0.000379
	$\lambda_{OP4} = 0.967$	0.056348	0.016485	0.006042	0.003911	0.002627	0.001305	0.000719
1173.15	$\lambda_{OP1} = 2.054$	0.001582	0.000982	0.000671	0.000571	0.000493	0.000382	0.000310
	$\lambda_{OP2} = 1.403$	0.004541	0.002068	0.001094	0.000832	0.000648	0.000419	0.000290
	$\lambda_{OP3} = 1.085$	0.021317	0.007281	0.003035	0.002078	0.001471	0.000801	0.000478
	$\lambda_{OP4} = 0.884$	0.138194	0.035367	0.011602	0.007152	0.004595	0.002106	0.001084
1223.15	$\lambda_{OP1} = 1.970$	0.001674	0.001011	0.000675	0.000568	0.000486	0.000371	0.000296
	$\lambda_{OP2} = 1.346$	0.005532	0.002417	0.001235	0.000925	0.000711	0.000448	0.000304
	$\lambda_{OP3} = 1.041$	0.029580	0.009576	0.003817	0.002564	0.001782	0.000940	0.000546
	$\lambda_{OP4} = 0.848$	0.219035	0.052467	0.016292	0.009805	0.006161	0.002715	0.001351
1273.15	$\lambda_{OP1} = 1.893$	0.001789	0.001050	0.000685	0.000571	0.000484	0.000343	0.000286
	$\lambda_{OP2} = 1.294$	0.006791	0.002847	0.001406	0.001038	0.000786	0.000484	0.000321
	$\lambda_{OP3} = 1.000$	0.041610	0.012759	0.004862	0.003202	0.002185	0.001116	0.000631
	$\lambda_{OP4} = 0.815$	0.349686	0.078430	0.023061	0.013551	0.008329	0.003529	0.001697
1373.15	$\lambda_{OP1} = 1.755$	0.002101	0.001164	0.000725	0.000592	0.000492	0.000357	0.000274
	$\lambda_{OP2} = 1.199$	0.010626	0.004095	0.001886	0.001350	0.000995	0.000582	0.000370
	$\lambda_{OP3} = 0.927$	0.084590	0.023272	0.008105	0.005130	0.003377	0.001616	0.000863
	$\lambda_{OP4} = 0.756$	0.917752	0.180401	0.047544	0.026628	0.015659	0.006132	0.002754

Fifth observation: The temperature range of our pyrometer is 975.15K up to 1473.15K, we see that the optimal lengths obtained at $T_s = 1073.15K$ have a better standard deviation compared to those obtained at $T_s = 1373.15K$. The worst standard deviation is 0.056348K at $T = 975.15K$ for $T_s = 1073.15K$, against 0.917752K at $T = 975.15K$ for $T_s = 1373.15K$.

4.4. Sensitivity of the flux at temperature and wavelength

The model called TNL.Tabc consists in making temperature measurements without mastering all the influencing factors. However, it is necessary to take certain precautions to minimize the measurement error on the temperature. However, our field of work is on the increasing part of the Planck curve because the reduced sensitivities of the flux at the temperature χ_T and at the wavelength χ_λ are all the better that we work at short wavelengths. The wavelengths obtained should give better sensitivity to temperature (Table-4) and wavelength (Table-5).

$$\chi_T = \frac{1}{L_\lambda(T)} \frac{dL_\lambda(T)}{dT}$$

$$\chi_\lambda = \frac{1}{L_\lambda(T)} \frac{dL_\lambda(T)}{d\lambda}$$

- For table-4:

First observation: The sensitivity of the flux to the temperature increases as the wavelength decreases.

Second observation: In the spectral band of our pyrometer, the sensitivity of the flux to the temperature applied to a wavelength decreases if the temperature increases.

Third observation: In the spectral band between 0.7µm and 3µm, the sensitivity of the flux to the temperature is more and better at 975.15K than at 1473.15K upper limit of our temperature to be measured.

Table-4: Sensitivity of the flux obtained from the optimal wavelengths at the temperature

$T_s[K]$	$\lambda_{OP} [\mu m]$	975.15K	1073.15K	1173.15K	1223.15K	1273.15K	1373.15K	1473.15K
1073.15	$\lambda_{OP1} = 2.246$	0.006773	0.005576	0.004674	0.004304	0.003978	0.003429	0.002990
	$\lambda_{OP2} = 1.535$	0.009898	0.008140	0.006813	0.006268	0.005786	0.004976	0.004326
	$\lambda_{OP3} = 1.186$	0.012810	0.010534	0.008815	0.008109	0.007485	0.006435	0.005591
	$\lambda_{OP4} = 0.967$	0.015711	0.012919	0.010811	0.009945	0.009179	0.007891	0.006856
1173.15	$\lambda_{OP1} = 2.054$	0.007402	0.006091	0.005102	0.004697	0.004339	0.003737	0.003255
	$\lambda_{OP2} = 1.403$	0.010829	0.008905	0.007452	0.006856	0.006328	0.005442	0.004729
	$\lambda_{OP3} = 1.085$	0.014002	0.115147	0.009635	0.008863	0.008181	0.007033	0.006111
	$\lambda_{OP4} = 0.884$	0.017186	0.014132	0.011826	0.010879	0.010041	0.008632	0.007500
1223.15	$\lambda_{OP1} = 1.970$	0.007716	0.006348	0.005317	0.004894	0.004520	0.003892	0.003389
	$\lambda_{OP2} = 1.346$	0.011287	0.009282	0.007767	0.007146	0.006596	0.005671	0.004929
	$\lambda_{OP3} = 1.041$	0.014594	0.012001	0.010042	0.009238	0.008527	0.007330	0.006369
	$\lambda_{OP4} = 0.848$	0.017916	0.014732	0.012328	0.011341	0.010467	0.008998	0.007818
1273.15	$\lambda_{OP1} = 1.893$	0.008029	0.006605	0.005531	0.005090	0.004701	0.004047	0.003522
	$\lambda_{OP2} = 1.294$	0.011741	0.009655	0.008079	0.007432	0.006861	0.005898	0.005126
	$\lambda_{OP3} = 1.000$	0.015193	0.012493	0.010454	0.009617	0.008876	0.007631	0.006630
	$\lambda_{OP4} = 0.815$	0.018641	0.015329	0.012827	0.011800	0.010891	0.009363	0.008135
1373.15	$\lambda_{OP1} = 1.755$	0.008658	0.007122	0.005962	0.005486	0.005066	0.004359	0.003792
	$\lambda_{OP2} = 1.199$	0.012671	0.010420	0.008719	0.008021	0.007403	0.006365	0.005531
	$\lambda_{OP3} = 0.927$	0.016389	0.013477	0.011277	0.010374	0.009575	0.008231	0.007152
	$\lambda_{OP4} = 0.756$	0.020096	0.016525	0.013828	0.012721	0.011741	0.010093	0.008769

