Optimization Of Flexural Strength And Split Tensile Strength Of Hybrid Polypropylene Steel Fibre Reinforced Concrete (HPSFRC)

K. C. Nwachukwu[1], C.C. Mathew [2], B.O.Mama[3] , O.Oguaghamba[4] and C.S.Uzoukwu[5]

[1,5]*Department Of Civil Engineering, Federal University Of Technology, Owerri, Imo State, Nigeria*

[2,]*Department Of Engineering Mechanics ,Virginia Polytechnic Institute and State University, USA*

[3,4] *Department Of Civil Engineering, University Of Nigeria, Nsukka, Enugu State, Nigeria*

Corresponding E- Mail: knwachukwu@gmail.com, Whatsapp: +234-8036670993

ABSTRACT

Replacing costly conventional reinforcement with inexpensive fibres is one of the techniques that can adequately guarantee fast reduction in the cost of concrete production. Sometimes, when two or more fibres are combined and added to concrete in various mixture designs as replacement for conventional reinforcement, then superior reinforced concrete is produced with promising mechanical properties that can control cracking due to plastic shrinkage and to drying shrinkage. This research work is aimed at using Scheffe's Second Degree Model for six component mixture to optimize the Flexural Strength and Split Tensile Strength of Hybrid – Polypropylene - Steel Fibre Reinforced Concrete (HPSFRC).Using Scheffe's Simplex method, the Flexural Strength and Split Tensile Strength of HPSFRC were determined for different mix proportions. Control experiments were also carried out and the flexural and split tensile strengths evaluated. The test statistics using the Student's t-test validated the results. Maximum design strengths recorded for the flexural test at 14 and 28 days were 6.22MPa and 9.84MPa respectively, while those recorded for the splitting tensile test were 4.38MPa and 6.08MPa respectively. HPSFRC controllable design strength values are capable of sustaining construction of light-weight and heavy-weight structures such as Bridges, Airports, maintenance hangars, access roads etc at the best possible economic and safety advantages.

Keywords: *Scheffe's (6,2) Optimization Model, HPSFRC, Flexural Strength, Split Tensile Strength, Mixture Design*

1.INTRODUCTION

Concrete, as the most widely used material in the construction industries has been undergoing changes both as a material and due to technological advancement. By definition, concrete, according to Oyenuga (2008), is a composite inert material comprising of a binder course (cement), mineral filter or aggregates and water. In terms of its strength, concrete, being a homogeneous mixture of cement, sand, gravel and water is very strong in carrying compressive forces and hence is gaining increasing importance as building materials throughout the world (Syal and Goel, 2007). Again, concrete, according to Neville (1990), plays an important part in all building structures owing to its numerous advantages that ranges from low built in fire resistance, high compressive strength to low maintenance. However, concrete has got its own disadvantages. According to Shetty (2006), concrete (especially the plain type) possesses a very low tensile strength, limited ductility, low shear strength and little resistance to cracking. As a result of these limitations, key players in the construction industries are always at the look out on how to remedy these situations, giving priority to safety, economy and sustainable environmentally friendly technology. Thus, through extensive research and development work, reinforcement of the tension zone of the concrete with conventional steel bars came to be. Due to the expensive nature of the conventional reinforcement, further researches have shown that incorporation of fibres into the concrete would act as crack arrester and would substantially improve its static as well as dynamic properties. This also led to a type of research known as Fibre reinforced concrete (FRC) research. FRC is a composite material consisting of mixtures of cement, mortar or concrete and discontinuous, discrete as well as uniformly dispersed fibre. Further researches have also shown that the use of two or more fibres is always better than one. In a nutshell, Hybrid Fibre Reinforced Concrete (HFRC) is the use of two or more fibres in a single concrete mixture matrix with the aim to improve its overall properties. In well-designed hybrid composites, there is always positive interaction between the fibres and hence, the resulting hybrid performance is expected to exceed the sum of individual fibres performances due to synergy between the fibres. HFRC can also reduce the permeability of concrete and thus reduce bleeding of water. Incorporation of Hybrid Fibres (HF) with concrete can produce a range of materials which possess enhanced tensile strength, compressive strength, elasticity, toughness, and durability etc. Hybrid – Polypropylene - Steel Fibre Reinforced Concrete (HPSFRC) is concrete mixture where the conventionally steel reinforcement in concrete production is replaced (wholly or partially) with polypropylene fibre and steel fibre. Special mechanical properties of HPSFRC under investigation in this present work are the flexural strength and the split tensile strength.. By definition, flexural strength is the ability of the material to withstand bending forces applied perpendicular to its longitudinal axis. It is also defined as the maximum bending stress that can be applied to the material before it yields. On the other hand, splitting tensile strength test on concrete cylinder is a method to determine the tensile strength of concrete. It is generally carried out to obtain the tensile strength of concrete, and the stress field in the tests is actually a biaxial stress field with compressive stress three times greater than the tensile stress. The split tensile strength test is an indirect method of testing tensile strength of concrete and is generally greater than direct tensile strength and lower than flexural strength (modulus of rupture). .

The process of combination of the fibres in order to iinvestigate the special mechanical properties of HPSFRC under consideration are sometimes called hybridization. However, the best possible way of combining the fibres is actualized through optimization, which is less laborious. By definition, an optimization problem is one requiring the determination of the optimal (maximum or minimum) value of a given function, called the objective function, subject to a set of stated restrictions, or constraints placed on the variables concerned. The objective of mix design, according to Shacklock (1974), is to determine the most appropriate proportions in which to use the constituent materials to meet the needs of construction work. Specifically, optimization of the concrete mixture design is a process of search for a mixture for which the sum of the costs of the ingredients is lowest, yet satisfying the required performance of concrete, such as strength, workability and durability etc. When focusing on the widely varying properties of the constituent materials, the conditions that prevail at the site of work, the exposure condition, and the conditions that are demanded for a particular work for which the mix is designed, the design of concrete mix according to (Shetty, 2006) is not a simple task. According to Jackson and Dhir (1996), concrete mix design remains the procedure which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. From the above definition, the cost of any concrete includes, in addition to that of the materials themselves, the cost of the mix design, of batching, mixing and placing the concrete and of the site supervision. In the context of the above guidelines, the empirical mix design methods and procedures proposed by Hughes (1971), ACI- 211(1994) and DOE (1988) appears to be a little bit complex as well as time consuming. This is because, they involve a lot of trial mixes and complex statistical calculations before the desired strength of the concrete can be reached. Thus, when considering the drawbacks associated with the above empirical methods, it be could be ascertained that optimization of the concrete mixture design is the fastest method, best option, most convenient and the most efficient way of selecting concrete mix ratios or proportions for better efficiency and better performance of concrete. A typical example of optimization model is Scheffe’s Model. It could be in the form of Scheffe’s Second Degree Model or Scheffe’s Third Degree Model. In this present study, Scheffe’s Second Degree Model for six components mixtures (namely Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate, Polypropylene Fibre and Steel Fibre are presented.

