Optimization of Compressive Strength of Hybrid Polypropylene – Nylon Fibre Reinforced Concrete (HPNFRC)

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ABSTRACT

It is an established fact that reinforcement of concrete with a single type of fiber may likely improve the desired properties to a limited level. In other to overcome this shortcoming, a composite fibre , otherwise referred as as hybrid (for two or more types of fibers) are rationally combined to produce a composite that derives benefits from each of the individual fibers and also exhibits a synergetic response, such as increase in the compressive strength of the new composite. Thus this research work is focused at using Scheffe's Second Degree Polynomial Model for six component mixtures, Scheffe's (6,2) to optimize the compressive strength of Hybrid – Polypropylene – Nylon Fibre Reinforced Concrete (HPNFRC). Using , Scheffe's (6,2) simplex model introduced by Nwachukwu and others (2022g) , the compressive strengths of HPNFRC were obtained for different mix proportions/ratio. The mix proportion of Polypropylene – Nylon was in 50% - 50% ratio. Control experiments were also carried out, leading to the evaluation of the compressive strengths at the experimental control points. Through the use of the Student's t-test statistics, the adequacy of the model was validated .The optimum (maximum) compressive strength of HPNFRC was 60.05 MPa . This maximum value is higher than the minimum value specified by the American Concrete Institute (ACI), as 20 MPa as well as the minimum value specified by ASTM C 469, as 30.75 for good concrete. Thus, the HPSFRC compressive strength value can sustain construction of ground –level application and basement foundation as well as supporting both commercial and industrial construction works as high performance concrete at the best possible economic, aesthetic and safety advantages.

Keywords: HPNFRC, Scheffe's (6,2) Polynomial Model, Compressive Strength, Mixture Design

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1.INTRODUCTION

In general, an optimization problem is one requiring the determination of the optimal (maximum or minimum) value of a given function, called the objective function, subject to a set of stated restrictions, limitations, boundaries or constraints placed on the concerned variables. It is important to note every optimization problem requires an objective (called an objective function) which might be to maximize profit or benefit, to minimize cost or to minimize the use of material resources. Specifically, optimization of the concrete mixture design is a process of search for a mixture for which the sum of the costs of the ingredients is lowest, yet satisfying the required performance of concrete, such as strength, workability and durability etc. According to Shacklock (1974), the objective of mix design is to determine the most appropriate proportions in which to use the constituent materials to meet the needs of construction work On the account of the widely varying properties of the constituent materials, the conditions that prevail at the site of work, the exposure condition, and the conditions that are demanded for a particular work for which the mix is designed, the design of concrete mix according to (Shetty, 2006) has not being a simple task. By definition, concrete mix design according to Jackson and Dhir (1996) is the procedure which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. Following the above definition, the cost of any concrete includes, in addition to that of the materials themselves, the cost of the mix design, of batching, mixing and placing the concrete and of the site supervision. In the context of the above guidelines, the mix design methods and procedures proposed by Hughes (1971), ACI-211(1994) and DOE (1988) appeared to be more complex and time consuming as they involve a lot of trial mixes and complex statistical calculations before the desired strength of the concrete can be reached. Therefore, optimization of the concrete mixture design remains the fastest method, best option, most convenient and the most efficient way of selecting concrete mix ratios /proportions for better efficiency and better performance of concrete when compared with usual empirical methods as listed above. An example of optimization model is Scheffe's Polynomial /Mathematical/Regression Model.it could be in the form of Scheffe's Second Degree Model or Scheffe's Third Degree Model. Thus, in this present study, Scheffe's Second Degree Model for six components mixtures (namely cement, fine aggregate, coarse aggregate, water, polypropylene fibre and nylon fibre) will be in focus.

By definition, concrete according to Oyenuga (2008) is a composite inert material comprising of a binder course (cement), mineral filter or aggregates and water. Concrete which is classified as the most widely used construction material has been undergoing changes both as a material and due to technological advancement.. Concrete, being a homogeneous mixture of cement, sand, gravel and water is very strong in carrying compressive forces and hence is gaining increasing importance as building materials throughout the world (Syal and Goel, 2007). Concrete, according to Neville (1990), plays an important part in all building structures owing to its numerous advantages that ranges from low built in fire resistance, high compressive strength to low maintenance. However, according to Shetty (2006) , plain concrete possesses a very low tensile strength, limited ductility and little resistance to cracking. That is, unreinforced (plain) concrete is brittle in nature, and is characterized by low tensile strength but high compressive strength. As a result of this situation, there have been continuous search for the upgrading of the concrete properties. In line with this, attempts have been made in the past to improve the tensile properties of concrete members by way of using conventional reinforced steel bars. Although both these methods provide tensile strength to the concrete members, they however, do not increase the inherent tensile strength of concrete itself. Sequel to further researches and recent developments in concrete technology, it has been established that the addition of fibres (either as glass fibre, polypropylene fibre, nylon fibre, steel fibre , plastic fibre, asbestor (mineral fibre), or carbon fibres , etc.) to concrete would act as crack arrester and would substantially improve its static as well as dynamic properties. This type of concrete is known as Fibre reinforced concrete (FRC). FRC is a composite material consisting of mixtures of cement, mortar or concrete and discontinuous, discrete, uniformly dispersed. . Combining fibres with concrete can produce a range of materials which possess enhanced tensile strength, elasticity, toughness, and durability etc. This is accomplished by limiting or controlling the start, spread, or spread persistence of cracks. Hybrid Fibre Reinforced Concrete (HFRC) is the use of two or more fibres in a single concrete mixture matrix with the aim of improving its overall properties Hybrid – Polypropylene - Nylon Fibre Reinforced Concrete (HPNFRC) is concrete mixture where the conventionally steel reinforcement in concrete production is replaced (wholly or partially) with polypropylene fibre and nylon fibre. Before now, works on optimization of compressive strength of PFRC and NFRC have been carryout out.

