PREDICTING VIRUS SPREAD IN A CITY OF RESIDENTS USING THE LOGISTIC FUNCTION

M.Dhanalakshmi[1], K.Bhanu Priya[2], G.Jyothi[3], D.Anusha[4], I.Tejaswini[5], B.Nisha[6], Ch.Durga Bhavani[7], D.Sowmya[8]

drjyothivellanki@gmail.com

1,2,3(Lecturers, Department of Mathematics, Sri Durga Malleswara Siddhartha Mahila Kalasala, Vijayawada, A.P, India).

4,5,6,7,8(Students, Mathematics, Sri Durga Malleswara Siddhartha Mahila Kalasala, Vijayawada, A.P, India).

ABSTRACT

The study employs the logistic growth model to predict the spread of viral outbreak, specifically virus, within a city with a population of 1,07,524 residents. Based on medical data documenting the virus's progression from 3 to 376 infection over a span of 5 days, the growth rate is determined. Using this growth rate, the model extrapolates the expected number of infected residents after 15 days. The findings after valuable insight into the potential trajectory of the outbreak, aiding public health planning and response strategies.

Keywords: *COVID-19, Growth Rate, Logistic Growth Model and Bernoulli's Equation.*

1. INTRODUCTION

Logistic differential equations, also known as the Verhulst equation, describe the growth of a population over time, taking into account limiting factors such as available resources. It is often used in biology and ecology to model population dynamics. Logistic differential equation are mathematical models used to describe population growth or the spread of certain phenomena over time, taking into account limiting factors. The most common form of a logistic differential equation is the logistic growth equation, which is often written as:

$$\frac{dy}{dt} = ky(1 - y/L) \quad \ldots\ldots\ldots\ldots \text{(i)}$$

Where $\frac{dy}{dt}$ represents the rate of change of the population with respect to time, k is the intrinsic growth rate of the population. L is carrying capacity of the environment, representing the maximum sustainable population size. There are two terms in the right side of the equation ky and $\frac{ky^2}{L}$. The first term describes the growth characteristic and the second term is providing the limitation in the model.

The equation model how a population grows rapidly when its small, then slows down as it approaches the carrying capacity. It's a fundamental concept in ecology and population dynamics.

Solving logistics differential equations can help predict population trends and understand the dynamics of various natural and artificial systems, such as the spread of diseases or the adoption of new technologies.

1.1 Now we can find the general solution of the above equation (i).

$$\frac{dy}{dt} = ky(1 - \frac{y}{L})$$

$$\frac{dy}{dt} - ky = \frac{-ky^2}{L}$$

Using Bernoulli's Equation

$$y'y^{-2} - ky^{-1} = \frac{-k}{L}$$

$$\frac{dy^{-1}}{dt} + ky^{-1} = \frac{k}{L}$$

Put $y^{-1} = z$

$$\frac{dz}{dt} + kz = \frac{k}{L}$$

$$I.F = e^{\int k\,dt} = e^{kt}$$

General solution is

$$z(I.F) = \int \frac{k}{L}\,(I.F)dt$$

$$ze^{kt} = \int \frac{k}{L}\,e^{kt}dt$$

$$z = \frac{1}{e^{kt}} \int \frac{k}{L}\,e^{kt}dt$$

$$z = \frac{1}{e^{kt}\,L}(e^{kt} + c)$$

$$y = \frac{e^{kt}L}{e^{kt} + c}$$

$$y = \frac{L}{1 + ce^{-kt}}$$

1.2 A virus epidemic was formally announced by the city government. According to medical data, it took 5 days for the virus to infect from 3 to 376 people. If the city has 107,524 residents, how many residents are predicted to be infected after 15 days. Assuming no measures were taken yet by city officials.

Using logistic function

$$y = \frac{L}{1 + ce^{-kt}}$$

y-number of people infected

t-time days

L-total population of city

Find constants c and k

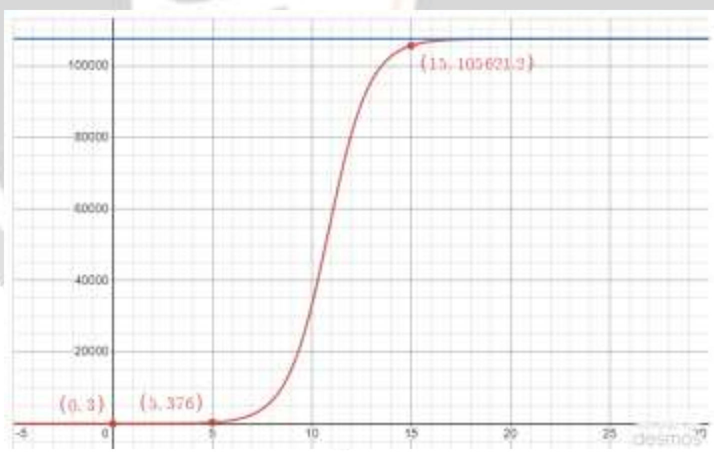
Initial condition: y=3 when t=0 days

$$3 = \frac{107524}{1 + ce^{-k(0)}}$$

$$c = 35840\frac{1}{3}$$

1.3 Now we get the logistic function.

Here L is the limiting capacity, C is the initial value, k is the growth rate and t is time. First, we find the constants c and k.



2.CONCLUSION: In this analysis, we utilized the logistic growth model to forecast the spread of COVID-19 in a city with a population of 107,524 residents. By examining empirical data demonstrating the virus's progression from 3 to 376 infections over a 5-day period, we calculated a growth rate that enabled us to project the number of infected individuals after 15 days. Our findings indicate that, under the current conditions and assumptions, the virus is likely to continue spreading within the city. The logistic growth model provides a useful tool for estimating the trajectory of infectious diseases, helping public health authorities prepare and allocate resources effectively. However, it is crucial to note that real-world factors, such as interventions and behavioral changes, can influence the actual outcome. To enhance the accuracy of such predictions, ongoing monitoring and adjustment of parameters are necessary. Additionally, the implementation of preventive measures and vaccination campaigns remains vital in mitigating the spread of contagious diseases and safeguarding public health. This study underscores the importance of data-driven decision-making and proactive measures in managing health crises.

3.REFERENCES:

[1].Braun, M., Differential Equations and their Applications, Springer-Verlag, Berlin(1975).
[2].Kaplan, W., Ordinary Differential Equations, Addison Welsey, Reading Mass.(1962).
[3].Kelly, L.M., Elementary Differential Equations, 6^{th} ed., McGraw Hill, New York(1965).
[4].Shepley L. Ross, Differential Equations, 3^{rd} ed., Wiley-India.
[5].Zafar Ahsan, Differential Equations and Their Applications,2^{nd} ed.