This present study examines the use of Scheffe’s Second Degree Model for six component mixture in the optimization of the Flexural Strength and Split Tensile Strength of HPSFRC. As a matter of fact many researchers have done related works on polypropylene fibre as well as steel fibre and HPSFRC, but none has addressed the subject matter sufficiently. For instance, on PFRC , Bayasi and Zeng (1993) and Patel and others (2012) have investigated the properties of PFRC. On SFRC and related works, Baros and others (2005) investigated the post – cracking behaviour of SFRC. Jean-Louis and Sana (2005) investigated the corrosion of SFRC from the crack. Lima and Oh (1999) carried out an experimental and theoretical investigation on the shear of SFRC beams. Similarly, Lau and Anson (2006) carried out research on the effect of high temperatures on high performance SFRC. The work of Lie and Kodar (1996) was on the study of thermal and mechanical properties of SFRC at elevated temperatures. Blaszczynski and Przybylska-Falek (2015) investigated the use of SFRC as a structural material. Huang and Zhao (1995) investigated the properties of SFRC containing larger coarse aggregate. Arube and others (2021) investigated the Effects of Steel Fibres in Concrete Paving Blocks. Again, Khaloo and others (2005) examined the flexural behaviour of small SFRC slabs. And Ghaffer and others (2014) investigated the use of steel fibres in structural concrete to enhance the mechanical properties of concrete. On HPSFRC and related works,.Yew and others (2011) have investigated the strength properties of Hybrid Nylon-Steel fibre-reinforced concrete in comparison to that of polypropylene-steel fibre-reinforced concrete. Singh and others (2010) have investigated the strength and flexural toughness of concrete reinforced with Steel – Polypropylene Hybrid Fibres. Kayalvizhi and others (2019) carried out a Test on the Behaviour of Hybrid Fiber Reinforced Concrete. In their contribution, Varma and Raji (2019) have presented an experimental investigation to quantity the improved mechanical properties of Hybrid - Polypropylene-Steel Fibre-Reinforced Concrete. Nuaklong and others (2020) investigated the effect of hybrid polypropylene- steel- fibres on strength characteristics of UHPFRC. Qian and Stroeven (2000) investigated the optimization of fibre size , fibre content, and fly ash content in hybrid polypropylene- steel fibre concrete based on general mechanical properties. Recent works on optimization show that many researchers have used Scheffe's method to carry out one form of optimization work or the other. For example, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere (2006) were also based on the use of Scheffe' mathematical model in the optimization of compressive strength of Perwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's (4,2) and Scheffe's (4,3). Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also, Nwachukwu and others (2022a) developed and used Scheffe's Third Degree Polynomial model, Scheffe's (5,3) to optimize the compressive strength of GFRC where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). Nwachukwu and others (2022b) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). Furthermore, Nwachukwu and others (2022e) used Scheffe's Third Degree Regression model, Scheffe's (5,3) to optimize the compressive strength of PFRC. Nwachukwu and others (2022f) applied Modified Scheffe's Third Degree Polynomial model to optimize the compressive strength of NFRC. Nwachukwu and others (2022g) applied Scheffe's Third Degree Model to optimize the compressive strength of SFRC. Nwachukwu and others (2022h) developed and use Scheffe's (6,2) Model to optimize the compressive strength of Hybrid- Polypropylene – Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2022 i) applied Scheffe's (6,2) model to optimize the Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2022j) applied Scheffe's (6,2) model in the Optimization of Compressive Strength of Hybrid Polypropylene – Nylon Fibre Reinforced Concrete (HPNFRC) .Finally, Nwachukwu and others (2022k) applied the use of Scheffe's Second Degree Polynomial Model to optimize the compressive strength of Mussel Shell Fibre Reinforced Concrete (MSFRC). Nwachukwu and others (2022 l) carried out an optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Periwinkle Shells Ash (PSA) Using Scheffe's Second Degree Model. Nwachukwu and others (2023a) applied Scheffe's Third Degree Regression Model to optimize the compressive strength of Hybrid- Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC). Finally, Nwachukwu and others (2023b) applied Scheffe's (6,3) Model in the Optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA)**.**

From the works reviewed so far, it can be envisaged that no work has been done on the use of Scheffe's Second Degree Model to optimize the flexural strength and split tensile strength of HPSFRC. Thus, there is urgent need for this present research work.

2. BACKGROUND IN SCHEFFE'S (6, 2) OPTIMIZATION THEORY

A simplex lattice is described as a structural representation of lines joining the atoms of a mixture where these atoms are constituent components of the mixture. For this present concrete mixture, the six constituent elements are, Water, Cement, Fine Aggregate, Coarse Aggregate. Polypropylene Fibre and Steel Fibre. According to Obam (2009), mixture components are usually subject to the constraint that the sum of all the components must be equal to 1 as stated in Eqn. (1)

$$X_1 + X_2 + X_3 + \ldots + X_q = 1 \; ; \quad \Rightarrow \sum_{i=1}^{q} X_i = 1 \qquad \textbf{(1)}$$

where $X_i \geq 0$ and $i = 1, 2, 3\ldots$ q, and q = the number of mixtures.

2.1. DESIGN POINTS FOR SCHEFFE'S (6,2) COMPONENT MIXTURES

The (q, m) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen mathematical equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains ${}^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \ldots, 1; \quad i = 1, 2, \ldots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degee, which in this present study is 2.

For example a (3, 2) lattice consists of ${}^{3+2-1}C_2$ i.e. ${}^4C_2$ = 6 points. Each X_i can take m+1 = 3 possible values; that is $x = 0, \frac{1}{2}, 1$ with which the possible design points are : $(1,0,0), (0,1,0), (0,0,1), \left(\frac{1}{2}, \frac{1}{2}, 0\right), \left(0, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, 0, \frac{1}{2}\right)$.
The general formula for evaluating the number of coefficients/terms/ design points required for a given lattice is always given by: $k = \frac{(q+m-1)!}{(q-1)!\,.\,m!}$ Or ${}^{q+m-1}C_m$ **2(a-b)**

Where k = number of coefficients/ terms / points, q = number of components = 6 in this study, m = number of deqree of polynomial = 2 in this present work. Using either of Eqn. (2), $k_{(6,2)} = 21$

Thus, the possible design points for Scheffe's (6,2) lattice can be as follows:

A_1 (1,0,0,0,0,0); A_2 (0,1,0,0,0,0); A_3 (0,0,1,0,0,0); A_4 (0,0,0,1,0,0), A_5 (0,0,0,0,1,0); A_6 (0, 0,0,0, 0, 1); A_{12} (0.67,0.33, 0, 0, 0, 0); A_{13} (0.67, 0, 0.33,0,0,0); A_{14} (0.67, 0, 0, 0.33,0,0); A_{15} (0.67, 0, 0, 0,0.33, 0); A_{16} (0.67, 0, 0, 0, 0, 0.33); A_{23} (0,0.50,0.50, 0,0,0); A_{24} (0, 0.50, 0, 0.50, 0,0); A_{25}, (0, 0.50, 0, 0,0.50, 0); A_{26} (0, 0.50,0,0, 0.50); A_{34} (0.50, 0.50, 0, 0,0,0); A_{35} (O.50, 0,0.50, 0,0,0); A_{36} (0.50,0, 0,0.50, 0, 0); A_{45} (0.50, 0, 0, 0,0.50, 0); A_{46}(0.50,0,0,0,0,0.50);A_{56}(0,0,0.50,0.50,0,0); **(3)**

According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable $X_1, X_2, X_3, X_4 \ldots X_q$ is given in the form of Eqn.(4): $P = b_0 + \sum bi\,xi + \sum bij xj + \sum bi\,jxjxk + + \sum bi^{\,j}{}_2 + \ldots\, i_n xi_2 xi_n$ **(4)**

where ($1 \leq i \leq q$, $1 \leq i \leq j \leq k \leq q$, $1 \leq i_1 \leq i_2 \leq \ldots \leq i_n \leq q$ respectively) , b = constant coefficients and P is the response which represents the property under investigation, which in this case is the Flexural Strength (P^F) or Split Tensile Strength (P^S) as the case may be. As this research work is based on the Scheffe's (6, 2) simplex, the actual form of Eqn. (4) for six component mixture, degree two (6, 2) has been developed by Nwachukwu and others (2022h) and will be applied subsequently in this work

2.2. PSEUDO AND ACTUAL COMPONENTS.

In Scheffe's mixture design, the relationship between the pseudo components and the actual components is given as: $Z = A * X$ **(5)**
where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging Eqn. (5) yields: $X = A^{-1} * Z$ **(6)**

2.3. FORMULATION OF MATHEMATICAL EQUATION FOR HPSFRC SCHEFFE'S (6,2) SIMPLEX LATTICE

The polynomial/Mathematical equation by Scheffe (1958), which is known as response is given in Eqn.(4) and for the Scheffe's (6,2) simplex lattice, the polynomial equation for six component mixtures has been formulated based on Eqn.(4) by the work of Nwachukwu and others (2022g) as stated under:

$P = ß_1X_1 + ß_2X_2 + ß_3X_3 + ß_4X_4 + ß_5X_5 + ß_6X_6 + ß_{12}X_1X_2 + ß_{13}X_1X_3 + ß_{14}X_1X_4 + ß_{15}X_1X_5 + ß_{16}X_1X_6 + ß_{23}X_2X_3 + ß_{24}X_2X_4 + ß_{25}X_2X_5 + ß_{26}X_2X_6 + ß_{34}X_3X_4 + ß_{35}X_3X_5 + ß_{36}X_3X_6 + ß_{45}X_4X_5ß45 + ß_{46}X_4X_6 + ß_{56}X_5X_6$ **(7)**

2.4 . COEFFICIENTS DETERMINATION OF THE HPSFRC SCHEFFE'S (6, 2) POLYNOMIAL

Based on the work of Nwachukwu and others (2022g), the coefficients of the Scheffe's (6, 2) polynomial have been evaluated as stated under. :