The major aim of engineering design is to ensure that the structure being designed will not reach a Serviceability Limit State (SLS), which is connected with deflection, cracking, vibration etc, and Ultimate Limit State (ULS), which is generally connected with collapse (Ettu, 2001). In all of the above, the concrete's compressive strength is one of the most important properties of concrete that require close examination because of its important role. Compressive strength of concrete is the strength of hardened concrete measured by the compression test. It is a measure of the concrete's ability to resist loads which tend to compress it. It is measured by crushing cylindrical concrete specimens in a universal testing machine (UTM). Further, the compressive strength of the concrete cube test also provides an idea about all the characteristics of concrete under examination.

This recent work examines the use of Scheffe's Second Degree Polynomial Model in optimizing the compressive strength of HPNFRC. Before now, a lot of researchers have done related works on polypropylene fibre as well as nylon fibre, but none has been able to address the subject matter sufficiently. For instance, on PFRC and HFRC , MK-Yew and others (2011) investigated the Strength Properties of Hybrid Nylon- Steel and Polypropylene –Steel Fiber-Reinforced High Strength Concrete. Bayasi and Zeng (1993) and Patel and others (2012) have investigated the properties of PFRC. Similarly, Kumbhar and others (2014) investigated the compressive strength of Hybrid Fibre Concrete. In his contribution, Richardson (2014) also investigated the compressive strength of concrete with polypropylene fibre addition.. On NFRC, Ganesh Kumar and others (2019) have carried out a study on waste nylon fibre in concrete.Samrose and Mutsuddy (2019) have investigated the durability of NFRC. Hossain and others (2012) have also investigated the effect of NF in concrete rehabilitation. Ali and others (2018) have carried out a study on NFRC through partial replacement of cement with metakaolin. Song and others (2005) also investigated the strength properties of NFRC and PFRC respectively.Recent works on optimization show that many researchers have used Scheffe's method to carry out one form of optimization work or the other. For example, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and

Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere (2006) were also based on the use of Scheffe' mathematical model in the optimization of compressive strength of Perwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's (4,2) and Scheffe's (4,3) where comparison was made between second degree model and third degree model. Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also, Nwachukwu and others (2022a) developed and used Scheffe's Third Degree Polynomial model, Scheffe's (5,3) to optimize the compressive strength of GFRC where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). Nwachukwu and others (2022b) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). Furthermore, Nwachukwu and others (2022e) used Scheffe's Third Degree Regression model, Scheffe's (5,3) to optimize the compressive strength of PFRC. Nwachukwu and others (2022f) applied Modified Scheffe's Third Degree Polynomial model to optimize the compressive strength of NFRC. Again, Nwachukwu and others (2022g) applied Scheffe's Third Degree Model to optimize the compressive strength of SFRC. In what is termed as introduction of six component mixture and its Scheffe's formulation ,Nwachukwu and others (2022h) developed and use Scheffe's (6,2) Model to optimize the compressive strength of Hybrid-Polypropylene – Steel Fibre Reinforced Concrete (HPSFRC). Finally , Nwachukwu and others (2022 i) applied Scheffe's (6,2) model to optimize the Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). From the works reviewed so far, it appears that the subject matter has not been wholly addressed as it can be envisaged that no work has been done on the use of Scheffe's Second Degree Model to optimize the compressive strength of HPNFRC. Henceforth, the need for this recent research work.

2. GENERAL BACKGROUND ON SCHEFFE'S THEORY

Generally, a simplex lattice is a structural representation of lines joining the atoms of a mixture. Expectedly, these atoms are constituent components of the mixture. For a HPNFRC mixture, the constituent elements are the following six components: water, cement, fine aggregate, coarse aggregate, polypropylene fibre and nylon fibre. This shows that a simplex of six-component mixture is a five -dimensional solid. Mixture components, according to Obam (2009) are subject to the constraint that the sum of all the components must be equal to 1. That is:

$$X_1 + X_2 + X_3 + \dots + X_q = 1 \; ; \quad \Rightarrow \sum_{i=1}^{q} X_i = 1 \qquad \textbf{(1)}$$

where $X_i \geq 0$ and $i = 1, 2, 3\dots$ q, and q = the number of mixtures.

