ROBUST ANALYSIS OF RRD MANOEUVRE OF THE MARINE CRAFT

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ABSTRACT

In this article, we will see the advantages of the proposed methods which reside on the use of the standard form. That allows great flexibility in formatting problem. The combination of the H_∞ *approach and the μ-analysis which is a tool to analyze the robustness of a large family of uncertain linear systems and finally the use of the robust analysis method associated with the development of the optimal modal control to find the placement range of the achievable poles in the stability region which is delimited by a generally open boundary that models the movements of the sea with respect to a boat.*

Keyword: *$\mathfrak{D}$ –Stability, Robust analysis, Marine craft,μ-analysis and* H_∞

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1. INTRODUCTION

We will devote ourselves to the application of robust analysis techniques of linear systems applied to the analytical model of marine craft (boat). The simulations presented aim to use a numerical tool for analyzing the robustness of linear systems. We present for the analytical model of the boat, the nominal model used, the uncertainties envisaged, the stability domain $\mathfrak{D}$ and the numerical results obtained during the simulation.

2. DEGREES OF FREEDOM AND MOTION OF A MARINE CRAFT

In maneuvering, a marine craft experiences motion in 6 degrees of freedom (DOFs). The DOFs are the set of independent displacements and rotations that specify completely the displaced position and orientation of the craft. The motion in the horizontal plane is referred to as surge (longitudinal motion, usually superimposed on the steady propulsive motion) and sway (sideways motion). Yaw (rotation about the vertical axis) describes the heading of the craft. The remaining three DOFs are roll (rotation about the longitudinal axis), pitch (rotation about the transverse axis) and heave (vertical motion).

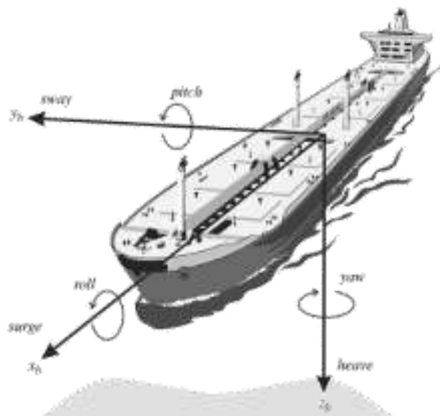

Fig-1: Motion in 6 degrees of freedom (DOF).

3. THEOREM OF ROBUSTNESS IN PERFORMANCE

The introduction between the input vector c and the output vector z_1 of a fictitious error Δ_f matrix with $\|\Delta_f\|_\infty < 1$ makes possible the transformation of the robustness analysis scheme into performance into a stability analysis diagram. We then obtain the block diagram of Fig-1.

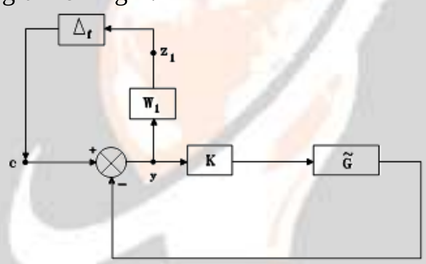

Fig -1: Block diagram of the disturbed system with introduction of the fictitious error model

By isolating respectively the model errors Δ_f and Δ, we obtain the block diagram of the Fig-2.

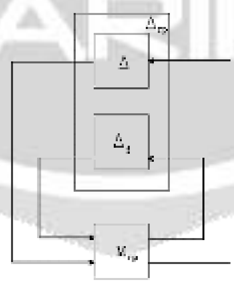

Fig -2: Block diagram of the disturbed system for performance robustness analysis

The system of Fig-2 is stable for any matrix $\Delta_{rp}(s) \in \underline{\Delta}(s)$ such as $\|\Delta_{rp}\|_\infty \leq 1$, if and only if:

$$\forall \omega \in \mathbb{R} \qquad \mu_{\underline{\Delta}}[M_{rp}(j\omega)] < 1 \tag{1}$$

Inequality (1) is equivalent to:

$$\forall \omega \in \mathbb{R} \quad \mu_{\Delta}[M_{pn}(j\omega)] < 1 \text{ et } \mu_{\Delta}[M_{rs}(j\omega)] < 1 \tag{2}$$

With

$$\Delta_{rp} = \{\mathrm{diag}\,[\Delta, \Delta_f]\}. \tag{3}$$

3. STRUCTURED SINGULAR VALUES

The notion of structured singular values is introduced to deal with robustness in the case of structured model uncertainties.