$\beta_1 = P_1$; $\beta_2 = P_2$; $\beta_3 = P_3$; $\beta_4 = P_4$; $\beta_5 = P_5$ and $\beta_6 = P_6$ **8(a-f)**

$\beta_{12} = 4P_{12} - 2P_1 - 2P_2$; $\beta_{13} = 4P_{13} - 2P_1 - 2P_3$; $\beta_{14} = 4P_{14} - 2P_1 - 2P_4$; **9(a-c)**

$\beta_{15} = 4P_{15} - 2P_1 - 2P_5$; $\beta_{16} = 4P_{16} - 2P_1 - 2P_6$; $\beta_{23} = 4P_{23} - 2P_2 - 2P_3$; $\beta_{24} = 4P_{24} - 2P_2 - 2P_4$; **10(a-d)**

$\beta_{25} = 4P_{25} - 2P_2 - 2P_5$; $\beta_{26} = 4P_{26} - 2P_2 - 2P_6$, $\beta_{34} = 4P_{34} - 2P_3 - 2P_4$; $\beta_{35} = 4P_{35} - 2P_3 - 2P_5$; **11(a-d)**

$\beta_{36} = 4P_{36} - 2P_3 - 2P_6$; $\beta_{45} = 4P_{46} - 2P_4 - 2P_6$, $\beta_{46} = 4P_{46} - 2P_4 - 2P_6$; $\beta_{56} = 4P_{56} - 2P_{35} - 2P_6$; **12(a-d)**

Where P_i = Response Function (Flexural Strength or Split Tensile Strength) for the pure component, i

2.5. HPSFRC SCHEFFE'S (6, 2) MIXTURE DESIGN MODEL

Substituting Eqns. (8)- (12) into Eqn. (7), we obtain the mixture design model for the **HPSFRC** mixture based on Scheffe's (6,2) lattice.

2.6. ACTUAL AND PSEUDO MIX RATIOS FOR THE HPSFRC SCHEFFE'S (6,2) DESIGN LATTICE AT INITIAL EXPERIMENTAL POINT AND CONTROL POINT

2.6.1. AT THE INITIAL EXPERIMENTAL TEST POINTS

Since the requirement of simplex lattice design based on Eqn. (1) criteria makes it impossible to use the conventional mix ratios such as 1:2:4 etc., at a given water/cement ratio for the actual mix ratio., there is need for the transformation of the actual components proportions to meet the above criterion. Based on experience previous knowledge from literature and other related work done on HPSFRC, the following arbitrary prescribed mix ratios are always chosen for the six vertices of Scheffe's (6,2) lattice when the percentage of Polypropylene Fibre and Steel Fibre mixture is **50: 50.**

A_1 (0.67:1:1.7:2:0.5:0.5); A_2 (0.56:1:1.6:1.8:0.8:0.8); A_3 (0.5:1:1.2:1.7:1:1); A_4 (0.7:1:1:1.8:1.2:1.2);

A_5 (0.75:1:1.3:1.2:1.5:1.5), and A_6 (0.80:1:1.3:1.2:0.9:0.9) **(13)**

which represent water/cement ratio, cement, fine aggregate, coarse aggregate, polypropylene fibre and steel fibre respectively.

For the pseudo mix ratio, the following corresponding mix ratios at the vertices for six component mixtures are always chosen:

A_1(1:0:0:0:0:0), A_2(0:1:0:0: 0:0), A_3(0:0:1:0:0:0), A_4(0:0:0:1:0:0), A_5(0:0:0:0:1:0) and A_6(0:0:0:0:0:1) **(14)**

For the transformation of the actual component, Z to pseudo component, X, and vice versa, Eqns. (5) and (6) are used. Substituting the mix ratios from point A_1 into Eqn. (5) yields:

$$\begin{pmatrix} 0.67 \\ 1.00 \\ 1.70 \\ 2.00 \\ 0.50 \\ 0.50 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \textbf{(15)}$$

Transforming the R.H.S matrix and solving, we obtain as follows:

$A_{11}(1) + A_{21}(0) + A_{31}(0) + A_{41}(0) + A_{51}(0) + A_{61}(0) = 0.67$. Thus , $A_{11} = 0.67$

Similarly, A_{21}= 1; A_{31}= 1.7; A_{41}= 2; A_{51}= 0.5; A_{61}= 0.5

The same approach is used to obtain the remaining values as shown in Eqn. (16)

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.50 & 0.50 & 0.75 & 0.75 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 1.70 & 1.60 & 1.20 & 1.00 & 1.30 & 1.30 \\ 2.00 & 1.80 & 1.70 & 1.80 & 1.20 & 1.20 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{pmatrix} \quad \textbf{(16)}$$

Now considering mix ratios at the mid points from Eqn.(3) and substituting these pseudo mix ratios in turn into Eqn.(16) yields the corresponding actual mix ratios. For instance, considering point A_{12} we have: A_{12} (0.67,0.33, 0, 0, 0, 0). This implies:

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.50 & 0.50 & 0.75 & 0.75 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 1.70 & 1.60 & 1.20 & 1.00 & 1.30 & 1.30 \\ 2.00 & 1.80 & 1.70 & 1.80 & 1.20 & 1.20 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \end{pmatrix} \begin{pmatrix} 0.67 \\ 0.33 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.63 \\ 1 \\ 1.67 \\ 1.90 \\ 1.60 \\ 1.60 \end{pmatrix} \quad \textbf{(17)}$$

Solving, $Z_1 = 0.63$; $Z_2 = 1.00$; $Z_3 = 1.67$’ $Z_4 = 1.90$; $Z_5 = 1.60$ and $Z_6 = 1.60$

The same approach goes for the remaining mid-point mix ratios and twenty-one (21) experimental tests tests (each for Flexural Strength and Split Tensile Strength) are needed to generate the 21 polynomial coefficients based on the corresponding mix ratios depicted in Table 1.

Table 1: Pseudo (X) and Actual (Z) Mix Ratio for HPSFRC based on Scheffe’s (6,2) Lattice At The Initial Experimental Test Points (For Flexural Strength And Split Tensile Strength).

S/N	**POINTS**	**PSEUDO COMPONENT**						**RESPONSE SYMBOL**	**ACTUAL COMPONENT**					
		X_1	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**		**Z_1**	**Z_2**	**Z_3**	**Z_4**	**Z_5**	**Z_6**
1	1	1	0	0	0	0	0	P_1	0.67	1.00	1.70	2.0	0.5	0.5
2	2	0	1	0	0	0	0	P_2	0.56	1.00	1.60	1.8	0.8	0.8
3	3	0	0	1	0	0	0	P_3	0.50	1.00	1.20	1.7	1.0	1.0
4	4	0	0	0	1	0	0	P_4	0.70	1.00	1.00	1.8	1.2	1.2
5	5	0	0	0	0	1	0	P_5	0.75	1.00	1.30	1.2	1.5	1.5
6	6	0	0	0	0	0	1	P_6	0.63	1.00	1.67	1.9	1.6	1.6
7	12	0.67	033	0	0	0	0	P_{12}	0.60	1.00	1.63	1.8	0.7	0.7
8	13	0.67	0	0.33	0	0	0	P_{13}	0.61	1.00	1.54	1.9	0.6	0.6
9	14	0.67	0	0	0.33	0	0	P_{14}	0.56	1.00	1.37	1.8	0.8	0.8
10	15	0.67	0	0	0	0.33	0	P_{15}	0.68	1.00	1.47	1.9	0.7	0.7
11	16	0.67	0	0	0	0	0.33	P_{16}	0.69	1.00	1.23	1.8	0.9	0.9
12	23	0	0.50	0.50	0	0	0	P_{23}	0.70	1.00	1.57	1.7	0.8	0.8
13	24	0	0.50	0	0.50	0	0	P_{24}	0.72	1.00	1.43	1.4	1.1	1.1

14	25	0	0.50	0	0	0.50	0	P_{25}	0.55	1.00	1.40	1.7	0.8	0.8
15	26	0	0.50	0	0	0	0.50	P_{26}	0.52	1.00	1.20	1.7	0.9	0.9
16	34	0.50	0.50	0	0	0	0	P_{34}	0.61	1.00	1.67	1.8	0.9	0.9
17	35	0.50	0	0.50	0	0	0	P_{35}	0.66	1.00	1.73	1.8	1.0	1.0
18	36	0.50	0	0	0.50	0	0	P_{36}	0.63	1.00	1.50	1.6	0.7	0.7
19	45	0.50	0	0	0	0.50	0	P_{45}	0.69	1.00	1.40	1.4	0.6	0.6
20	46	0.50	0	0	0	0	0.50	P_{46}	0.57	1.00	1.13	1.7	1.0	1.0
21	56	0	0	0.50	0.50	0	0	P_{56}	0.64	1.00	1.07	1.7	1.1	1.1

2.6.2. AT THE EXPERIMENTAL (.CONTROL) POINT

For the purpose of this research, twenty- one (21) different control test (each for Flexural Strength and Split Tensile Strength) were predicted which according to Scheffes, their summation should not be more than one. The same approach for component transformation adopted for the initial experimental points are also adopted for the control points and the results are shown in Table 2.