2.1. SCHEFFE'S (6, 2) MIXTURES SIMPLEX LATTICE DESIGN

The Scheffe's (q, m) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen regression equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains $^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \dots, 1; \quad i = 1, 2, \dots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degee, which in this present study is 2.

For example a (3, 2) lattice consists of $^{3+2-1}C_2$ i.e. $^4C_2$ = 6 points. Each X_i can take m+1 = 3 possible values; that is $x = 0, \frac{1}{2}, 1$ with which the possible design points are: $(1,0,0), (0,1,0), (0,0,1), \left(\frac{1}{2}, \frac{1}{2}, 0\right), \left(0, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, 0, \frac{1}{2}\right)$. To evaluate the number of coefficients/terms/points required for a given lattice , the following general formula is employed:

$$k = \frac{(q+m-1)!}{(q-1)!\,.\,m!} \quad \text{Or} \quad ^{q+m-1}C_m \qquad \textbf{2(a-b)}$$

Where k = number of coefficients/ terms / points

q = number of components/mixtures = 6 in this present study

m = number of deqree of polynomial = 2 in this present work

Using either of Eqn. (2), $k_{(6,2)}$ = 21

This implies that the possible design points for Scheffe's (6,2) lattice can be as follows:

A_1 (1,0,0,0,0,0); A_2 (0,1,0,0,0,0); A_3 (0,0,1,0,0,0); A_4 (0,0,0,1,0,0), A_5 (0,0,0,0,1,0); A_6 (0, 0,0,0, 0, 1); A_{12} (0.67,0.33, 0, 0, 0, 0); A_{13} (0.67, 0, 0.33,0,0,0); A_{14} (0.67, 0, 0, 0.33,0,0); A_{15} (0.67, 0, 0, 0,0.33, 0); A_{16} (0.67, 0, 0, 0, 0, 0.33); A_{23} (0,0.50,0.50, 0,0,0); A_{24} (0, 0.50, 0, 0.50, 0,0); A_{25}, (0, 0.50, 0, 0,0.50, 0); A_{26} (0, 0.50,0,0, 0.50); A_{34} (0.50, 0.50, 0, 0,0,0); A_{35} (O.50, 0,0.50, 0,0,0); A_{36} (0.50,0, 0,0.50, 0, 0); A_{45} (0.50, 0, 0, 0,0.50, 0); A_{46}(0.50,0,0,0,0,0.50);A_{56}(0,0,0.50,0.50,0,0); **(3)**

Again according to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable X_1, X_2, X_3, X_4 … X_q is given in the form of Eqn.(4) under.

$$N = b_0 + \sum b_i x_i + \sum b_{ij} x_j + \sum b_{ijk} x_j x_k + \sum b_{i_1 i_2 \ldots i_n} x_{i_1} x_{i_2} x_{i_n} \qquad (4)$$

where ($1 \le i \le q$, $1 \le i \le j \le k \le q$, $1 \le i_1 \le i_2 \le \ldots \le i_n \le q$ respectively) , b = constant coefficients and N is the response which represents the property under investigation, which ,in this case is the compressive strength.

As this research work is based on the Scheffe's (6, 2) simplex, the actual form of Eqn. (4) for six component mixture , degree two (6, 2) will be developed subsequently.

2.2. PSEUDO AND ACTUAL COMPONENTS IN SCHEFFE'S MIX DESIGN

In Scheffe's mix design, the relationship between the pseudo components and the actual components has been established as

$$Z = A * X \qquad (5)$$

where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging Eqn. (5) yields: $X = A^{-1} * Z$ **(6)**

2.3. ESTABLISHMENT OF HPNFRC OPTIMIZATION EQUATION FOR SCHEFFE'S (6,2) LATTICE

The Optimization/polynomial equation by Scheffe (1958), which is also known as response is given in Eqn.(4). But Eqn.(4) has been developed by Nwachukwu and others (2022h) to accommodate six component mixture for Scheffe's second degree model .Hence, the formulated polynomial equation for Scheffe's (6,2) simplex lattice based on Eqn.(4) is shown in Eqn.(7):

$$N = ß_1X_1 + ß_2X_2 + ß_3X_3 + ß_4X_4 + ß_5X_5 + ß_6X_6 + ß_{12}X_1X_2 + ß_{13}X_1X_3 + ß_{14}X_1X_4 + ß_{15}X_1X_5 + ß_{16}X_1X_6 + ß_{23}X_2X_3 + ß_{24}X_2X_4 + ß_{25}X_2X_5 + ß_{26}X_2X_6 + ß_{34}X_3X_4 + ß_{35}X_3X_5 + ß_{36}X_3X_6 + ß_{45}X_4X_5 + ß_{46}X_4X_6 + ß_{56}X_5X_6 \qquad (7)$$

