RUNGE-LENZ VECTOR AND EIGEN VALUES FOR DYON SYSTEM IN SIX-SPACE

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ABSTRACT

The Maxwell, Lorentz force equations are expressed in terms of generalized potentials, fields and six-currents in symmetric, orthogonal six-dimensional space-time. The Lagrangian and canonical Hamiltonian for a two-dyon system have been shown to depend on dual charge source strength and coupling strength of dyons. The gauge invariant, rotationally symmetric Runge-Lenz vector have been expressed in six-space and constructing raising operators, their commutations have been explored and it has been shown that third component of angular momentum commutes with Hamiltonian. Characterizing the eigen basis, the minimum angular momentum in the system is retained and integral value of chirality leads to degeneracy in eigen values.

Keywords - *Six-dimensional space-time, Extended relativity, Hamiltonian, Runge-Lenz operators.*

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1. INTRODUCTION:

The particles with simultaneous electric and magnetic source, were first proposed by Schwinger [1] and today, monopoles and dyons, abelian or non-abelian, are part of all current grand unified theories [2,3], superconductivity [4,5] and supersymmetric models [6,7]. The dyonic solutions have been found intrinsically associated with hadronic structure of matter [8], black-holes [9] and gravitation [10]. Their potential theoretical presence has inspired many physicists to detect these particles experimentally and recently Acharya et al [11] have reported for experimental evidence for dyon captured in the trapping detector through a superconducting quantum interference device (SQUID) magnetometer.

In search of a consistent theory of all particles of sub- or superluminal nature in extended relativity [12-15], the psudo-Eucledian, orthogonal, six-dimensional space-time [16-22] has drawn considerable interest. Besides extended relativity, the concept of higher dimensional space-time has been widely accepted in unification of fundamental forces and current superstring theories. At high energies, the temporal fluctuations become prominent and may effect particle stability [23]. The radiation trajectory of the particles also depends on the choice of time trajectory in six-space and studying charge-field interaction, we [24] have shown that considerable amount of energy is required to turn the time orientation of the radiation. The space-time dependent structural mappings [25] bring an event in six-space to four-dimensional observational manifold i.e. usual Minkowski space or tachyonic four-dimensional space, where event may be detected. The space-time behavior menifests itself in terms of Lorentz group [26] and discrete symmetries associated with formulation in six-space [27].

In the present paper, Using the definitions of six generalized potentials, fields and currents, classical Lagrangian has been chosen to investigate Maxwell Lorentz equations for dyons in six space. The field equations and Lorentz force equations for dyon with generalized potential have been constructed. The canonical Hamiltonian commutes with gauge invariant angular momentum operators in six space and coupling parameters are shown to be present in each observational subspace of the six-manifold. The Runge-Lenz vector and then raising operators have been constructed and their commutation have been identified. The Runge-Lenz operators with eigen basis have been shown to create degeneracy in eigen values.

2. BACKGROUND: MAXWELL, LORENTZ EQUATIONS FOR DYON:

The pseudo-Euclidian, symmetric, six-dimensional space-time, with equal number of space and time coordinates is characterized by ;

$$P\{x^{\mu}\} \text{ in } D(3 \oplus 3) \equiv (\vec{r}, \vec{t})^{T} \equiv (x^1, x^2, x^3, x^4 = t^1, x^5 = t^2, x^6 = t^3)^{T} \quad (1)$$

With six coordinates or a spatial vector $\vec{r}$ and a temporal vector $\vec{t}$. The bracket { } specifies six-vector in six-space. The electromagnetic fields associated with spin-1 dyon with mass *m*, say, carry generalized charge $q = e - ig$, where e is electric charge strength and g is magnetic charge strength. We define the electric potential A_μ and magnetic potential B_μ, as six-vectors given by;

$$\{A_\mu\} = \{A, \phi\} \quad (2)$$

$$\{B_\mu\} = \{B, \omega\} \quad (3)$$

Where A, ϕ and B, ω represent electric potential, and magnetic potential respectively, with spatial and temporal constituents. The generalized six potential for dyon is expressed as;

$$\mathcal{V}_\mu = \{A_\mu\} - \{iB_\mu\} = \{V, \Omega\} \quad (4)$$

And the six-current for dyon may consequently be described as,

$$\mathcal{J}_\mu = j_\mu - ik_\mu \quad (5)$$

Where $\mathcal{J}^d$ is six current for dyon and j_μ is electric (or k_μ is magnetic) six current respectively. We can construct electromagnetic field tensor with six potential (4) as,