Let us define $\underline{\Delta}$ the set of complex matrices presenting the same structure as the uncertainty matrix $\Delta(s)$ and satisfying $\overline{\sigma}(\Delta) < 1$:

$$\underline{\Delta} = \left\{ \Delta = \mathrm{diag}\ [\Delta_1, \ldots, \Delta_F, \delta_1 I_{r1}, \ldots, \delta_r I_{rr}, \delta_1 I_{c1}, \ldots, \delta_c I_{cc}] \right\} \tag{4}$$

The uncertainty matrix Δ comprises F stable transfer matrices of any structure, r real so-called scalar repetitive blocks (the scalar δ_i is repeated once to take account of the corresponding uncertainty) and complex c scalars repeated with:

- $\Delta_j \in C^{m_j \times m_j}, 1 < j < F$ and checking the normalization condition $\|\Delta_j\|_\infty < 1$;
- $\delta_i \in R, 1 < i < r$ and checking the normalization condition $\|\delta_i\|_\infty < 1$;
- $\delta_k \in R, 1 < k < c$ and checking the normalization condition $\|\delta_k\|_\infty < 1$;
- $\Delta \in C^{n \times n}$

4. $\mathfrak{D}$- ROBUST STABILITY

4.1 General

The system is said to be $\mathfrak{D}$ -stable if all its poles closed-loop $I + GK$ transfer matrix are within a region $\mathfrak{D}$ belonging to the left complex half-plane.

We will say that the state matrix A is $\mathfrak{D}$ -stable if and only if all its eigenvalues are strictly contained within the region $\mathfrak{D}$ of the left complex half-plane.

The system is said to be robust $\mathfrak{D}$ -stable if all the poles of the transfer matrix $I + \tilde{G}K$ are within a region $\mathfrak{D}$ belonging to the left complex half-plane.

4.2 The various stability regions

We present in this section some regions of stability $\mathfrak{D}$. A stability region is shown in Fig- 3. It is an arbitrary set of points of the left complex plane bounded on the right by a boundary defined by:

$$\partial \mathfrak{D} = \left\{ s \in \mathbf{C} \,\middle|\, s = \gamma + j\omega, \gamma = f(\omega), \omega \in \mathbf{R} \right\} \tag{5}$$

In this case the set can be described by inequality:

$$\mathfrak{D} = \left\{ s \in \mathbf{C} \,\middle|\, s = \gamma + j\omega, \gamma - f(\omega) < 0, \omega \in \mathbf{R} \right\} \tag{6}$$

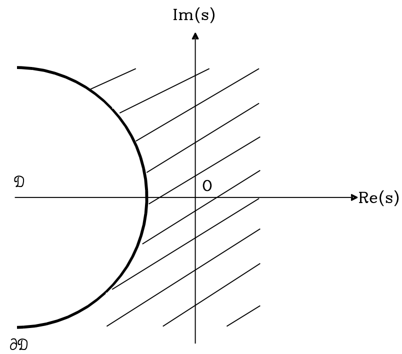

Fig -3: Region of stability

4.3 Stability regions 𝔇 where stability margins are desired

The simple condition of stability does not guarantee the proper operation of the slave system. An ideal looped system is defined as that for which the true difference remains practically nil in all circumstances, that is, for which the transitory regimes must be rapid and well amortized. Under these conditions, therefore it must be avoided that the dominant poles of the system are too close to the origin of the complex plane. The transient behavior of a looped system strongly depends on the nature and position of its poles (real or complex). For a stable system, we are looking for:

- a short response time;
- a sufficient damping so that the index overruns are low: it is then necessary that the damping factor ξ is between 0 and 1 (the problem of the damping concerns the systems whose dominant mode of the answer is governed by a pair of complex poles conjugates).

To satisfy the above conditions, we impose margins of security in the position of the poles of the looped system in the zone of the negative reals of the complex plane: these are the margins of absolute stability and relative stability.