2.4. COEFFICIENTS DETERMINATION OF THE SCHEFFE'S (6, 2) POLYNOMIAL

From the work of Nwachukwu and others (2022h), the coefficients of the Scheffe's (6, 2) polynomial are expressed as under. :

$\beta_1 = N_1$; $\beta_2 = N_2$; $\beta_3 = N_3$; $\beta_4 = N_4$; $\beta_5 = N_5$ and $\beta_6 = N_6$ **8(a-f)**

$\beta_{12} = 4N_{12} - 2N_1 - 2N_2$; $\beta_{13} = 4N_{13} - 2N_1 - 2N_3$; $\beta_{14} = 4N_{14} - 2N_1 - 2N_4$; **9(a-c)**

$\beta_{15} = 4N_{15} - 2N_1 - 2N_5$; $\beta_{16} = 4N_{16} - 2N_1 - 2N_6$; $\beta_{23} = 4N_{23} - 2N_2 - 2N_3$; $\beta_{24} = 4N_{24} - 2N_2 - 2N_4$; **10(a-d)**

$\beta_{25} = 4N_{25} - 2N_2 - 2N_5$; $\beta_{26} = 4N_{26} - 2N_2 - 2N_6$, $\beta_{34} = 4N_{34} - 2N_3 - 2N_4$; $\beta_{35} = 4N_{35} - 2N_3 - 2N_5$; **11(a-d)**

$\beta_{36} = 4N_{36} - 2N_3 - 2N_6$; $\beta_{45} = 4N_{45} - 2N_4 - 2N_5$, $\beta_{46} = 4N_{46} - 2N_4 - 2N_6$; $\beta_{56} = 4N_{56} - 2N_{35} - 2N_6$; **12(a-d)**

Where N_i = Response Function (or Compressive Strength) for the pure component, i

2.5. HPNFRC SCHEFFE'S (6, 2) MIXTURE DESIGN MODEL

By substituting Eqns. (8)-(12) into Eqn. (7), yields the mixture design model for the HPNFRC Scheffe's (6,2) lattice.

2.6. EVALUATING PSEUDO AND ACTUAL MIX PROPORTIONS FOR THE HPNFRC SCHEFFE'S (6, 2) DESIGN LATTICE AT INITIAL EXPERIMENTAL TEST POINTS AND CONTROL POINTS.

2.6.1. AT INITIAL EXPERIMENTAL TEST POINTS

The requirement for conventional mix ratio is usually in the form of 1:2:4. However this requirement is impossible to use since the requirement of simplex lattice design is based on Eqn. (1). Thus , Eqn.(1) criteria makes it impossible to use the conventional mix ratios such as 1:2:4 etc., at a given water/cement ratio for the actual mix ratio and there is therefore need for the transformation of the actual components proportions to meet the above criterion. Based on experience and previous knowledge from literature, the following arbitrary prescribed mix ratios are always chosen for the six vertices of Scheffe's (6,2) lattice. They are as follows :

A_1 (0.67:1:1.7:2:0.5:0.5); A_2 (0.56:1:1.6:1.8:0.8:0.8); A_3 (0.5:1:1.2:1.7:1:1); A_4 (0.7:1:1:1.8:1.2:1.2);

A_5 (0.75:1:1.3:1.2:1.5:1.5), and A_6 (0.80:1:1.3:1.2:0.9:0.9) **(13)**

which represent water/cement ratio, cement, fine aggregate, coarse aggregate, polypropylene fibre and nylon fibre respectively.

For the pseudo mix ratio, the following corresponding mix ratios at the vertices for six component mixtures are always chosen:

A_1(1:0:0:0:0:0), A_2(0:1:0:0: 0:0), A_3(0:0:1:0:0:0), A_4(0:0:0:1:0:0), A_5(0:0:0:0:1:0) and A_6(0:0:0:0:0:1) **(14)**

For the transformation of the actual component, Z to pseudo component, X, and vice versa, Eqns. (5) and (6) are used. By substituting the mix ratios from point A_1 into Eqn. (5), we obtain :

$$\begin{Bmatrix} 0.67 \\ 1.00 \\ 1.70 \\ 2.00 \\ 0.50 \\ 0.50 \end{Bmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \textbf{(15)}$$

Transforming the R.H.S matrix and solving, we obtain as follows:

$A_{11}(1) + A_{21}(0) + A_{31}(0) + A_{41}(0) + A_{51}(0) + A_{61}(0) = 0.67$. Thus , $A_{11} = 0.67$

Similarly, $A_{21}= 1$; $A_{31}= 1.7$; $A_{41}= 2$; $A_{51}= 0.5$; $A_{61}= 0.5$

The same approach is used to obtain the remaining values as shown in Eqn. (16)