$$\mathcal{G}_{\mu\nu} = \mathcal{V}_{\mu,\nu} - \mathcal{V}_{\nu,\mu}, \quad (6)$$

and using six current (5), the electromagnetic field tensor leads to the six- covariant field equations given by,

$$\mathcal{G}_{\mu\nu}{}^{,\nu} = \mathcal{J}_\mu \quad (7)$$

Let us define a new, duality invariant generalized field ψ such that,

$$\psi^r = E_r - iH_r, \text{ and } \psi^t = E_t - iH_t \quad (8)$$

Here, superscript d represents dimensionality and ψ^r, ψ^t are generalized fields associated with dyons in spatial and temporal dimensions, respectively. The dyonic fields associated with six space may also be expressed in terms of generalized six potentials as,

$$\psi^r = -\partial_{t_m}.\mathcal{V} - \nabla_r.\Omega_j + (i^{-1}).(\nabla_r \text{ x } \mathcal{V}) \quad (9)$$

$$\psi^t = -\nabla_t.\mathcal{V}_j - \partial r_j.\Omega + (i^{-1}).(\nabla_t \times \Omega), \quad (10)$$

and corresponding field equations, representing the six-dimensional fields and six-current relations may be given by,

$$(-i)(\nabla_r \times \psi_r) - \frac{\partial \psi_r}{\partial t_m} = \mathcal{J}_r \quad (11)$$

$$(-i)(\nabla_t \times \psi_t) - \frac{\partial \psi_t}{\partial r_j} = \mathcal{J}_t \,. \quad (12)$$

where, ψ^r, ψ^t, $\mathcal{J}_r$ and $\mathcal{J}_t$ represent spatial and temporal parts of the generalized fields and currents. The charge field interaction through generalized quantities also carries specific space-time structure dependence in six-space. The generalized fields and generalized current reproduce symmetric, duality invariant Maxwell equations for dyons. In presence of generalized six-fields, the most general expression for Lorentz force acting on dyon, in covariant form is given by [28];

$$m\frac{dU_\mu}{d\tau} - q\mathcal{G}_{\mu\nu}U_\nu = 0 \quad (13)$$

where $d\tau$ is the infinitesimal increment along the world line. The Lorentz force may also be expressed in terms of generalized fields, equation (8), in six-space as;

$$m\frac{dU_r}{d\tau} = q\,[\,n\,\psi^r + i\,(\,v \text{ x } \psi^r)] \quad (14)$$

$$m\frac{dU_t}{d\tau} = q\,[\,v\,\psi^t + i\,(\,n \text{ x } \psi^t)] \quad (15)$$

In Lorentz force equations, (14,15), U_r and U_t represent the spatial and temporal velocity vectors and n is unit time vector along the particle trajectory.

3. RUNGE-LENZ VECTOR AND EIGEN VALUES FOR TWO DYON SYSTEM:

In order to investigate the dynamics of a dyon of mass m and charge q in generalized fields, the suitable classical Lagrangian may be chosen as;

$$£ = L_P + L_F + L_I \quad (16)$$

$$L_P = -m_o \quad (17)$$

$$L_F = -(1/4)\, K\, [\, \alpha \{ \mathcal{A}^2 - \mathcal{B}^2 \} - 2\beta\, \mathcal{A}.\mathcal{B}\,] \quad (18)$$

$$L_I = [\, (\alpha A_\mu - \beta B_\mu)\, j^\mu - (\alpha B_\mu - \beta A_\mu) k^\mu\,] \quad (19)$$

Such that L_P , L_F , L_I represent free particle Lagrangian, free field Lagrangian and interacting Lagrangian which arises due to interactions among six-dimensional fields and particles. In equation (18), $\mathcal{A}$ and $\mathcal{B}$ are given by;

$$\mathcal{A} \equiv \mathcal{A}_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu} \quad (20)$$

$$\mathcal{B} \equiv \mathcal{B}_{\mu\nu} = B_{\mu,\nu} - B_{\nu,\mu} \quad (21)$$

and K is the strength of interaction and α, β are coupling parameters given by,

$$\alpha - i\,\beta = \exp(-2i\theta)\, ; \quad \alpha^2 + \beta^2 = 1 \quad (22)$$

$$\tan\theta = g/e = \frac{A_\mu}{B_\mu} = = \frac{j_\mu}{k_\mu} \quad (23)$$

where θ is the angle of rotation in complex charge space. Applying the usual variational technique, the variation of six-potential (keeping particle trajectory fixed) yields field equations for dyons given by equation (14,15). The Lagrangian also enjoys invariance under extended Lorentz group [26] and under gauge transformations of the second type,

$$\mathrm{V} \rightarrow V' = \partial_\mu \chi\, , \quad (24)$$

where χ is arbitrary gauge potential. To derive the equation of motion from the Lagrangian density (16), we consider the variation of dyonic trajectory, without altering the generalized fields associated with dyons and relations (11,12) may be arrived.