First case: asymptotic stability margin Fig-4

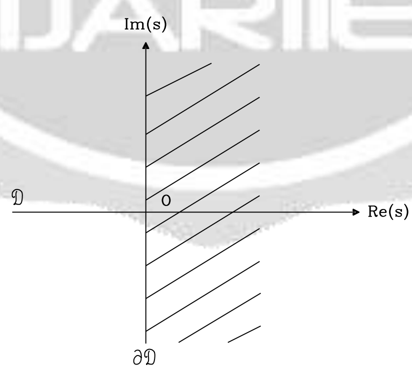

Fig -4: Region of stability 𝔇 to ensure asymptotic stability margin

$$\mathfrak{D} = \left\{ \mathbf{s} \in \mathbf{C} \,\middle|\, , \gamma = \mathbf{0}, \omega \in \mathbf{R} \right\} \tag{7}$$

Second case: Margin of absolute stability

$$\mathfrak{D} = \left\{ \mathbf{s} \in \mathbf{C} \,\middle|\, , \gamma - \gamma_0 < \mathbf{0}, \omega \in \mathbf{R} \right\} \tag{8}$$

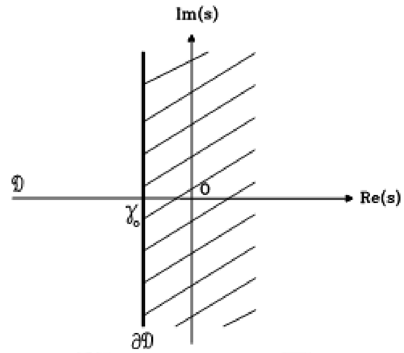

Fig -5: Stability region to ensure absolute stability margin

To avoid too long response times, we set a line $\delta\mathfrak{D}$ parallel to the axis $Im\ (s)$, to the right of which we must not find poles.

Third case: Relative stability margin

$$\mathfrak{D}=\left\{\mathbf{s}\in\mathbf{C}\,\middle|\,,\gamma+|\omega|.\tan\varphi<0,\omega\in\mathbf{R}\right\} \quad (9)$$

The relative stability margin relates to the damping factor, and that only occurs in the case of complex poles.

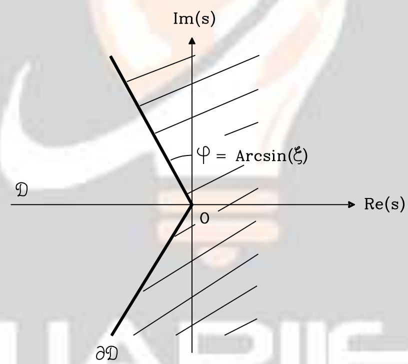

Fig -6: Stability region to provide relative stability margin

Fourth case: Absolute and relative stability margins

$$\mathfrak{D}=\left\{\mathbf{s}\in\mathbf{C}\,\middle|\,\gamma-\gamma_0<0\ \textbf{ et }\ \gamma+|\omega|.\tan\varphi<0,\omega\in\mathbf{R}\right\} \quad (10)$$

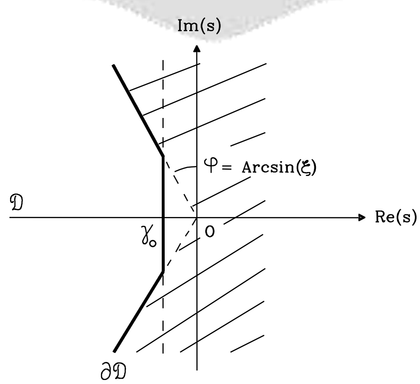

Fig -7: Stability region 𝔇 to ensure margins of absolute stability and relative stability

5. ROBUST ANALYSIS OF THE ANALYTICAL MODEL OF MANEUVERING A BOAT BY THE μ-ANALYSIS WITH A CORRECTOR K OBTAINED BY SYNTHESIS H_∞

Matrix of transfer of the maneuver of the boat is represented by:

$\dot{x} = Ax + Bu$

A and B being a partitioned matrix, we get:

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{13} & 0 \\ a_{31} & a_{33} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} b_1 \\ b_3 \\ 0 \end{bmatrix} u$$

The linearized model is useful for frequency analysis of rudder-roll damping (RRD) systems. For simplicity consider a ship with one rudder $u = \delta$ and $b = [b_2]$

$$G(p) = \frac{\psi}{\delta}(s) = \frac{c_3 s^3 + c_2 s^2 + c_1 s + c_0}{s(s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0)} \approx \frac{K_{yaw}\,(1 + T'_3 s)}{s(1 + T_1 s)(1 + T_2 s)(1 + T_3 s)} = \frac{K}{p(1 + Tp)(1 + T_e p)}$$

With: $T'_3 \approx T_3$, $K_{yaw} = K$, such as: T_e is the electric constant; and $T = T_m$(Constant mass of the boat) $+ T_M$(Mechanical constant)

5.1 Transfer function of the nominal system

The nominal values of the parameters of the electric motor are: $k = 300\ rd/s/V$, $T = 25ms$, and $T_e = 25ms$. The transfer function of the analytical model of maneuver of a boat is written in the following form:

$$\mathbf{G(s)} = \frac{\mathbf{k}}{\mathbf{s(1+Ts)(1+T_e s)}} \quad (11)$$

5.2 Calculating the Corrector by H-infinite synthesis

The calculation of the corrector K is obtained by using a classical synthesis method which is the resolution of the standard problem H_∞ by the Riccati matrix equations.