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{Bmatrix} = \begin{Bmatrix} 0.67 & 0.56 & 0.50 & 0.50 & 0.75 & 0.75 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 1.70 & 1.60 & 1.20 & 1.00 & 1.30 & 1.30 \\ 2.00 & 1.80 & 1.70 & 1.80 & 1.20 & 1.20 \end{Bmatrix} = \begin{pmatrix} 1\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} \quad \textbf{(16)}$$

$$Z_5 \quad 0.50 \quad 0.80 \quad 1.00 \quad 1.20 \quad 1.50 \quad 1.50 \quad X_5$$
$$Z_6 \quad 0.50 \quad 0.80 \quad 1.00 \quad 1.20 \quad 1.50 \quad 1.50 \quad X_6$$

Considering mix ratios at the mid points from Eqn.(3) and substituting these pseudo mix ratios in turn into Eqn.(16) will yield the corresponding actual mix ratios.

For instance, considering point A_{12} we have: A_{12} (0.67,0.33, 0, 0, 0, 0). Thus,

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.50 & 0.50 & 0.75 & 0.75 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 1.70 & 1.60 & 1.20 & 1.00 & 1.30 & 1.30 \\ 2.00 & 1.80 & 1.70 & 1.80 & 1.20 & 1.20 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \end{pmatrix} \begin{pmatrix} 0.67 \\ 0.33 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.63 \\ 1 \\ 1.67 \\ 1.90 \\ 1.60 \\ 1.60 \end{pmatrix} \quad \textbf{(17)}$$

Solving, $Z_1 = 0.63$; $Z_2 = 1.00$; $Z_3 = 1.67$' $Z_4 = 1.90$; $Z_5 = 1.60$ and $Z_6 = 1.60$

The same approach goes for the remaining mid-point mix ratios.

Hence,in order to generate the 21 polynomial coefficients, twenty-one (21) experimental tests will be carried out and the corresponding mix ratios are depicted in Table 1.

Table 1: Pseudo (X) and Actual (Z) Mix Ratio for HPNFRC based on Scheffe's (6,2) Lattice

S/N	**POINTS**	**PSEUDO COMPONENT**						**RESPONSE**	**ACTUAL COMPONENT**					
		X_1	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**	**SYMBOL**	**Z_1**	**Z_2**	**Z_3**	**Z_4**	**Z_5**	**Z_6**
1	1	1	0	0	0	0	0	N_1	0.67	1.00	1.70	2.0	0.5	0.5
2	2	0	1	0	0	0	0	N_2	0.56	1.00	1.60	1.8	0.8	0.8
3	3	0	0	1	0	0	0	N_3	0.50	1.00	1.20	1.7	1.0	1.0
4	4	0	0	0	1	0	0	N_4	0.70	1.00	1.00	1.8	1.2	1.2
5	5	0	0	0	0	1	0	N_5	0.75	1.00	1.30	1.2	1.5	1.5
6	6	0	0	0	0	0	1	N_6	0.63	1.00	1.67	1.9	1.6	1.6
7	12	0.67	033	0	0	0	0	N_{12}	0.60	1.00	1.63	1.8	0.7	0.7
8	13	0.67	0	0.33	0	0	0	N_{13}	0.61	1.00	1.54	1.9	0.6	0.6

9	14	0.67	0	0	0.33	0	0	N_{14}	0.56	1.00	1.37	1.8	0.8	0.8
10	15	0.67	0	0	0	0.33	0	N_{15}	0.68	1.00	1.47	1.9	0.7	0.7
11	16	0.67	0	0	0	0	0.33	N_{16}	0.69	1.00	1.23	1.8	0.9	0.9
12	23	0	0.50	0.50	0	0	0	N_{23}	0.70	1.00	1.57	1.7	0.8	0.8
13	24	0	0.50	0	0.50	0	0	N_{24}	0.72	1.00	1.43	1.4	1.1	1.1
14	25	0	0.50	0	0	0.50	0	N_{25}	0.55	1.00	1.40	1.7	0.8	0.8
15	26	0	0.50	0	0	0	0.50	N_{26}	0.52	1.00	1.20	1.7	0.9	0.9
16	34	0.50	0.50	0	0	0	0	N_{34}	0.61	1.00	1.67	1.8	0.9	0.9
17	35	0.50	0	0.50	0	0	0	N_{35}	0.66	1.00	1.73	1.8	1.0	1.0
18	36	0.50	0	0	0.50	0	0	N_{36}	0.63	1.00	1.50	1.6	0.7	0.7
19	45	0.50	0	0	0	0.50	0	N_{45}	0.69	1.00	1.40	1.4	0.6	0.6
20	46	0.50	0	0	0	0	0.50	N_{46}	0.57	1.00	1.13	1.7	1.0	1.0
21	56	0	0	0.50	0.50	0	0	N_{56}	0.64	1.00	1.07	1.7	1.1	1.1

2.6.2. AT THE CONTROL POINTS

Twenty- one (21) different controls were predicted which according to Scheffe's (1958) ,their summation should not be greater than one. The same approach for component transformation adopted for the initial experimental points are also adopted for the control points and the results are shown in Table 2.

