River Flood Propagation Simulation

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ABSTRACT

In many cases, the shallow water equation give a very sufficient account of the evolution of flood waves in the rivers. The numerical resolution of these equations was achieved using the MATLAB calculator, using the finite differences method. Initial and boundary conditions were varied to compare results in flood calculations and the impact on the river's flood height. Also, it is assumed that the flood waves diffuse, hence the use of the model of Barré Saint Venant on the principle of the diffusing wave. The benefits of these simplified processes can be seen primarily in the development of flood forecasting systems. The results obtained take account of the variation.

Keyword : *shallow water equation, finite difference method, Matlab*

1. INTRODUCTION

Understanding flows in the natural environment is an important scientific and human issue. Indeed the majority of humans live along rivers or coasts, hence the interest to approach the study of floods in rivers.

The theory of propagation in watercourses is based on shallow water equation which describe the various, non-permanent and non-uniform flows. Although the shallow water equation imply approximations in order to simplify the mathematical formulation of the phenomen, propagation is still a complex problem for free-surface rivers. [1]

Free surface flows often return to the field of investigation and intervention of the environmental engineer. In this case, the depth of the water layer is low in front of the horizontal extension of the observed phenomena. This is the case with flows in rivers or canals, runoff on the ground or large-scale movements of a lake.

A flood can be considered a wave propagating into a river. A wave is characterized by a propagation speed, amplitude, wavelength, frequency and damping rate.

In nature, there are several types of flood wave: progressive flood wave, kinetic flood wave, dynamic flood wave and diffusing flood wave. In this study, we will address the principle of the diffusing wave, assuming that there are no lateral inputs or losses and that the terms of inertia are negligible before the terms of gravity. Thus, in a first step, we will solve the shallow water equation, in order to know the variation of the flood flow according to the initial conditions and the limits. We will see the impact of the flood depending on time and distance.

2. METHODS

The models of the predominant hydraulic propagation aim to predict the propagation downstream of the rise of the waters according to shallow water equation.

2.1 shallow water equation

The almost one-dimensional flows of the rivers are described by the Barré – Saint Venant equations [2].
Based on a series of hypotheses, these equations are two in number:
A partial differential equation that expresses mass conservation. That's the continuity equation.

$$\frac{\partial S}{\partial t}+\frac{\partial Q}{\partial x}=q \qquad (1.1)$$

with:
- q linear flow rate representing inputs (q>O) or lateral samples (q<O).
- S wet section.
- Q flow rate.
- x curvilinear abscissa along the river.
- t time.

A second partial differential equation that expresses the preservation of the amount of motion is the dynamic equation.

$$\frac{dV}{dt}+V\frac{dV}{dt}+g\frac{dz}{dx}+gJ=0 \qquad (1.2)$$

with the first two terms called inertia terms representing variations in the amount of motion, the third term the gravity and variation of the pressure forces and the fourth term friction.
- g acceleration of gravity.
- J load loss per unit length (slope of the load line at steady state).
- z side of the free surface.
- v flow rate.

2.2 Principles of diffusive flood wave

In addition to the conservation of mass resulting from the conservation of volume, the incompressibility in particular, and the preservation of the quantity of movement, in which it is taken into account that the terms of inertia are negligible before the terms of gravity, It is also assumed that the flow is quasi-horizontal (slight curvature of the current lines, otherwise the pressure in the flow is assumed to be hydrostatic as in a fluid at rest.
Therefore, it is necessary for us to choose a form of stream that meets all these conditions, that is, the hypothesis of unidimensionality.
The vertical cut of the stream is considered to be a rectangle according to chart-1. The free surface is assumed to be horizontal. B represents the width

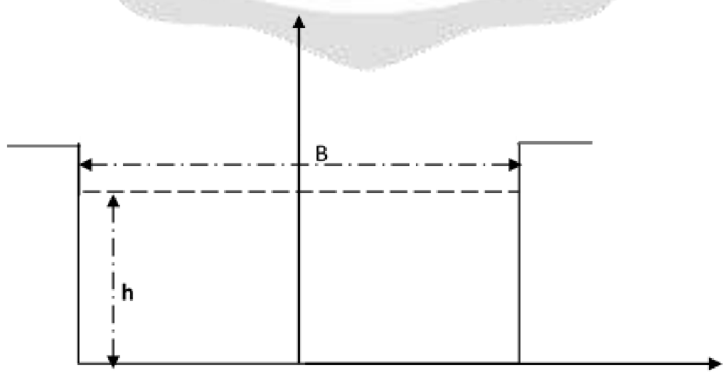

Fig -1: Cross-section of a river

The wet area is B.h, so the continuity equation is written:

$$\frac{\partial Q}{\partial x} + B\frac{\partial h}{\partial t} = 0 \qquad (1.3)$$

2.2.1 Diffusing wave speed

Let C be the speed of the wave and Q(x,t) the flow, we have

$$dQ = \frac{\partial Q}{\partial x}dx + \frac{\partial Q}{\partial t}dt = 0 \qquad (1.4)$$

in an uniform regime.