The momenta canonically conjugate to generalized field tensor, given by equation (6), may be expressed as;

$$p_\mu = \frac{\partial L}{\partial \mathcal{G}_{\mu\nu}} = = \frac{\partial L}{\partial \mathcal{J}_\mu} = \partial V_\mu^* \quad (25)$$

Where,

$$V_\mu^* = A_\mu + iB_\mu \quad (26)$$

is the complex potential conjugate to generalized potential V_μ.

The gauge invariant linear momentum associated with generalized fields [ref-pc] associated with dyons π^d may be expressed as;

$$\pi^d = (\, p^\mu - \mu_{12}\, V^d) \quad (27)$$

The angular momentum operators, in terms of linear angular momentum, may also be written as;

$$\mathrm{J}^r = \vec{r}\ x\ \pi^r + \mu_{12}\frac{\vec{r}}{r} \quad (28)$$

$$\mathrm{J}^t = \vec{t}\ x\ \pi^t + \mu_{12}\frac{\vec{t}}{t}. \quad (29)$$

The Hamiltonian for classical dyon in six-space, may be written as [ref pc];

$$H_d = [\{\pi^d \pi_d\}/2m] - [\alpha_{12}/(x_d] + [\mu_{12}/2\mathrm{m}\, x_d x^d] \quad (30)$$

Where m is the dyon mass and electronic coupling parameter α_{12},is given by,

$$\alpha_{12} = e_1 e_2 + g_1 g_2\ . \quad (31)$$

sub-, superscript d chosen to be either r or t corresponding to respective subspace in equation (30). The classical Hamiltonian for dyons is therefore modified in six-space and it contains coupling parameters and gauge invariant linear momentum operators. One can verify that the additional term in Hamiltonian is justified to retain following commutation rule;

$$[(\mathrm{J}^d)^2\, , H_d\,] = 0. \quad (32)$$

In order to compute the eigen values of the angular momentum operators, we may construct gauge invariant, rotationally symmetric Runge-Lenz vectors, say A_d of the system in respective planes as follows;

$$A_d = (1/\mathrm{m})\, [\, \underline{i}\, \pi_d - (J_d \times \pi_d) + \alpha_{12}\, (\vec{d}/d\,) \quad (33)$$

The Runge-Lenz vector satisfies following commutation relations;

$$[\, \mathrm{J}_a\, , A_b]^d = -\underline{i}\ \epsilon_{abc}\, \mathrm{A}_c^d \ \ (\forall\, a, b, c = 1,2,3 \text{ spatial coordinates}) \quad (34)$$

$$[\, A_a\, , A_b]^d = -\underline{i}\ \epsilon_{abc}\, \mathrm{j}_c^d\, (-2\,\hat{H}/m\,) \quad (35)$$

$$A_d . J_d = J_d . A_d = \alpha_{12} . \mu_{12} \quad (36)$$

$$(A_d)^2 = \alpha_{12}{}^2 + \{(2\,\hat{H}/m\,)\,(A_d)^2 - \mu_{12}{}^2 + 1\}. \quad (37)$$

The contribution from magnetic coupling parameter μ_{12} is irrespective of space (time) plane and hence magnetic charge contribution of two-dyon system through μ_{12}, appears in both spatial and temporal sectors of the theory.

Let us construct following operators in six-space,

$$Z_1^d = (\,J_d/2) + (1/2)\,(-m/2H)^{1/2}\,A_d \quad (38)$$

$$Z_2^d = (\,J_d/2) - (1/2)\,(-m/2H)^{1/2}\,A_d \quad (39)$$

Which satisfy following commutation relations;

$$[\,(\,Z_1^d)_a\,,(\,Z_1^d)_b] = -\,\underline{i}\;\in_{abc}\;(\,Z_1^d)_c \quad (40)$$

$$[\,(\,Z_2^d)_a\,,(\,Z_2^d)_b] = -\,\underline{i}\;\in_{abc}\;(\,Z_2^d)_c \quad (41)$$

$$[\,(\,Z_1^d)_a\,,(\,Z_2^d)_b] = 0 \quad (42)$$

$$[\,(\,Z_1^d)_a\,,(J_d)^2\,] = [\,(\,Z_2^d)_a\,,(J_d)^2\,] = 0 \quad (43)$$

From the identities, represented by equations (40-43), it can be readily seen that the angular momentum operators commute with addition or subtraction of Z_1^d , Z_2^d, i.e.,

$$[\,J_d, (Z_1^d\text{-}\,Z_2^d)\,] = [\,J_d, (Z_1^d + Z_2^d)\,] = 0 \quad (44)$$