Corrector K obtained by synthesis H_∞ has the function of transfer:

$$K(s) = \frac{10\ (1+\frac{s}{20})\ (1+\frac{s}{70})\ (1+\frac{s}{2000})}{0.050(1+\frac{s}{0.050})\ (1+\frac{s}{250})\ (1+\frac{s}{500})\ (1+\frac{s}{19500})} \quad (12)$$

Note: We can note that the corrector is relatively high (order 4) and has a pole in -0.050, so very close to the origin and a pole very far in the complex left half-plane, in -19500. Its influence is negligible on frequency responses.

5.3 Nominal stability of the boat

The shape of the amplitude and phase diagrams of the frequency response of the open-loop transfer function $GK\ (j\omega)$ shows that the looped system is nominally stable and that the responses of the open-loop transfer comply with the specifications. . The corrector $K\ (s)$ adjusts the gain so that the transfer in open loop passes to $0dB$ for $100\ rd/s$ and ensures correct margins of stability.

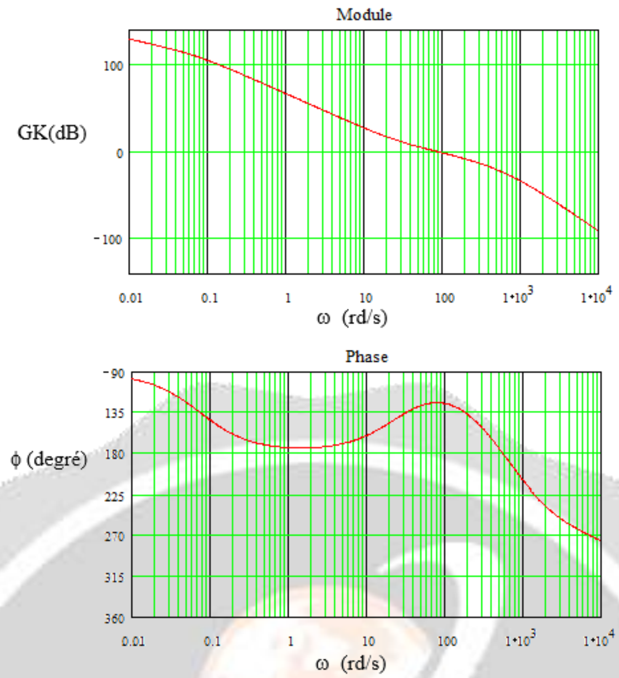

Chart -1: Frequency responses of $GK\ (j\omega)$

The gain and phase margins of the corrected open-loop transfer function $GK\ (j\omega)$ are equal to $20\ dB$ and $50\ °$ respectively.

Table -1: Characteristics of the nominal loop system with $T_\varepsilon\ = 0$

Real part	Imaginary Part	Frequency	Damping
-29,379	0	29,379	1
-41,085	-60,989	73,536	0,55870
-41.085	60,989	73,536	0,55870
-145,41	0	262,47	1
-262,47	0	972,07	1
-19500	0	19500	1

The poles of the nominal looped system are given in Table-1. We can notice that all the poles have a real part lower than -29,379 and that the complex poles have a damping higher than $\xi = 0.558$. The looped system is therefore nominally stable.