From this equation we deduce the velocity V_{obs} , the maximum speed of the flood wave:

$$V_{obs} = \frac{dx}{dt} = -\frac{\left(\frac{\partial Q}{\partial t}\right)}{\left(\frac{\partial Q}{\partial x}\right)} \qquad (1.5)$$

With continuity equation (1.3), we have:

$$V_{obs} = \frac{1}{B}\frac{dQ}{dh} \qquad (1.6)$$

We consider that the floods are very slow, that is to say a steady regime, and the formula of Stickler gives

$$V = Kh^{\frac{2}{3}}I^{\frac{1}{2}} \qquad (1.7)$$

And the logarithmic derivative gives:

$$\frac{dV}{V} = \frac{2}{3}\frac{dh}{h} \qquad (1.8)$$

As we can write:

$$Q = BhV\ , \qquad (1.9)$$

So we get $V_{obs} = \frac{5}{3}V$, otherwise it is the speed of the maximum flood or the maximum speed :

$$V_{obs} = C = \frac{5}{3}V = 1,67V \qquad (1.10)$$

Table -1: *V depending particle size [3]*

Nature du matériau	Granulométrie représentative	V (m/s)
Fine sediment	0,06 -0,20	0,20 – 0,30
Sand	0,20 – 0,60	0,30 – 0,55
Gross sand	0,60 – 2,00	0,55 – 0,65
Fine gravel	2 – 6	0,65 – 0,80
Medium gravel	6 – 20	0,80 – 1,00
Large gravel	20 – 30	1,00 – 1,40
Small pebbles	30 – 50	1,40 – 1,80
Medium pebbles	50 – 75	1,80 – 2, 40
Rubbles	75- 100	2,40 – 2,70

2.2.2 Propagation equation

Starting from the equations of continuity (1.3) and Saint-Venant and drifting successively in relation to x and t, we have:

$$\sigma \frac{\partial^2 Q}{\partial x^2} = c\frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial t} \tag{1.11}$$

where σ represents the attenuation coefficient, as:

$$\sigma = \frac{1}{B\frac{\partial J}{\partial Q}} \text{, and } J = \frac{Q^2}{B.K.h^{10/3}}, \tag{1.12}$$

The expression of $\sigma = \frac{Q}{2.J.B}$ (1.13)

2.2.3 Boundary conditions

To solve the equation, here are the initial conditions and limits used:

- ✓ For t = 0, we have $\frac{\partial Q}{\partial t} = 0$.

  Therefore equation (1.4) becomes $\sigma \frac{\partial^2 Q}{\partial x^2} = c\frac{\partial Q}{\partial x}$, (1.14)

  Solving equation (1.11) give us $Q(x) = Q_0 \exp\left(\frac{c}{\sigma}x\right)$ for t=0. (1.15)

- ✓ For x=0, h=h_0 .
- ✓ And for x=L, we have $\frac{\partial Q}{\partial x} = 0$ (1.16).

2.2.3 Attenuation coefficient σ

Remember expression $\sigma = \frac{Q}{2.J.B}$ (1.17)

We see that is proportional to the flow Q, but inversely proportional to the width B and the slope J.

In chart-2, we have taken Q=68m3/s, and we plot according to the slope J. The representative curve of σ is decreasing function of J, for different values of the width of the mirror B. The same is verified for Q=18m3/s (chart-3).

The difference between these two sets of curves is that for Q=68m3/s and B=2m, σ can reach 2428.57m²/s, while for Q=18m3/s and for the same value of B=2m, σ is 642.86m²/s.

Again for these 2 series of curves, σ is larger for low B values. The two flow values Q=68m3/s and Q=68m3/s correspond to the maximum and minimum high flood values of the Ivondro river, Madagascar from 1954 to 1982 [4].