Table 2 : Actual and Pseudo Component of HPNFRC Based on Scheffe (6,2) Lattice for Control Points

S/N	POINTS	PSEUDO COMPONENT						CONTROL POINTS	ACTUAL COMPONENT					
		X_1	X_2	X_3	X_4	X_5	X_6		Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
1	1	0.25	0.25	0.25	0.25	0	0	C_1	0.61	1	1.38	1.83	0.5	0.50
2	2	0.25	0.25	0.25	0	0.25	0	C_2	0.62	1	1.45	1.68	0.8	0.8
3	3	0.25	0.25	0	0.25	0.25	0	C_3	0.67	1	1.40	1.70	1	1

4	4	0.25	0	0.25	0.25	0.25	0	C_4	0.66	1	1.30	1.68	1.2	1.2
5	5	0	0.25	0.25	0.25	0.25	0	C_5	0.63	1	1.28	1.63	1.5	1.5
6	6	0.20	0.20	0.20	0.20	0.20	0	C_6	0.64	1	1.36	1.70	0.65	0.65
7	12	0.30	0.30	0.30	0.10	0	0	C_{12}	0.59	1	1.45	1.83	0.75	0.75
8	13	0.30	0.30	0.30	0	0.10	0	C_{13}	0.59	1	1.48	1.77	0.85	0.85
9	14	0.30	0.30	0	0.30	0.10	0	C_{14}	0.65	1	1.42	1.80	1	1
10	15	0.30	0	0.30	0.30	0.10	0	C_{15}	0.64	1	1.30	1.77	0.9	0.9
11	16	0	0.30	0.30	0.30	0.10	0	C_{16}	0.60	1	1.27	1.71	1	1
12	23	0.10	0.30	0.30	0.30	0	0	C_{23}	0.60	1	1.31	1.79	1.55	1.55
13	24	0.30	0.10	0.30	0.30	0	0	C_{24}	0.62	1	1.33	1.83	1.1	1.1
14	25	0.30	0.10	0.30	0.30	0	0	C_{25}	0.63	1	1.41	1.85	1.25	1.25
15	26	0.10	0.20	0.30	0.40	0	0	C_{26}	0.61	1	1.25	1.79	1.35	1.35
16	34	0.30	0.20	0.10	0.40	0	0	C_{34}	0.64	1	1.35	1.85	0.89	0.89
17	35	0.20	0.20	0.10	0.10	0.40	0	C_{35}	1.40	1	1.04	1.59	1.08	1.08
18	36	0.30	0.10	0.30	0.20	0.10	0	C_{36}	0.62	1	1.36	1.77	0.92	0.92
19	45	0.25	0.25	0.15	0.15	0.20	0	C_{45}	0.61	1	1.51	3.16	0.91	0.91
20	46	0.30	0.30	0.20	0.10	0.10	0	C_{46}	0.68	1	1.56	1.96	0.98	0.98
21	56	0.10	0.30	0.30	0.30	0	0	C_{56}	1.30	1	1.31	1.79	0.95	0.95

The actual component as transformed from Eqn. (17) , Table (1) and (2) were used to measure out the quantities of water/cement ratio (Z_1), cement (Z_2), fine aggregate (Z_3), coarse aggregate (Z_4), polypropylene fibre (Z_5) and nylon fibre (Z_6) in their respective ratios for the concrete cube strength test.

3. MATERIALS AND METHODS

3.1 MATERIALS

In this present work, the constituent materials under investigation in line with Scheffe's (6,2) model are Water/Cement ratio, Cement, Fine and Coarse Aggregates, Polypropylene and Nylon Fibres . The water is obtained from potable water from the clean water source. The cement is Dangote cement, a brand of Ordinary Portland Cement obtained from local distributors, which conforms to British Standard Institution BS 12 (1978). Fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite (as a coarse aggregate) of 20mm size was obtained from a local stone market and was downgraded to 4.75mm. The same size and nature of polypropylene fibre and nylon fibre used previously by Nwachukwu and others (2022c) and Nwachukwu and others (2022d) respectively, are the same as the one being used in this present work.

3.2. METHOD

3.2.1. SPECIMEN PREPARATION / BATCHING/ CURING

The specimen used for the compressive strength is concrete cubes. They were cast in steel mould measuring 15cm*15cm*15cm. The mould and its base were damped together during concrete casting to prevent leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 42 mix ratios were to be used to produce 84 prototype concrete cubes. Twenty- one , out of the 42 mix ratios were as control mix ratios to produce 42 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (12). Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28 days of curing the specimens were taken out of the curing tank.