5.4 Nominal performance of the boat

Chart-2 shows the frequency responses of the sensitivity function $S\ (j\omega)$, the complementary sensitivity function $T\ (j\omega)$, the inverse of the weighting function and the structured singular value $\mu_{\Delta_{pn}} = [M_{pn}(j\omega)]$

Conclusion: The nominal performance condition is verified:

$$|\mathbf{S(j\omega)}| < \frac{\mathbf{1}}{|\mathbf{w_1(j\omega)}|} \tag{13}$$

Thus, the nominal performance is considered satisfactory because according to figure-9, we have:

$$\forall \quad \omega \in \mathbf{R} \qquad \mu_{\Delta_{pn}}[\mathbf{M_{pn}(j\omega)}] < \mathbf{1} \tag{14}$$

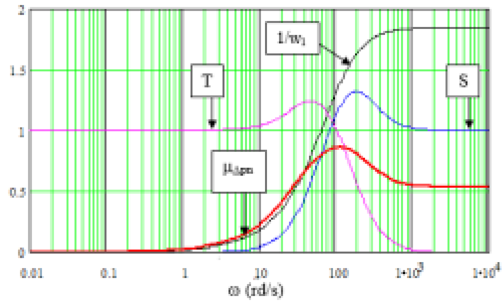

Chart -2: Frequency responses

5.5 Robustness in stability

Simulation diagram of the disturbed looped system:

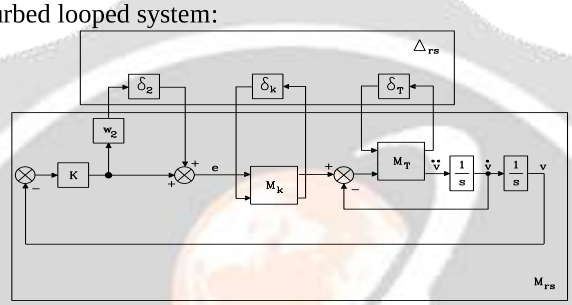

Fig-8: Block diagram of the perturbed system based on the with unstructured uncertainties and structured uncertainties

Figure-10 represents the simulation diagram of the analytical model of maneuvering a ship with unstructured uncertainties and structured uncertainties. By isolating the uncertainties δ_2, δ_k and δ_T, we obtain the analysis scheme of the robustness in stability with structured uncertainties of the figure-9.

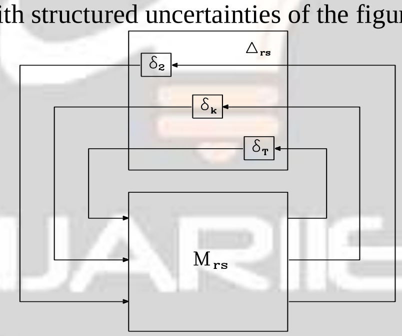

Fig-9: Stability robustness analysis diagram

By collecting the three uncertainties in a single block, we obtain the following uncertainty matrix:

$$\Delta_{rs}(s) = \mathbf{diag}\left\{ \delta_2 ; \delta_k ; \delta_T \right\}$$

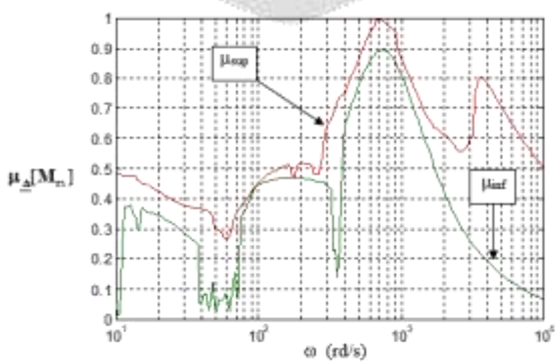

Chart-3: Robustness of the position of the poles ensuring an asymptotic stability margin (μ-analysis):

Conclusion: The computation of an upper bound and a lower bound of the structured singular value $\mu_{\underline{\Delta}}[M_{rs}(j\omega)]$ of the looped system ensuring an asymptotic margin, for s traversing the imaginary axis leads to the curves of figure-12. When the parameters $\tilde{k}$ and $\tilde{T}$ of the analytical model of maneuver of a ship with direct current present $\pm 52.25\%$ of uncertainties ($\omega_k = \omega_T = 0.911$), the upper bound μ of the singular value structured is equal to 1 for a pulsation of $\omega = 700 rd/s$, we deduce that the robustness in stability is guaranteed for all $\|\Delta_{rs}(s)\|_\infty < 1$, thus for the following domain of uncertainties:

$$22\ rd/s/V \leq \tilde{k} \leq 500\ rd/s/V \quad (15)$$

and $$1.5ms \leq \tilde{T} \leq 30ms. \quad (16)$$

5.5 Robustness in performance

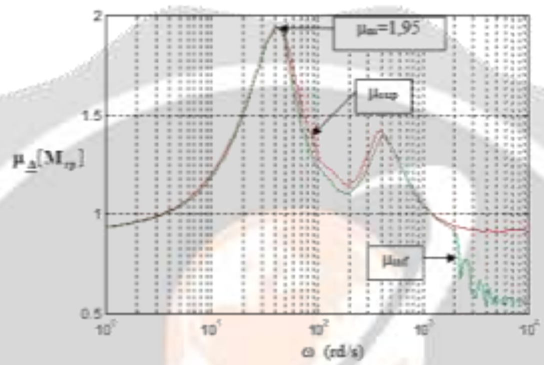

Chart-4: Robustness in the performance of the perturbed looped system (μ-analysis)