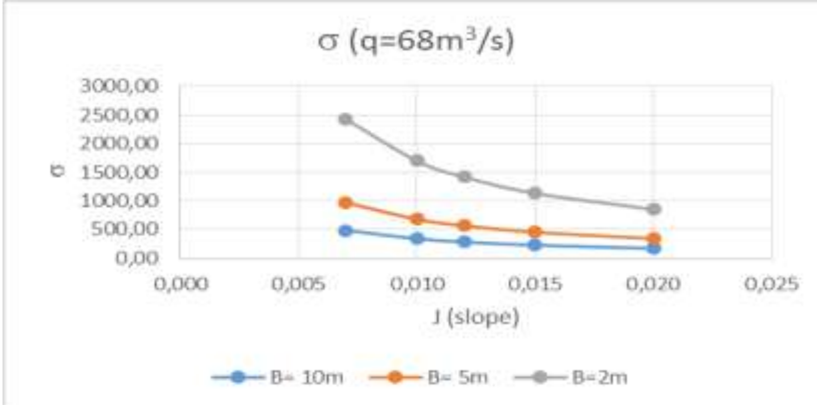

Chart -2: σ variations with J, for different B, for Q=68m^3/s

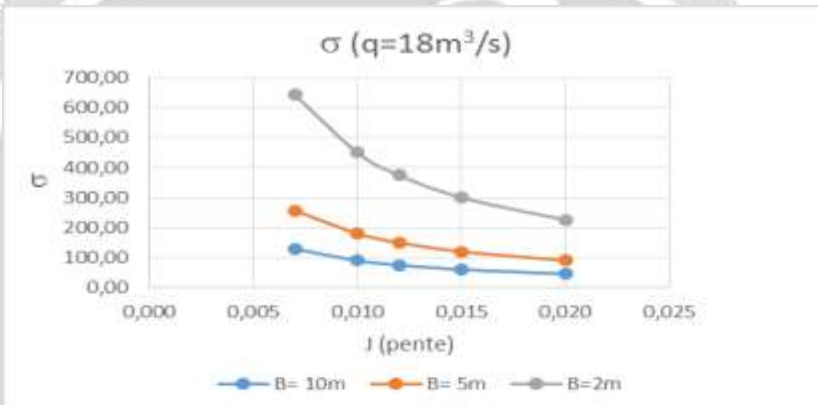

Chart -3: σ variations with J, for different B, for Q=18m^3/s

In the suite, the width of the mirror B is fixed at 5 meters. table 2 gives values of σ according to slope J, corresponding to Q=68m3/s.

Table-2: Values of as a function of slope J

σ (m²/s)	B	J
971,43	5m	0,007
680,00	5m	0,010
485,71	5m	0,015
309,09	5m	0,022

In the numerical resolution of the motion equation, we will take 3 initial flow values, at time t=0, q0=5m3/s, q0=10m3/s et q0=15m3/s.

The curves below show the flow rate evolution and the impact on the flood height over time. The σ value was set to 971.43m²/s, corresponding to a slope of 0.007.

In chart-4, we see that the Q curves decrease hyperbolically over space, for all the initial values of q0.

For the different initial values of q0 and for σ=971.43m²/s, the representative curves of Q are all the larger if q0 is large.
After 15 minutes, at the distance 2000m from the origin, the flood rate is 33,90% of q0, whatever the value of q0.

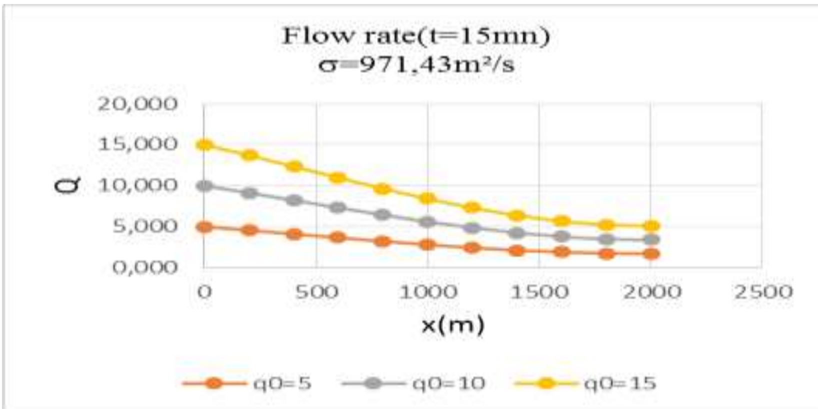

Chart -4: Flow rate evolution over 2000m after 15mn, for different values of q_0.

In chart-5, we represent the same curves in chart-4, but after 1 hour. We see that the Q curves are almost horizontal. And at 2000m of the origin, the flood rate Q is 92.44% of q_0, whatever the values of q_0.