3.2.2. COMPRESSIVE STRENGTH TEST

Compressive strength testing was done in accordance with BS 1881 – part 116 (1983) - Method of determination of compressive strength of concrete cube and ACI (1989) guideline. As it is customary, two samples were crushed for each mix ratio and in each case, the compressive strength was calculated using Eqn.(18)

$$\text{Compressive Strength} = \frac{\text{Average failure Load,P (N)}}{\text{Cross- sectional Area, A (mm}^2\text{)}} \qquad \textbf{(18)}$$

4. RESULTS PRESENTATION AND DISCUSSION

4.1. HPNFRC RESPONSES FOR THE INITIAL EXPERIMENTAL TESTS POINTS.

The results of the compressive strength ($R_{esponse,}$ N_i) of HPNFRC based on a 28-days strength is presented in Table 3. These are calculated from Eqn..(18)

Table 3: 28th Day Compressive Strength (Responses) Test Results for HPNFRC Based on Scheffe's (6, 2) Model for the Initial Experimental Tests.

S/N	POINTS	EXPERIMENTAL NUMBER	RESPONSE N_i, MPa	RESPONSE SYMBOL	$\sum N_i$	AVERAGE RESPONSE N, MPa
1	1	1A 1B	**43.56** **44.32**	N_1	**87.88**	**43.94**
2	2	2A 2B	**52.21** **53.11**	N_2	**105.32**	**52.66**

3	3	3A 3B	**54.65** **54.68**	N_3	**109.33**	**54.67**
4	4	4A 4B	**45.87** **46.43**	N_4	**92.30**	**46.15**
5	5	5A 5B	**60.06** **60.04**	N_5	**120.10**	**60.05**
6	6	6A 6B	**40.76** **41.32**	N_6	**82.08**	**41.04**
7	12	7A 7B	**43.88** **43.65**	N_{12}	**87.53**	**43.77**
8	13	8A 8B	**32.52** **32.44**	N_{13}	**64.96**	**32.48**
9	14	9A 9B	**42.12** **43.09**	N_{14}	**85.21**	**42.61**
10	15	10A 10B	**39.98** **40.12**	N_{15}	**104.64**	**52.31**
11	16	11A 11B	**44.32** **43.65**	N_{16}	**80.10**	**40.05**
12	23	12A 12B	**45.86** **44.96**	N_{23}	**90.82**	**45.41**
13	24	13A 13B	**41.76** **41.75**	N_{24}	**83.51**	**41.76**
14	25	14A 14B	**33.54** **33.68**	N_{25}	**67.22**	**33.61**
15	26	15A 15B	**43.87** **44.76**	N_{26}	**88.63**	**44.32**
16	34	16A 16B	**50.45** **51.08**	N_{34}	**101.53**	**50.77**
17	35	17A 17B	**50.33** **51.12**	N_{35}	**101.45**	**50.73**
18	36	18A 18B	**42.86** **43.21**	N_{36}	**86.07**	**43.04**

19	45	19A 19B	**39.98** **40.21**	N_{45}	**80.19**	**40.10**
20	46	20A 20B	**51.33** **51.22**	N_{46}	**102.55**	**51.28**
21	56	21A 21B	**45.34** **44.88**	N_{56}	**90.22**	**45.11**

4.2. HPNFRC RESPONSES FOR THE EXPERIMENTAL (CONTROL) TEST.

Table 4 shows the 28th day Compressive strength results for the Experimental (Control) Test

Table 4: 28TH Day Compressive Strength (Responses) Results for HPNFRC Based on Scheffe's (6,2) Model for the Experimental (Control) Tests.

S/N	CONTROL POINTS	EXPERIMENTAL NUMBER	RESPONSE, MPa	AVERAGE RESPONSE, MPa
1	C_1	1A 1B	**49.33** **50.43**	**49.88**
2	C_2	2A 2B	**54.43** **53.98**	**54.21**
3	C_3	3A 3B	**53.56** **54.47**	**54.02**
4	C_4	4A 4B	**47.54** **47.43**	**47.49**
5	C_5	5A 5B	**59.65** **59.44**	**54.55**
6	C_6	6A 6B	**39.37** **40.34**	**39.86**
7	C_{12}	7A 7B	**42.43** **43.32**	**42.87**
8	C_{13}	8A 8B	**29.54** **30.45**	**30.00**
9	C_{14}	9A 9B	**43.56** **42.54**	**43.05**

10	C_{15}	10A 10B	**40.45** **40.13**	**40.29**
11	C_{16}	11A 11B	**44.21** **44.23**	**44.22**
12	C_{23}	12A 12B	**40.56** **40.45**	**40.51**
13	C_{24}	13A 13B	**39.34** **40.34**	**39.84**
14	C_{25}	14A 14B	**35.43** **36.54**	**35.99**
15	C_{26}	15A 15B	**42.76** **43.54**	**43.15**
16	C_{34}	16A 16B	**54.78** **54.76**	**54.77**
17	C_{35}	17A 17B	**49.87** **50.76**	**50.32**
18	C_{36}	18A 18B	**41.32** **42.32**	**41.82**
19	C_{45}	19A 19B	**42.32** **42.43**	**42.38**
20	C_{46}	20A 20B	**50.34** **51.23**	**50.79**
21	C_{56}	21A 21B	**46.34** **47.12**	**46.73**