Conclusion: The computation of an upper bound and a lower bound of the structured singular value $\mu_{\underline{\Delta}}[M_{rp}(j\omega)]$ of the looped system providing an asymptotic margin for the analysis of the robustness in performance leads to the curves of Chart-4. When the parameters $\tilde{K}$ and $\tilde{T}$ of the analytical model of maneuver of a ship present uncertainties ($\omega_k = \omega_T = 0.911$), the upper bound μm of the singular value structured is 1.95 for a pulsation of $\omega_0 = 40 rd/s$, from this we deduce that robustness in performance is not guaranteed.

In the case where the parameters k and T have an uncertainty of the order $\pm 5\%$ (Chart-5), then the looped system is robust in performance for all $\|\Delta_{rp}(s)\|_\infty < 1$, thus for the following domain of uncertainties:

$$220\ rd/s/V \leq \tilde{k} \leq 260\ rd/s/V \quad (17)$$

and $$14ms \leq \tilde{T} \leq 16{,}5ms. \quad (18)$$

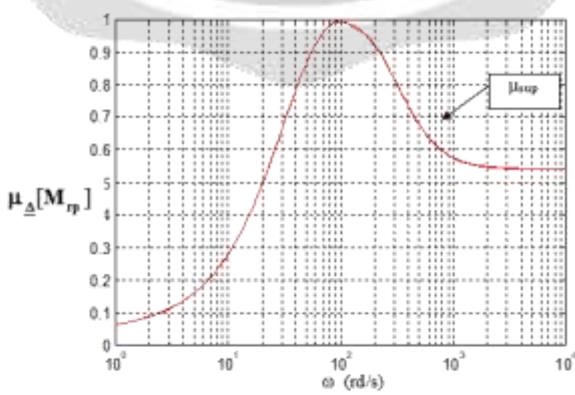

Chart-5: Robustness in looped system performance (μ-analysis)

Note: In the case of Chart-4, as the μ_m value is greater than 1, we can not guarantee the performance robustness of the disturbed looped system for the proposed template ($w_1(j\omega)$But this robustness in performance is ensured by reducing the range of uncertainties of the parameters of the analytical model of maneuvering a boat (Chart-5).

5.6 Absolute stability margin of the analytical model of a boat

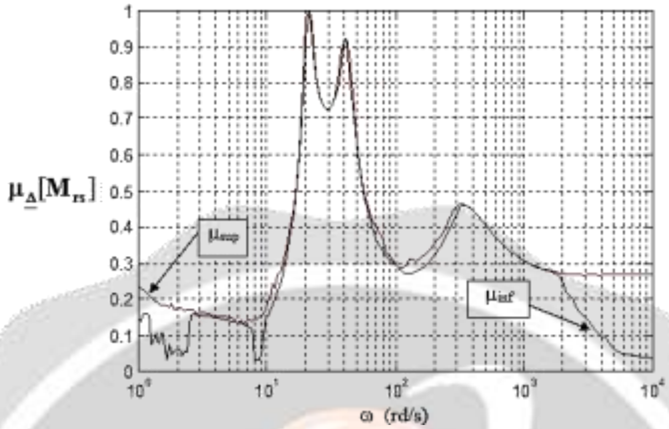

Chart-6: Robustness of the position of the poles ensuring a margin of absolute stability (µ-analysis)