Table 2 : Actual and Pseudo Component of HPSFRC Based on Scheffe (6,2) Lattice for Control Points (For Flexural Strength And Split Tensile Strength).

S/N	POINTS	PSEUDO COMPONENT						CONTROL POINTS	ACTUAL COMPONENT					
		X_1	X_2	X_3	X_4	X_5	X_6		Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
1	1	0.25	0.25	0.25	0.25	0	0	C_1	0.61	1	1.38	1.83	0.5	0.50
2	2	0.25	0.25	0.25	0	0.25	0	C_2	0.62	1	1.45	1.68	0.8	0.8
3	3	0.25	0.25	0	0.25	0.25	0	C_3	0.67	1	1.40	1.70	1	1
4	4	0.25	0	0.25	0.25	0.25	0	C_4	0.66	1	1.30	1.68	1.2	1.2
5	5	0	0.25	0.25	0.25	0.25	0	C_5	0.63	1	1.28	1.63	1.5	1.5
6	6	0.20	0.20	0.20	0.20	0.20	0	C_6	0.64	1	1.36	1.70	0.65	0.65
7	12	0.30	0.30	0.30	0.10	0	0	C_{12}	0.59	1	1.45	1.83	0.75	0.75
8	13	0.30	0.30	0.30	0	0.10	0	C_{13}	0.59	1	1.48	1.77	0.85	0.85
9	14	0.30	0.30	0	0.30	0.10	0	C_{14}	0.65	1	1.42	1.80	1	1
10	15	0.30	0	0.30	0.30	0.10	0	C_{15}	0.64	1	1.30	1.77	0.9	0.9
11	16	0	0.30	0.30	0.30	0.10	0	C_{16}	0.60	1	1.27	1.71	1	1
12	23	0.10	0.30	0.30	0.30	0	0	C_{23}	0.60	1	1.31	1.79	1.55	1.55
13	24	0.30	0.10	0.30	0.30	0	0	C_{24}	0.62	1	1.33	1.83	1.1	1.1
14	25	0.30	0.10	0.30	0.30	0	0	C_{25}	0.63	1	1.41	1.85	1.25	1.25
15	26	0.10	0.20	0.30	0.40	0	0	C_{26}	0.61	1	1.25	1.79	1.35	1.35
16	34	0.30	0.20	0.10	0.40	0	0	C_{34}	0.64	1	1.35	1.85	0.89	0.89

17	35	0.20	0.20	0.10	0.10	0.40	0	C_{35}	1.40	1	1.04	1.59	1.08	1.08
18	36	0.30	0.10	0.30	0.20	0.10	0	C_{36}	0.62	1	1.36	1.77	0.92	0.92
19	45	0.25	0.25	0.15	0.15	0.20	0	C_{45}	0.61	1	1.51	3.16	0.91	0.91
20	46	0.30	0.30	0.20	0.10	0.10	0	C_{46}	0.68	1	1.56	1.96	0.98	0.98
21	56	0.10	0.30	0.30	0.30	0	0	C_{56}	1.30	1	1.31	1.79	0.95	0.95

The actual component as transformed from Eqn. (17) , Table (1) and (2) were used to measure out the quantities of water/cement ratio (Z_1), cement (Z_2), fine aggregate (Z_3), coarse aggregate (Z_4), polypropylene fibre (Z_5) and steel fibre (Z_6) in their respective ratios for the concrete cube and cylindrical specimen strength tests.

3. MATERIALS AND METHODS

3.1 MATERIALS

The constituent materials for laboratory examination in this present study are Water/Cement ratio, Cement, Fine and Coarse Aggregates as well as Polypropylene Fibre and Steel Fibre. The cement is Dangote cement, a brand of Ordinary Portland Cement, conforming to British Standard Institution BS 12 (1978). The fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite of 20mm size was obtained from a local stone market and was downgraded to 4.75mm. The same size and nature of polypropylene fibre and steel fibre used previously by Nwachukwu and others (2022c) and Nwachukwu and others (2022b) respectively are the same as the one being used in this present work. Potable water from the clean water source was sourced and used in this experimental investigation.

3.2. METHOD

3.2.1. SPECIMEN PREPARATION / BATCHING/ CURING FOR FLEXURAL STRENGTH TEST

In this work, the standard size of specimen (mould) for the Flexural Strength measures 15cm*15cm*60cm . The mould is of steel metal with sufficient thickness to prevent spreading or warping. The mould is constructed with the longer dimension horizontal and in such a manner as to facilitate the removal of the moulded specimen without damage. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 42 mix ratios were to be used to produce 84 prototype concrete cubes. Twenty- one (21) out of the 42 mix ratios were as control mix ratios to produce 42 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (12). Twenty-four (24) hours after moulding, curing commenced. Test specimens are stored in water at a temperature of 24^0 to 30^0 for 48 hours before testing. They are tested immediately on removal from the water whilst they are still in a wet condition. After 14 days and 28 days of curing respectively, the specimens were taken out of the curing tank for flexural strength determination.

3.2.2. FLEXURAL STRENGTH TEST PROCEDURE/CALCULATION

Flexural strength testing was done in accordance with BS 1881 – part 118 (1983) - Method of determination of Flexural Strength and ACI (1989) guideline. In this present study, two samples were crushed for each mix ratio. In each case, the Flexural Strength of each specimen/sample which is expressed as the Modulus of Rupture (MOR) was then calculated to the nearest 0.05 MPa using Eqn.(18)

$$MOR = \frac{PL}{bd^2} \quad \textbf{(18)}$$

where b = measured width in cm of the specimen, d = measured depth in cm of the specimen at the point of failure, where L = Length in cm of the span on which the specimen was supported and P = maximum load in kg applied to the specimen.

3.2.3. SPECIMEN PREPARATION / BATCHING/ CURING FOR SPLIT TENSILE STRENGTH TEST

The specimen for the Split Tensile Strength is Concrete Cylindrical specimen measuring diameter 150 mm and length 300 mm. They were cast with plastic fibres and the specimen was loaded for ultimate compressive load under Universal Testing Machine (UTM) for each mix. A total number of 42 mix ratios were to be used to produce 84 prototype concrete cubes. Twenty- one (21) out of the 42 mix ratios were as control mix ratios to produce 42 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (12). After 28 days of curing the specimens were taken out of the curing tank for the Split Tensile Strength determination.

3.2.4. SPLIT STRENGTH TEST PROCEDURE/CALCULATION

The cylindrical split tensile test was done using the universal testing machine in accordance with BS EN 12390-6:2009 and ASTM C 496/ C 496 M-11 (2011). Two samples were crushed for each mix ratio and each case, the Split Tensile Strength of each specimen/sample was then calculated using Eqn. (19)

$$F_t = \frac{2P}{\pi D L} \quad \textbf{(19)}$$

Where, F_t = Split Tensile Strength, MPa , P = maximum applied load (that is Load at failure, N) ; D = diameter of the cylindrical specimen (Dia. Of cylinder, mm); and L = Length of the specimen (Length of cylinder, mm),

4. RESULTS PRESENTATION AND DISCUSSION

4.1 HPSFRC RESPONSES (FLEXURAL STRENGTH) FOR THE INITIAL EXPERIMENTAL TEST

The results of the Flexural Strength (responses) test based on Eqn. (18) are shown in Table 3

Table 3: HPSFRC Flexural Strength (Response) Test Results Based on Eqn.(18)