Explicitly, the operators J_d become diagonal when the basis Z_1^d- Z_2^d to Z_1^d + Z_2^d is used to characterize them. Equations (22,23) yield;

$$(Z_1^d\text{-}\,Z_2^d)\,(\,Z_1^d + Z_2^d + 1) = (-m/2H)^{1/2}\,\alpha_{12}.\,\mu_{12} \quad (45)$$

Where,

$$Z_1^d\text{ - }Z_2^d = \mu_{12} \quad (46)$$

$$Z_1^d + Z_2^d = (-m/2H)^{1/2}\,\alpha_{12} - 1 \quad (47)$$

Using relation, the eigen values of the total angular momentum J_d, may directly be written as;

$$|\alpha_{12}|, |\alpha_{12}| + 1, \ldots.\quad, |\alpha_{12}| + n, \ldots., (-m/2H)^{1/2}\,\alpha_{12} - 1 \quad (48)$$

With the condition;

$$|\,J_d\,| \;\geq\; \mu_{12}, \quad (49)$$

or more specifically

$$|\,J_r\,| \;\geq\; \mu_{12} \text{ and } |\,J_t\,| \;\geq\; \mu_{12} \quad (50)$$

In respective 3D spatial or temporal sectors of the six-dimensional manifold. The third component of total angular momentum i.e., $(J_d)^3$ may execute following eigen values;

$$-\,(\,|\mu_{12}| + n)\;\ldots\ldots + (\,|\mu_{12}| + n) \quad (51)$$

In respective planes, where n chooses the integer values.

4. DISCUSSION:

The symmetric six-dimensional space-time is constructed with three space and three time coordinates with signature $g_{\mu\nu}$ = (1,1,1,-1,-1,-1). The generalized Maxwell field equations for a dyon have been expressed in terms of generalized potential, current and dual fields in six-space by equation (7). Using generalized potentials given by equation (4), a dual field in terms of existing generalized electromagnetic fields in six-space is defined through equation (8) and the Maxwell set of equations are expressed through equation (11,12). The Lorentz force acting on a dually charged dyon is expressed through equations (14,15). These equations may be used to find force equations in respective observational slices of the six-space. The Lagrangian for a two-dyon system is expressed in terms of generalized potentials and coupling parameters, which depend on dual charge source strength and interaction strength of dyons. The momenta canonically conjugate to the generalized electromagnetic field tensor is expressed by equation (25). The gauge invariant linear momentum leads to angular momentum given by equation (27). The Hamiltonian for the dyon system is expressed is expressed by equation (30), which commutes with angular momentum operators and carries temporal dependence in six-space. The total angular momentum and its components commute with the Hamiltonian, implies the spherical symmetry, and generated degeneracy in energy level can be connected by raising operators. An analogous operator may be constructed in terms of Runge-Lenz vector, which commutes with Hamiltonian but does not commutes with total angular momentum. The angular momentum and Runge-Lenz, both separately constitute rotation invariance in three dimensions, may be combined to give complete set of operators in respective four-dimensions i.e. in $[R]^4$ or $[T]^4$ subspace of six-space.

Gauge invariant rotationally symmetric Runge-Lenz vector has been constructed in terms of equations (33) which satisfy commutations given by equations (34-36). These relations are rotationally symmetric and similar to conventional Coulomb problem of electric charge except that (i)- the contribution of magnetic charge be excluded and (ii)- the treatment be restricted to M^R slice of D^6 space. These relations may therefore, be used to investigate Coulomb problem with dyonic charge in respective observational slices of the theory. Defining new operators Z_1^d, Z_2^d through equations (38,39) in respective space-time manifold, the commutation relations (40-43), have been used to construct a simultaneous state for these operators satisfying the eigen value equations which characterize the basis Z_1^d- Z_2^d to $Z_1^d + Z_2^d$. The relations represented by equations (34,35) agree with the result that $x(J_d) = \mu_{12}$ must be the minimum angular momentum in the system. Conditions given by equations (44,45) lead to eigen values given by equation (51), for the operator $(J_d)^3$, where $|\mu_{12}|$ is the integral value of the chirality as result of the quantization of residual angular momentum. The increase in the degeneracy in the eigen value of the third component of angular momentum operators in respective subspaces may lead to an effective change in the scattering of dyons.

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