Conclusion: The calculation of an upper bound and a lower bound of the structured singular value $\mu_\Delta[M_{rs}(j\omega)]$ of the looped system providing an absolute margin, for traversing the boundary D leads to the curves. When the parameters $\tilde{k}$ and $\tilde{T}$ of the analytical model of maneuver of a boat present $\pm 25\%$ of uncertainties ($\omega_k = \omega_T = 0.25$), the upper bound μ_m of the singular value structured is equal to 1 for a pulsation de $w = 20\ rd\ /\ s$, from this we deduce that robustness in stability is guaranteed for all $\|\Delta_{rs}(s)\|_\infty < 1$, therefore for the following range of uncertainties:

$$180 rd/s/V \leq \tilde{k} \leq 300 rd/s/V \tag{19}$$

and

$$10\ ms \leq \tilde{T} \leq 20\ ms \,. \tag{20}$$

5.7 Relative stability margin of the analytical model of a boat

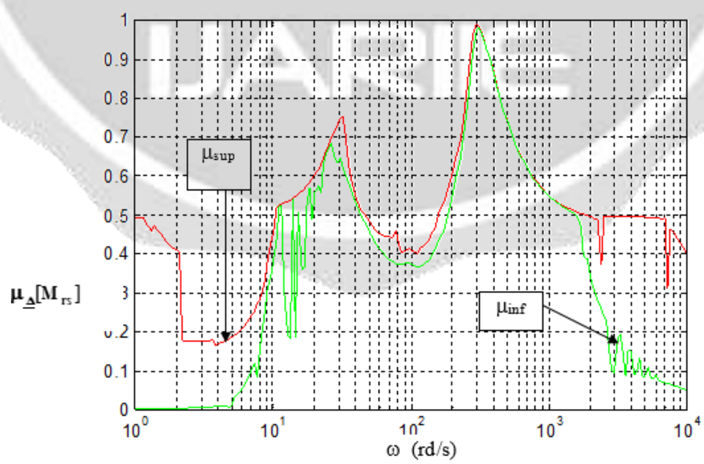

Chart-7: Robustness of the position of the poles ensuring a margin of relative stability (µ-analysis)

From this we deduce that robustness in stability is guaranteed for $\|\Delta_{rs}(s)\|_\infty < 1$, so for the following uncertainties:

$$130 rd/s/V \leq \tilde{k} \leq 380 rd/s/V \tag{21}$$

and

$$8\, ms \leq \tilde{T} \leq 20{,}5\, ms\,. \tag{22}$$

5.8 Absolute and relative stability margins of the analytical model of a vessel

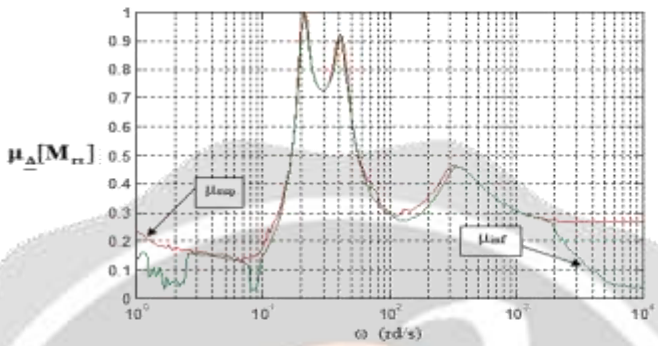

Chart-8: Robustness of the position of the poles ensuring an absolute and relative stability margin (μ-analysis)

From this we deduce that robustness in stability is guaranteed for $\|\Delta_{rs}(s)\|_\infty < 1$, so for the following uncertainties:

$$175 rd/s/V \leq \tilde{k} \leq 300 rd/s/V \tag{23}$$

and

$$11 ms \leq \tilde{T} \leq 18 ms\,. \tag{24}$$

5.9 Stability robustness analysis by the analysis of the analytical model of a boat

We then obtain the structured singular value curve of Chart-9 by scanning on and for the w_k=0.045 and $w_T = 0.085$ weighting functions.

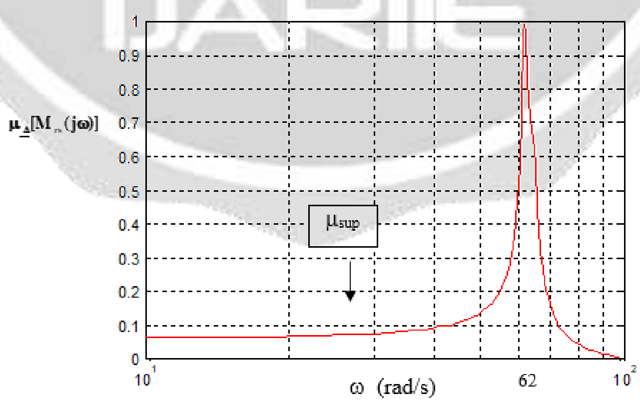

Chart-9: Robustness of the position of the poles of the disturbed system (μ-analysis)