S/N	POINTS	EXPERIMENTAL NO.	RESPONSE SYMBOL	RESPONSE P_i, MPa		$\sum P_i$		AVERAGE RESPONSE P, MPa	
				14th day Results	28th day Results	14th day Results	28th day Results	14th day Results	28th day Results
1	1	1A 1B	P_1	3.98 4.00	5.88 5.69	7.98	11.57	3.99	5.79
2	2	2A 2B	P_2	4.44 4.49	6.43 6.56	8.93	12.99	4.47	6.50
3	3	3A 3B	P_3	4.54 4.65	7.54 7.48	9.19	15.02	4.60	7.51
4	4	4A 4B	P_4	4.78 4.86	4.98 5.00	9.64	9.98	4.82	4.99
5	5	5A 5B	P_5	4.88 4.89	6.98 6.70	9.77	13.68	4.89	6.84
6	6	6A 6B	P_6	4.78 4.87	5.86 5.88	9.65	11.74	4.83	5.87
7	12	7A 7B	P_{12}	5.08 5.04	6.76 6.88	10.12	13.64	5.04	6.82
8	13	8A 8B	P_{13}	6.12 6.18	7.23 7.32	12.30	14.55	6.15	7.28
		9A		5.34	5.46	10.72	11.14	5.36	5.57

9	14	9B	P_{14}	5.38	5.68				
10	15	10A 10B	P_{15}	4.22 4.32	6.54 6.56	8.54	13.10	4.27	6.55
11	16	11A 11B	P_{16}	4.89 4.78	5.67 5.70	9.67	11.37	4.84	5.69
12	23	12A 12B	P_{23}	3.84 3.88	4.86 4.82	7.72	9.68	3.86	4.84
13	24	13A 13B	P_{24}	4.54 4.59	5.34 5.54	9.13	10.88	4.57	5.44
14	25	14A 14B	P_{25}	5.86 5.84	7.88 7.68	11.7	15.56	5.85	7.78
15	26	15A 15B	P_{26}	5.43 5.53	6.68 6.79	10.96	13.47	5.48	6.74
16	34	16A 16B	P_{34}	3.98 3.41	7.11 7.21	7.39	14.32	3.70	7.16
17	35	17A 17B	P_{35}	5.86 5.85	6.89 6.86	11.71	13.75	5.86	6.88
18	36	18A 18B	P_{36}	5.78 5.98	6.45 6.43	11.76	12.88	5.88	6.44
19	45	19A 19B	P_{45}	6.24 6.20	9.86 9.82	12.44	19.68	6.22	9.84
20	46	20A 20B	P_{46}	6.00 6.10	6.32 6.45	12.10	12.77	6.05	6.39
21	56	21A 21B	P_{56}	5.68 5.69	5.56 5.62	11.37	11.18	5.69	5.59

4.2 HPSFRC RESPONSES (SPLIT TENSILE STRENGTH) FOR THE INITIAL EXPERIMENTAL TEST

The results of the Split Tensile Strength (response) test based on Eqn. (19) are shown in Table 4

Table 4: HPSFRC Split Tensile Strength (Response) Test Results Based on Eqn.(19)

S/N	POINTS	EXPERIMENTAL NO.	RESPONSE SYMBOL	RESPONSE P_i, MPa		$\sum P_i$		AVERAGE RESPONSE P, MPa	
				14th day Results	28th day Results	14th day Results	28th day Results	14th day Results	28th day Results
1	1	1A 1B	P_1	3.45 3.54	3.44 3.48	6.99	6.92	3.50	3.46
2	2	2A 2B	P_2	3.34 3.38	3.69 3.72	6.72	7.41	3.36	3.71
3	3	3A 3B	P_3	3.45 3.51	3.94 4.01	6.96	7.95	3.48	3.98
4	4	4A 4B	P_4	3.65 3.57	4.56 4.64	7.22	9.20	3.61	4.60
5	5	5A 5B	P_5	3.43 3.46	5.34 5.56	6.89	10.90	3.45	5.45
		6A		3.54	4.78	7.10	9.64	3.55	4.82

6	6	6B	P_6	3.56	4.86				
7	12	7A 7B	P_{12}	4.00 4.03	5.68 5.65	8.03	11.33	4.02	5.67
8	13	8A 8B	P_{13}	3.78 3.84	5.67 5.48	7.62	11.15	3.81	5.58
9	14	9A 9B	P_{14}	3.67 3.76	3.56 3.66	7.43	7.22	3.72	3.61
10	15	10A 10B	P_{15}	3.86 3.84	4.54 4.65	7.70	9.10	3.85	4.55
11	16	11A 11B	P_{16}	4.12 4.16	4.34 4.42	8.28	8.76	4.14	4.38
12	23	12A 12B	P_{23}	3.00 3.04	3.40 3.36	6.04	6.76	3.02	3.38
13	24	13A 13B	P_{24}	4.05 4.06	4.12 4.21	8.11	8.33	4.06	4.17
14	25	14A 14B	P_{25}	4.21 4.19	4.23 4.30	8.40	8.53	4.20	4.27
15	26	15A 15B	P_{26}	3.98 4.00	5.76 5.68	7.98	11.44	3.99	5.72
16	34	16A 16B	P_{34}	3.54 3.58	4.53 4.54	7.12	9.07	3.56	4.54
17	35	17A 17B	P_{35}	3.98 3.86	4.79 4.81	7.84	9.60	3.92	4.80
18	36	18A 18B	P_{36}	3.76 3.74	5.65 5.76	7.50	11.41	3.75	5.71
19	45	19A 19B	P_{45}	4.40 4.36	6.06 6.10	8.76	12.16	4.38	6.08
20	46	20A 20B	P_{46}	4.34 4.32	5.47 5.53	8.66	11.00	4.33	5.50
21	56	21A 21B	P_{56}	4.11 4.14	5.45 5.48	8.25	10.93	4.13	5.47

4.3. HPSFRC RESPONSES (FLEXURAL STRENGHT) FOR THE EXPERIMENTAL (CONTROL) TEST POINTS

The response (Flexural strength) from experimental (control) tests is shown in Table 5.

Table 5: HPSFRC Response (Flexural strength) of Control Points from Experimental (control) Tests

S/N	POINTS	EXPEIMENTAL NO.	RESPONSE MPa		Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	AVERAGE RESPONSE, MPa	
			14th day Results	28th day Results							14th day Results	28th day Results

1	C_1	1A 1B	3.67 3.76	5.81 5.76	0.61	1	1.38	1.83	0.5	0.50	3.72	5.79
2	C_2	2A 2B	4.49 4.34	6.35 6.45	0.62	1	1.45	1.68	0.8	0.8	4.42	6.40
3	C_3	3A 3B	4.51 4.54	7.50 7.51	0.67	1	1.40	1.70	1	1	4.53	7.51
4	C_4	4A 4B	4.72 4.81	4.90 5.04	0.66	1	1.30	1.68	1.2	1.2	4.77	4.97
5	C_5	5A 5B	4.86 4.82	6.90 6.92	0.63	1	1.28	1.63	1.5	1.5	4.84	6.91
6	C_6	6A 6B	4.72 4.80	5.80 5.81	0.64	1	1.36	1.70	0.65	0.65	4.76	5.81
7	C_{12}	7A 7B	5.00 5.01	6.72 6.73	0.59	1	1.45	1.83	0.75	0.75	5.01	6.73
8	C_{13}	8A 8B	6.18 6.12	7.22 7.21	0.59	1	1.48	1.77	0.85	0.85	6.15	7.22
9	C_{14}	9A 9B	5.32 5.34	5.40 5.42	0.65	1	1.42	1.80	1	1	5.33	5.41
10	C_{15}	10A 10B	4.19 4.17	6.51 6.51	0.64	1	1.30	1.77	0.9	0.9	4.18	6.51
11	C_{16}	11A 11B	4.81 4.82	5.60 5.62	0.60	1	1.27	1.71	1	1	4.82	5.61
12	C_{23}	12A 12B	3.80 3.80	4.80 4.79	0.60	1	1.31	1.79	1.55	1.55	3.80	4.80
13	C_{24}	13A 13B	4.49 4.46	5.31 5.34	0.62	1	1.33	1.83	1.1	1.1	4.48	5.33
14	C_{25}	14A 14B	5.80 5.81	7.81 7.82	0.63	1	1.41	1.85	1.25	1.25	5.81	7.82
15	C_{26}	15A 15B	5.38 5.42	6.61 6.62	0.61	1	1.25	1.79	1.35	1.35	5.40	6.62
16	C_{34}	16A 16B	3.78 3.86	7.32 7.34	0.64	1	1.35	1.85	0.89	0.89	3.82	7.33
17	C_{35}	17A 17B	5.81 5.81	6.80 6.80	1.40	1	1.04	1.59	1.08	1.08	5.81	6.80
18	C_{36}	18A 18B	5.72 5.75	6.53 6.49	0.62	1	1.36	1.77	0.92	0.92	5.74	6.72
19	C_{45}	19A 19B	6.20 6.18	9.54 9.63	0.61	1	1.51	3.16	0.91	0.91	6.19	9.59
20	C_{46}	20A	6.08	6.27	0.68	1	1.56	1.96	0.98	0.98	6.06	6.31

		20B	6.03	6.34								
21	C_{56}	21A 21B	5.61 5.62	5.51 5.54	1.30	1	1.31	1.79	0.95	0.95	5.62	5.53

4.4. HPSFRC RESPONSES (SPLIT TENSILE STRENGHT) FOR THE EXPERIMENTAL (CONTROL) TEST POINTS

The response (Split Tensile Strength) from experimental (control) tests is shown in Table 6.