Table-5: Flux sensitivity at the wavelength in the temperature range of the pyrometer

T _s [K]	λ _{OP} [μm]	975.15K	1073.15K	1173.15K	1223.15K	1273.15K	1373.15K	1473.15K
1073.15	λ _{OP1} = 2.246	708783	438419	215437	118132	28815.5	-129307	-264699
	λ _{OP2} = 1.535	3017940	2433730	1949550	1737360	1541990	1194490	894948
	λ _{OP3} = 1.186	6295360	5315970	4503650	4147350	3819100	3234490	2729580
	λ _{OP4} = 0.967	10640700	9167330	7945180	7409060	6915060	6035040	5274610
1173.15	λ _{OP1} = 2.054	1072800	748273	480169	363010	255370	64539.6	-99164.8
	λ _{OP2} = 1.403	3947540	3247910	2667830	2413500	2179270	1762380	1402700
	λ _{OP3} = 1.085	7950900	6780620	5809910	5384100	4991770	4292950	3689210
	λ _{OP4} = 0.884	13263700	11500700	10038200	9396680	8805530	7752420	6842330
1223.15	λ _{OP1} = 1.970	1273700	920437	628392	500698	383333	175134	-3618.14
	λ _{OP2} = 1.346	4446190	3685950	3055540	2779110	2524490	2071220	1680020
	λ _{OP3} = 1.041	8840230	7568920	6514380	6051800	5624470	4866320	4210320
	λ _{OP4} = 0.848	14664100	12748200	11158900	10461700	9819330	8674880	7685850
1273.15	λ _{OP1} = 1.893	1486270	1103270	786471	647884	520461	294298	99976.3
	λ _{OP2} = 1.294	4965930	4143300	3461080	3161900	2886300	2395610	1971990
	λ _{OP3} = 1.000	9784980	8407280	7264470	6763160	6301240	5478390	4767390
	λ _{OP4} = 0.815	16124000	14049800	12329300	11574500	10879000	9639990	8569220
1373.15	λ _{OP1} = 1.755	1952330	1506060	1136600	974840	826027	561657	334207
	λ _{OP2} = 1.199	6114380	5156120	4361330	4012740	3691580	3119630	2625670
	λ _{OP3} = 0.927	11811500	10208300	8878370	8294970	7757400	6799760	5972200
	λ _{OP4} = 0.756	19255100	16844500	14844900	13967800	13159500	11719500	10475000

- For table-5 :

First observation: The sensitivity of the flux to the wavelength increases when the wavelength decreases.

Second observation: The sensitivity of the flux at the wavelength of the wavelengths obtained from the higher temperature is more and better than that obtained at the lower limit in the temperature range to be measured.

Third observation: The existence of negative values justifies that the use of these wavelengths obtained at T_s between 1073.15K and 1223.15K can give serious errors if the temperature to be measured is greater than 1373.15K. Therefore these wavelengths do not cover, in terms of sensitivity on flow, the temperatures necessary for the heat treatment of steels.

5. CONCLUSIONS

The TNL.Tabc model based on Planck's law gives the possibility of finding temperature and spectral emissivity at the same time. This model allowed us to sequentially select optimal wavelengths for a multi spectral pyrometer for the heat treatment of steels.

Among the optimal wavelengths obtained from 1073.15K, 1173.15K, 1223.15K, 1273.15K and 1373.15K, only a group of four wavelengths respects the criterion of minimum distance between two successive wavelengths. The choice of these wavelengths starting from the minimum standard deviation is then an insufficient criterion.

All optimal wavelengths have good temperature sensitivity between 975.15K and 1473.15K. But the fluxes at these optimal wavelengths are not at all sensitive if the temperature is between 1373.15K and 1473.15K. In addition, their standard deviation is very far from exceeding the 5% error limit.

To conclude, only the optimal wavelengths obtained from the temperature set at 1273.15K meet all the criteria for selecting the four optimal wavelengths for a multi spectral pyrometer for the heat treatment of steels. Therefore, with the method of sequential selection of the optimal wavelengths of the model TNL.Tabc by the ordinary least square method, it is better to use the temperature towards the upper limit of the temperature range (975.15K and 1473.15K) to be measured than the temperature towards the lower limit. And the closer you get to the upper limit, the longer the wavelengths reach the visible part.

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