Conclusion: We obtain a maximum singular value equal to the unit which is the limit of the robustness in D-stability for a pulsation of 62 rad / s. From this we deduce that the stability of the disturbed looped system is guaranteed for everything $\|\Delta_{rs}(s)\|_\infty < 1$, therefore for the following domain of uncertainties:

$$230 rd/s/V \leq \tilde{k} \leq 245\, rd/s/V \tag{25}$$

and $$14{,}25 ms \leq \tilde{T} \leq 15{,}95 ms. \tag{26}$$

5.9 Uncertainties on the parameters $\tilde{k}$, $\tilde{T}$ and $\tilde{T_e}$

By collecting the uncertainties in a single block, the uncertainty matrix of stability stability is now:

$$\Delta_{rs}(s) = \mathbf{diag}\{\ \delta_k;\ \delta_T I_2;\ \delta_{Te} I_2\ \} \tag{27}$$

By isolating these uncertainties we obtain the stability stability analysis schema with structured uncertainties

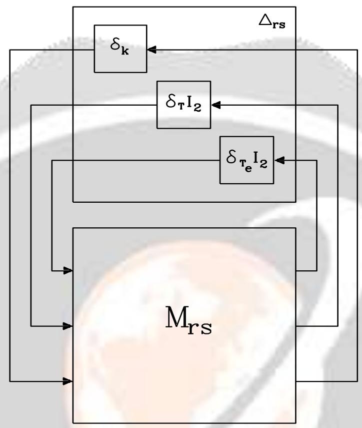

Fig-10: Scheme of analysis of the robustness in stability of the disturbed system

The calculation of the upper bound of the structured singular value $\mu_{\Delta}[M_{rs}(j\omega)]$of the looped system with uncertainties on the parameters $\tilde{k}$, $\tilde{T}$ and $\tilde{Te}$ leads to the curve.

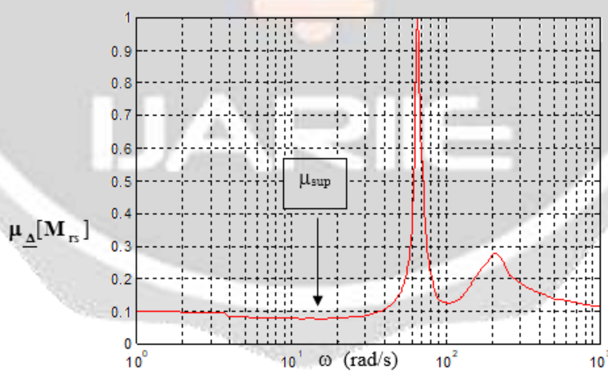

Chart-10 : Robustesse du placement des pôles (µ-analyse)

The robustness in stability is then studied by considering the possible cases of parametric variations $\mu_{\Delta}[M_{rs}(j\omega)]$. We assume that I_{m1} and I_{m2} are identical in each case.

The results obtained are summarized in Table -2

Table-2: Robustness of the placement of the poles with respect to the parameters of the electric motor

	W_K	W_T	W_{TE}	μ_M	$\omega_0\,(\frac{rd}{s})$
$\tilde{k}$	1	-	-	1	0

$\tilde{T}$	-	0.088	-	1	66.69
$\tilde{T_e}$	-	-	1	0.82	63.09
$\tilde{k}$ et $\tilde{T}$	0.05	0.076	-	1	65.93
$\tilde{k}$ et $\tilde{T_e}$	0.05	-	0.99	1	62.95
$\tilde{T}$ et $\tilde{T_e}$	-	0.0846	0.042	1	66.69
$\tilde{k}$, $\tilde{T}$ et $\tilde{T_e}$	0.05	0.064	0.12	1	65.17

Conclusion: Table-2 gives the complete results of the effect of the uncertainties on the parameters $\tilde{k}$, $\tilde{T}$ **and** $\tilde{T_e}$ of the analytical model of maneuver of a ship with direct current. Our study reveals that the looped system has a different sensitivity depending on the type and number of parameters affected.

6. CONCLUSIONS

We presented the analysis of the robustness of the system of the analytical model of a ship, the analytical model of maneuver of a boat, when the corrector is obtained by synthesis H_∞.

To use the μ-analysis of looped systems in which a corrector has been calculated for a nominal model, we isolate the uncertainties of the model in a structure block Δ and we group the rest in a transfer matrix M. We then calculate the singular value curve $\mu_\Delta(M)$ as a function of frequency.

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