Table 6: HPSFRC Response (Split Tensile Strength) of Control Points from Experimental (control) Tests

S/N	POINTS	EXPERIMENTAL NO	RESPONSE MPa		Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	AVERAGE RESPONSE, MPa	
			14th day Results	28th day Results							14th day Results	28th day Results
1	C_1	1A 1B	3.40 3.51	3.41 3.42	0.61	1	1.38	1.83	0.5	0.50	3.46	3.42
2	C_2	2A 2B	3.31 3.33	3.62 3.70	0.62	1	1.45	1.68	0.8	0.8	3.32	3.66
3	C_3	3A 3B	3.42 3.48	3.88 4.25	0.67	1	1.40	1.70	1	1	3.46	4.07
4	C_4	4A 4B	3.60 3.51	4.50 4.54	0.66	1	1.30	1.68	1.2	1.2	3.56	4.54
5	C_5	5A 5B	3.41 3.40	5.30 5.42	0.63	1	1.28	1.63	1.5	1.5	3.41	5.36
6	C_6	6A 6B	3.51 3.52	4.70 4.74	0.64	1	1.36	1.70	0.65	0.65	3.52	4.72
7	C_{12}	7A 7B	4.00 4.01	5.60 5.60	0.59	1	1.45	1.83	0.75	0.75	.4.01	5.60
8	C_{13}	8A 8B	3.70 3.81	5.61 5.53	0.59	1	1.48	1.77	0.85	0.85	3.76	5.57
9	C_{14}	9A 9B	3.61 3.71	3.50 3.60	0.65	1	1.42	1.80	1	1	3.66	3.55
10	C_{15}	10A 10B	3.80 3.82	4.50 4.54	0.64	1	1.30	1.77	0.9	0.9	3.81	4.52
11	C_{16}	11A 11B	4.09 4.11	4.43 4.39	0.60	1	1.27	1.71	1	1	4.10	4.41
12	C_{23}	12A 12B	3.00 3.09	3.45 3.43	0.60	1	1.31	1.79	1.55	1.55	3.05	3.44
13	C_{24}	13A 13B	4.09 4.03	4.23 4.42	0.62	1	1.33	1.83	1.1	1.1	4.06	4.33
14	C_{25}	14A 14B	4.20 4.12	4.43 4.45	0.63	1	1.41	1.85	1.25	1.25	4.16	4.44

15	C_{26}	15A 15B	3.92 4.00	5.74 5.73	0.61	1	1.25	1.79	1.35	1.35	3.96	5.74
16	C_{34}	16A 16B	3.50 3.51	4.43 4.34	0.64	1	1.35	1.85	0.89	0.89	3.51	4.39
17	C_{35}	17A 17B	3.88 3.81	4.70 4.72	1.40	1	1.04	1.59	1.08	1.08	3.85	4.71
18	C_{36}	18A 18B	3.70 3.72	5.61 5.53	0.62	1	1.36	1.77	0.92	0.92	3.71	5.57
19	C_{45}	19A 19B	4.39 4.31	6.00 6.08	0.61	1	1.51	3.16	0.91	0.91	4.35	6.04
20	C_{46}	20A 20B	4.30 4.31	5.40 5.39	0.68	1	1.56	1.96	0.98	0.98	4.31	5.40
21	C_{56}	21A 21B	4.09 4.13	5.41 5.40	1.30	1	1.31	1.79	0.95	0.95	4.11	5.41

4.5. SCHEFFE' S (6,2) POLYNOMIAL MODEL FOR THE HPSFRC RESPONSES (FLEXURAL STRENGHT AND SPLIT TENSILE STRENGHT).

A. FLEXURAL STRENGHT

By substituting the values of the flexural strengths (responses) from Table 3 into Eqns.(8) through (10), we obtain the coefficients (β_1 , β_2 ... β_{34} ,β_{35}…. B_{56}) of the Scheffe's second degree polynomial for HPSFRC Substituting the values of these coefficients into Eqn. (7) yields the polynomial model for the optimization of the flexural strength of HPSFRC (at 14th day or 28th day) based on Scheffe's (6,2) lattice as given under:

$$P^F = ß_1X_1 + ß_2X_2 + ß_3X_3 + ß_4X_4 + ß_5X_5 + ß_6X_6 + ß_{12}X_1X_2 + ß_{13}X_1X_3 + ß_{14}X_1X_4 + ß_{15}X_1X_5 + ß_{16}X_1X_6 + ß_{23}X_2X_3 + ß_{24}X_2X_4 + ß_{25}X_2X_5 + ß_{26}X_2X_6 + ß_{34}X_3X_4 + ß_{35}X_3X_5 + ß_{36}X_3X_6 + ß_{45}X_4X_5ß45 + ß_{46}X_4X_6 + ß_{56}X_5X_6 \quad (20)$$

B. SPLIT TENSILE STRENGHT

By substituting the values of the split tensile strengths (responses) from Table 4 into Eqns.(8) through (10), we obtain the coefficients (β_1 , β_2 ... β_{34} ,β_{35}…. B_{56}) of the Scheffe's Second degree polynomial for HPSFRC. Substituting the values of these coefficients into Eqn. (7) yield the polynomial model for the optimization of the split tensile strength of HPSFRC (at 14th day or 28th day) based on Scheffe's (6,2) lattice as given under:

$$P^S = ß_1X_1 + ß_2X_2 + ß_3X_3 + ß_4X_4 + ß_5X_5 + ß_6X_6 + ß_{12}X_1X_2 + ß_{13}X_1X_3 + ß_{14}X_1X_4 + ß_{15}X_1X_5 + ß_{16}X_1X_6 + ß_{23}X_2X_3 + ß_{24}X_2X_4 + ß_{25}X_2X_5 + ß_{26}X_2X_6 + ß_{34}X_3X_4 + ß_{35}X_3X_5 + ß_{36}X_3X_6 + ß_{45}X_4X_5ß45 + ß_{46}X_4X_6 + ß_{56}X_5X_6 \quad (21)$$

4.6. SCHEFFE'S (6,2) MODEL RESPONSES (FLEXURAL STRENGHT AND SPLIT TENSILE STRENGHT) FOR HPSFRC AT CONTROL POINTS.

A. FLEXURAL STRENGHT

By substituting the pseudo mix ratio of points C_1, C_2, C_3, C_4, C_5, … C_{56} of Table 5 into Eqn.(21), we obtain the Scheffe's second degree model responses (flexural strength) for the control points of HPSFRC

B. SPLIT TENSILE STRENGHT

By substituting the pseudo mix ratio of points C_1, C_2, C_3, C_4, C_5, … C_{56} of Table 6 into Eqn.(21), we obtain the second degree model responses (split tensile strength) for the control points of HPSFRC

4.7. VALIDATION AND TEST OF ADEQUACY OF HPSFRC MODEL RESULTS (FOR FLEXURAL STRENGHT AND SPLIT TENSILE STRENGHT) USING STUDENT'S – T -TEST

The test of adequacy is performed here in order to know if there is a correlation between the flexural and split tensile strengths results (lab responses) given in Tables 5 and 6 and model responses from the control points based on Eqns.(21 and 22). By using the Student's – T – test as the means of validation, the result shows that there are no significant differences between the experimental results and model responses. The procedures/steps involved in using the Student's – T - test have been explained by Nwachukwu and others (2022 c). Therefore, the models are adequate for predicting the flexural and split tensile strengths of HPSFRC based on Scheffe's (6,2) simplex lattice.