4.3. SCHEFFE'S (6,2) MODEL FOR THE HPNFRC RESPONSES

By substituting the values of the compressive strengths (responses) from Table 3 into Eqns.(8) through (12), we obtain the coefficients (in MPa) of the Scheffe's second degree polynomial as follows:

$\beta_1 = 43.94$; $\beta_2 = 52.66$; $\beta_3 = 54.67$; $\beta_4 = 46.15$; $\beta_5 = 60.05$; $\beta_6 = 41.04$; $\beta_{12} = -18.12$; $\beta_{13} = -67.30$; $\beta_{14} = -9.83$; $\beta_{15} = 0.36$; $\beta_{16} = -9.76$; $\beta_{23} = -33.02$; $\beta_{24} = -30.58$.; $\beta_{25} = -91.88$; $\beta_{26} = -10.12$; $\beta_{34} = 5.4$; $\beta_{35} = -19.42$; $\beta_{36} = -19.26$; $\beta_{45} = -52.90$; $\beta_{46} = 30.74$; $\beta_{56} = -22.64$ **(19)**

Substituting the values of these coefficients in Eqn.(19) into Eqn. (9), we obtain the polynomial model for the optimization of the compressive strength of HPNFRC based on Scheffe's (6,2) lattice as given in Eqn.(20)

$N = 43.94X_1 + 52.66X_2 + 54.67X_3 + 46.15X_4 + 60.05X_5 + 41.04X_6 - 18.12X_1X_2 - 67.30X_1X_3 - 9.84X_1X_4$

$$+ 0.36X_1X_5 - 9.76X_1X_6 - 33.02X_2X_3 - 30.58X_2X_4 - 91.88X_2X_5 - 10.12X_2X_6 + 5.4X_3X_4 - 19.42X_3X_5$$

$$-19.26 X_3X_6 - 52.90X_4X_5 + 30.74X_4X_6 - 22.64 X_5X_6 \quad \mathbf{(20)}$$

4.4. TEST OF ADEQUACY OF THE SCHEFFE'S (6,2) MODEL FOR HPNFRC

Here, the test of adequacy is performed to check the correlation between the compressive strength results (lab responses) given in Table 4 and model responses from the control points based on Eqn.(20). Here, the Student's – T - test is adopted as the means of validating the Scheffe's Model. The procedures for using the Student's – T - test have been explained by Nwachukwu and others (2022 c). The result of the test shows that there is no significant difference between the experimental results and model responses. Therefore, the model is very adequate for predicting the compressive strength of HPNFRC based on Scheffe's (6,2) lattice.

4.5. RESULTS DISCUSSION

The Optimum (maximum) compressive strength of HPNFRC based on Scheffe's (6,2) lattice is 60.05MPa . This corresponds to mix ratio of 0.75:1.00:1.30:1.20:1.50:1.50 for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate, Polypropylene Fibre and Nylon Fibre respectively. Similarly, the optimum (minimum) compressive strength is 32.48MPa which also correspond to the mix ratio of 0.61:1.00:1.54:1.90: 0.60:0.60 for W Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate, Polypropylene Fibre and Nylon Fibre respectively. The maximum value from the model was found to be greater than the minimum value specified by the American Concrete Institute for the compressive strength of good concrete and also minimum standard (of 4500psi or 30.75MPa) specified by the American Society of Testing and Machine, ASTM C 39 and ASTM C 469. Thus, the model can be used to obtain the HPNFRC compressive strength of all points (1 - 56) in the simplex based on Scheffe's Second Degree Model**.**

5. CONCLUSION

So far in this work, Scheffe's Second Degree Optimization / polynomial Model for HPNFRC has been presented . The Scheffe's Method was used to predict the mix ratios as well as a model for predicting the compressive strength of HPNFRC. By using Scheffe's (6,2) simplex model, the values of the compressive strength were obtained at all 21 points(1- 56). The result of the student's t-test confirmed that there is a good correlation between the strengths predicted by the models and the corresponding experimentally observed results. The optimum attainable compressive strength predicted by the model based on Scheffe's (6,2) model was 60.05MPa. As expected, the maximum value meet the minimum standard requirement (of 20 MPa and 30.75MPa) stipulated by American Concrete Institute (ACI) and American Society of Testing and Machine, ASTM C 39 and ASTM C 469 respectively, for the compressive strength of good concrete. Thus, with the Scheffe's (6,2) model, any desired strength, given any mix proportions can be easily predicted and evaluated and vice versa. Thus, the utilization of this Scheffe's optimization model has solved the problem of having to go through vigorous, time-consuming and laborious mixture design procedures in other to obtain the desired strength.

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