4.8. RESULTS DISCUSSION

The results show that the maximum flexural strengths of HPSFRC based on Scheffe's (6,2) lattice are **9.84** MPa and **6.22** MPa respectively for 28th and 14th day results. Similarly the maximum split tensile strengths of HPSFRC based on Scheffe's (6,2) lattice are **6.08** MPa and **4.38**MPa respectively for 28th and 14th day results .The corresponding optimum mix ratio is **0.69:1.00: 1.40:1.4:0.6:0.6**for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate , Polypropylene Fibre and Steel Fibre respectively. The minimum flexural strength and split tensile strength are **4..84** MPa, **3.86**MPa, **3.38**MPa and **3.02**MPa respectively for the 28th day and 14th day results. The minimum values correspond to the mix ratio of **0.70: 1.00:1.57:1.7:0.8:0.8** for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate , Polypropylene Fibre and Steel Fibre respectively. Thus, the Scheffe's model can be used to determine the HPSFRC flexural and spilt tensile strength of all points (1 - 56) in the simplex based on Scheffe's Second Degree Model for six component mixture.

5. CONCLUSION

So far, Scheffe's Second Degree Polynomial (6,2) has been presented and used to formulate a model for predicting the flexural and split tensile strengths of HPSFRC. Firstly, the Scheffe's model was used to predict the mix ratio for predicting both flexural and split tensile strengths of HPSFRC Through the use of Scheffe's (6,2) simplex model, the values of both strengths were determined at all 21 points (1- 56). The results of the student's t-test validated the strengths predicted by the models and the corresponding experimentally observed results. The optimum attainable strengths predicted by the model based on Scheffe's (6,2) model are as stated in the results discussion session, likewise the minimum values. Thus, with the Scheffe's (6,2) model, any desired strength, given any mix ratio can be easily predicted and evaluated and vice versa. Thus, the application of this Scheffe's optimization model has reduced the problem of having to go through vigorous, time-consuming and laborious empirical mixture design procedures in order to obtain the desired strengths of HPSFRC mixture.

REFERENCES

[1] ACI Committee 211. (1994): "Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete (ACI 211.1R-91)"; American Concrete Institute, ACI Manual of Concrete Practice, Detroit, Michigan, USA.

[2] ACI Committee 544. (1982): "State-of-the-Report on Fibre Reinforced Concrete, (ACI 544.1R-82)"; *Concrete International: Design and Construction*. Vol. 4, No. 5: Pp. 9-30, American Concrete Institute, Detroit, Michigan, USA.

[3] ACI Committee 544. (1989): "Measurement of Properties of Fibre Reinforced Concrete, (ACI 544.2R-89)";American Concrete Institute, Detroit, Michigan, USA.

[4] Aggarwal, M.L. (2002): "Mixture Experiments: Design Workshop Lecture Notes", University of Delhi, India.

[5]. Arube, G.E, Akusobo, I.S and Damini, R.G. (2021): "Analyzing the Effects of Steel Fibres in Concrete Paving Blocks" International Journal of Civil Engineering Construction and Estate Management, Vol.9, No.2, Pp.38- 51.

[6] ASTM C496/C 496 M-11 (2011), Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimen , American Society for Testing and Material, West Conshohocken, PA, USA.

[7]. Baros, J.A.O.., Cunha, V.M., Ribeiro, A.F. and Antunes, J.A.B. (2005):"Post- Cracking Behaviour of Steel Fibre Reinforced Concrete"; Journal of Materials and Structures , Vol. 38. Pp 47-56.

[8] Bayasi, Z., and Zeng, J. (1993):"Properties of Polypropylene Fibre Reinforced Concrete "; ACI Materials Journal.

[9]. Blaszczynski, T, and Przybylska-Falek, M. (2015):"Steel Fibre Reinforced Concrete As a Structural Material "; Procedia Engineering, Science Direct, Vol. 122. Pp 282-289.

[10]British/European code (BS EN 12390-6) 2009.Testing concrete - Split tensile strength of test specimens . BSI -London.

[11] British Standards Institution, BS 12 (1978): Ordinary and Rapid – Hardening Portland cement; London.

[12]British Standards Institution, BS 1881-Part 118 (1983). Methods of Determination of Flexural Strength of Concrete Cube, London .

[13] Department of Environment, DOE(1988): Design of Normal Concrete Mixes , HMSO, London.

[14] Ezeh, J.C.& Ibearugbulam, O.M. (2009):"Application of Scheffe's Model in Optimization of Compressive Cube Strength of River Stone Aggregate Concrete";*International Journal of Natural and Appllied Sciences;*Vol. 5, No. 4,Pp 303 – 308 .

[15] Ezeh, J.C., Ibearugbulam, O.M.& Anyaogu, L. (2010a):"Optimization of Compressive Strength of Cement-Sawdust Ash Sandcrete Block using Scheffe's Mathematical Model"; *Journal of Applied Engineering Research.* Vol.4, No.4, Pp 487–494.

[16] Ezeh, J.C., Ibearugbulam, O.M.&Anya, U. C (2010b):"Optimization of aggregate composition of laterite/sand hollow Block Using Scheffe's Simplex Method";*International Journal of Engineering.*Vol.4, No.4, Pp 471 – 478.

[17]. Ghaffar,, .A.., Chavhan, A.S., and Tatwawadi, R.S. (2014):"Steel Fibre Reinforced Concrete"; International Journal of Engineering Trends And Technology (IJETT), Vol..9., No. 15, Pp 791-797.

[18]. Huang, C., Cunha, and Zhao, G. (1995):"Properties of Steel Fibre Reinforced Concrete Containing Larger Coarse Aggregate"; Science Direct.

[**19]** Hughes, B.P. (197I): The Economic Utilization of Concrete Materials, Proceedings of the Symposium on Advances in Concrete , Concrete Society, London

[20] Ibearugbulam, O.M. (2006):"Mathematical Model for Optimization of Compressive Strength of Periwinkle Shell-Granite Aggregate Concrete";*M.Eng.. Thesis,* Federal University of Technology, Owerri, Nigeria.

[21] Jackson,N. and Dhir, R.K. (1996): Civil Engineering Materials, 5th Edition, Palgrave, New York.

[22]. Jean-Louis, G. and Sana, U.B. (2005):"Corrosion of Steel Fibre Reinforced Concrete From The Crack"; Journal of Cement and Concrete Research , Vol. 35. Pp 572 -577.

[23]Kayalvizhi, M. Sasikala, K. and Vijaya Pandi, A.(2019): Test on the Behaviour of Hybrid Fiber Reinforced Concrete ; SSRG International Journal of Civil Engineering (SSRG – IJCE) – Special Issue ICTER , Available Online at www.internationaljournalssrg.org Page 56

[24]. khaloo, A. R. and Afshari, M. (2005):" Flexural Behaviour of Small Steel Fibre Reinforced Concrete Slabs "; Science Direct, Vol. 27, No.1. Pp 141 -149.

[25]. Lau, A.and Anson, M. (2006):"Effect of High Temperatures on High Performance Steel Fibre Reinforced Concrete"; Science Direct, Vol. 36, No.9, Pp 1698 -1707.

[26]. Lie, T.T. and Kodur, V.K.R. (1996):"Thermal and Mechanical Properties of Steel Fibre Reinforced Concrete At Elevated Temperatures ";Canadian Journal of Civil Engineering , Vol. 23, No. 4, Pp 511 -517.

[27]. Lima, D.H. and Oh, B.H. (1999):"Experimental and Theoretical Investigation On The Shear Of Steel Fibre Reinforced Concrete Beam "; Journal of Engineering Structures , Vol. 21, No. 10, Pp 937 -944.

[28] Neville, A.M. (1990): Properties of Concrete; 3rd edition, Pitman, England.

[29] Nwachukwu,, K. C., Njoku, ,K.O., Okorie ,P. O, .Akosubo,, I.S. , Uzoukwu , C. S.,Ihemegbulem, E.O.and .Igbojiaku , A.U (2022a):" Comparison Between Scheffe's Second Degree (5,2) And Third Degree (5,3) Polynomial Models In The Optimization Of Compressive Strength Of Glass Fibre Reinforced Concrete (GFRC)"; American Journal of Computing and Engineering (AJCE), Vol. 5, No. 1, Pp 1– 23.

[30] Nwachukwu, K.C., Okafor, M. ,Thomas, B., Oputa, A.A., Okodugha,D.A. and Osaigbovo, M.E. (2017): "An Improved Model For The Optimization Of The Compressive Strength Of Glass Fibre Reinforced Concrete

(GFRC) Using Scheffe's Second Degree Polynomials"; Researchjournal's Journal of Civil Engineering, Vol. 3, No. 2

[31] Nwachukwu, K.C.,Okodugha, D.A., Uzoukwu , C. S.,Igbojiaku, A.U. and Okafor, M. (2022b): "Application Of Scheffe's Second Degree Mathematical Model For The Optimization Of Compressive Strength Of Steel Fibre Reinforced Concrete (SFRC)"; International Journal of Advances in Engineering and Management (IJAEM) , Vol. 4, No. 3.

[32] Nwachukwu, K.C., Okorie, P.O.. Ihemegbulem , E.O., Kemebiye Dimaro , Uzoukwu , C. S., and Maduagwu, J.C.. (2022c): "The Use Of Scheffe's Model For The Optimization Of Compressive Strength Of Polypropylene Fibre Reinforced Concrete (PFRC)"; International Journal of Innovative Science and Research Technology (IJISRT), Vol. 7, No. 1

[33] Nwachukwu, K.C.,Ozioko, H.O., Okorie, P.O, and Uzoukwu , C. S. (2022d): "Application Of Scheffe's Mathematical Model For The Optimization Of Compressive Strength Of Nylon Fibre Reinforced Concrete (NFRC)"; International Journal of Advances in Engineering and Management (IJAEM) , Vol. 4, No. 2, Pp 850-862

[34] Nwachukwu, K. C., Okodugha, D.A., Igbojiaku , A.U. , Attah, U.G. And Ihemegbulem, E.O. (2022e): Application Of Scheffe's Third Degree Regression Model For The Optimization Of Compressive Strength Of Polypropylene Fibre Reinforced Concrete (PFRC); International Journal Of Advance Research And Innovative Ideas In Education (IJARIIE) , Vol. 8, No. 2.

[35] Nwachukwu, K. C., Okodugha,, D.A., Edike, O., Okafor, M.. And Attah, U.G. (2022f): Application Of Modified Scheffe's Third Degree Polynomial Model For The Optimization Of Compressive Strength Of Nylon Fibre Reinforced Concrete (NFRC); International Journal of Trend In Scientific Research And Development (IJTSRD) , Vol. 6, No. 3.

[36] Nwachukwu, K.C., Okodugha, D.A, Akosubo, I.S. and Atulomah, F.K (2022g): Optimization of Compressive Strength of Steel Fibre Reinforced Concrete (SFRC) Using Scheffe's Third-Degree Regression Model; Iconic Research And Engineering Journals (IRE), Vol. 5, No. 11.

[37] Nwachukwu, K.C**.,**Onwuegbuchulem,U.C., Atulomah' ,F.K. and Ihemegbulem, E.O.(2022h): Optimization of Hybrid- Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC) ; Iconic Research And Engineering Journals (IRE), Vol. 5, No. 12

[38] Nwachukwu, K.C**.,**Thomas , B., Oguaghamba, O. ,Onwuegbuchulem, U.C., Igbojiaku, A.U., Atulomah, F.K and Okafor. M..(2022i): Optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA) Using Scheffe's Second Degree Model; international Journal of Engineering Inventions (IJEI), Vol.11 , No.8,Pp.40-50.

[39] Nwachukwu, K.C , Oguaghamba, O, Akosubo, I.S., Atulomah, F.K. and Igbojiaku, A.U. (2022j): Optimization of Compressive Strength of Hybrid Polypropylene – Nylon Fibre Reinforced Concrete(HPNFRC); International Journal Of Advanced Research And Innovative Ideas In Education (IJARIIE), Vol. 8, No. 5. Pp. 376 -392.

[40] Nwachukwu, K.C.,.Edike, O. Oguaghamba, O., Onwuegbuchulem, U.C and Egbulonu, B.A. (2022k): The Application Of Scheffe's Model For The Optimization Of Mussel Shells Fibre Reinforced Concrete(MSFRC); International Journal of Engineering Inventions (IJEI), Vol. 11, No. 9. Pp. 47- 57

[41]. Nwachukwu,K.C., Ozioko,H.O., Akosubo, I.S.,Oguaghamba, O.,Okafor, M.,Onyechere, I.C. and Ikpa, P.N (2022l): Optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Periwinkle Shells Ash (PSA) Using Scheffe's Second Degree Model; Paper presented at the Proceedings of the 1ST International Conference on Advances in cement and concrete research(ICACCR-2022) Organized by Ladoke Akintola University of Technology (LAUTECH), Nigeria. Theme: Cement and Concrete Innovative Materials For Sustainable Development.

[42] Nwachukwu, K.C., Ozioko, H. O., Mama, B.O., Oguaghamba, O., and Atulomah, F.K. (2023a): The Application of Scheffe's Third Degree Regression Model In The Optimization Of Compressive Strength of Hybrid- Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC); International Journal of Research and Analytical Reviews (IJRAR), Vol. , No.. Pp.

[43]. Nwachukwu, K.C., Oguaghamba, O., Ozioko, H. O. and Mama, B.O. (2023b): Optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA) Using Scheffe's (6,3) Model; International Journal of Trend In Scientific Research and Development (IJTSRD), Vol.7 , No.2, Pp. 737-754

[44] Nwaklong, P, Chitanurak , J, Jongvivatsakul, P, Pansuk, W, Lenwa, A and Likithesuange, S (2020): Effect Of Hybrid Polypropylene –Steel Fibers On Strength Characteristics Of UHPFRC; Advances in Concrete Construction, Vol. 10, No. 1, Pp. 1-11.

[45] Nwakonobi, T.U and Osadebe, N.N (2008):"Optimization Model for Mix Proportioning of Clay-Ricehusk- Cement Mixture for Animal Buildings"; *Agricultural Engineering International:the CIGREjournal,Manuscript BC 08 007*, Vol x

[46] Obam, S.O. (2006). The Accuracy of Scheffe's Third Degree over Second Degree Optimization Regression Polynomials, Nigerian Journal of Technology, Vol. 2, No.25, Pp 1 – 10.

[47] Obam, S.O.(2009):"A Mathematical model for Optimization of Strength of concrete : A case study forshear modulus of Rice Husk Ash Concrete." *Journal of Industrial Engineering International l;* Vol.5, No.9,Pp 76 – 84.

[48] Okere, C.E., (2006):"Mathematical Models for Optimization of the Modulus of rupture of concrete"; M.Eng. Thesis, Civil Engineering Department, Federal University of Technology, Owerri.

[49] Onwuka,D.O, Okere, C.E.,Arimanwa, J.I. and Onwuka, S.U.(2011):"Prediction of Concrete Mix ratios using Modified Regression Theory, *Computer Methods in Civil Engineering*, Vol. 2. No.1 Pp.95- 107.

[50] Oyenuga, V.O. (2008): Simplified Reinforced Concrete Design, 2nd Edition, Asros Ltd, Lagos, Nigeria.

[51] Patel, P.A., Desai, A.K. .& Desai, J. A (2012):"Evaluation of Engineering Properties for Polypropylene Fibre Reinforced Concrete ";International Journal of Advanced Engineering Technology(IJAET) Vol.3, No.1, Pp 42 – 45.

[52]Qian, C.X. and Stroeven, P. (2000): Development Of Hybrid Polypropylene-Steel Fibre-Reinforced Concrete; Elsevier Science, Vol. 30, No. 1, Pp. 61-69.

[53] Scheffe, H. (1958):Experiment with Mixtures"; *International Journal of Royal Statistics Society*, Series B, Vol.20, Pp. 344-360.

[54] Shetty, M.S. (2006): Concrete Technology (Theory and Practice), S. Chand and Company Ltd, India.

[55] Syal, I.C. and Goel, A.K. (2007): Reinforced Concrete Structures, S.Chand , India.

[56]Varma, S.M.S.and Raji, P.M. (2019): Mechanical Properties of Hybrid Polypropylene –Steel Fiber Reinforced Concrete Composite; International Conference on Sustainable Systems and Structures, IOP Publishing.

[57]Yew, M.K., Othman, I. , Yew, M.C., Yeo, S.H and Mahmud, H.B.(2011): Strength properties of hybrid nylon steel and polypropylene-steel fibre-reinforced high strength concrete at low volume fraction; International Journal of the Physical Sciences Vol. 6, No. 33, Pp. 7584 – 7588., Available online at http://www.academicjournals.org/IJPS.