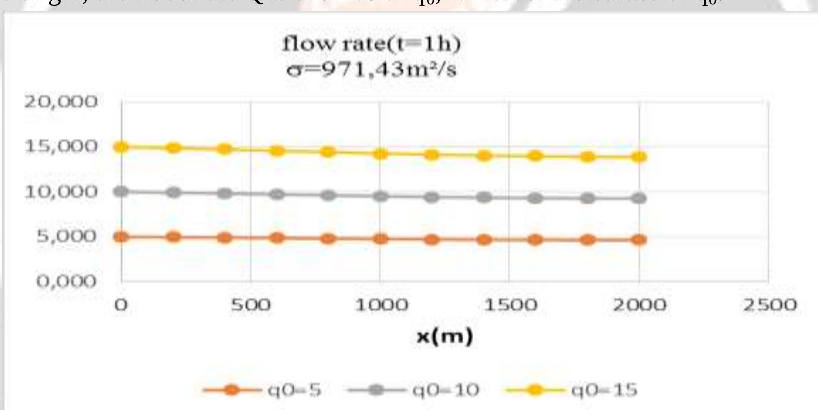

Chart -5: Flow rate evolution over 2000m after 1 hour, for differents values of q_0.

At the moment 2 hours, chart-6, the three curves of Q, corresponding to q_0=5m^3/s, q_0=10m^3/s and q_0=15m^3/s are horizontal. The calculation shows that 99.58% of the flood reaches 2000m.

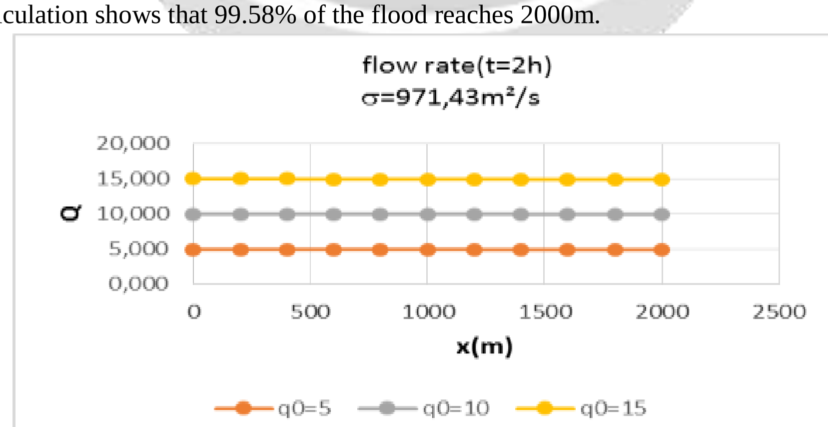

Chart -6: Flow rate evolution over 2000m after 2 hours, for different values of q_0.

In a second step, the coefficient of attenuation σ will be varied, which is equivalent to the decrease in the slope of the river (Table-2).

And with an accuracy of 10-3 of the flow, the calculation gives us the arrival time of the downstream flow, x = 2000m, depending on the value of σ.

Chart-7 shows that the increase in the attenuation coefficient only decreases the time taken to reach downstream. And naturally, it increases according to the value of q0.

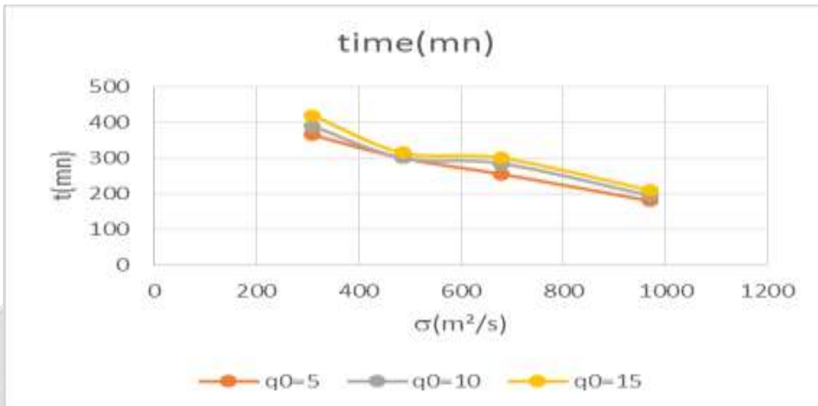

Chart-7: Evolution of the total transmission time of the initial flow rate, for different values of q_0

Now, to get the height, we solve the equation, using the different values of Q, obtained previously. The results are summarized in the curves below. As for the flow rate, several parameters will be taken into account: the attenuation coefficient, the initial flow rate q0.
In chart-8, we assume that the initial stream height is 1m, for the different initial flow values, q0=5m3/s, q0=10m3/s and q0=15m3/s keeping a constant value of σ=971.43m²/s.
Originally, x=0m, after 1h45mn, the height of the stream stabilizes at:

- h=6,24m for q0=5m3/s
- h=11,48m for q0=10m3/s
- h=16,72m for q0=15m3/s

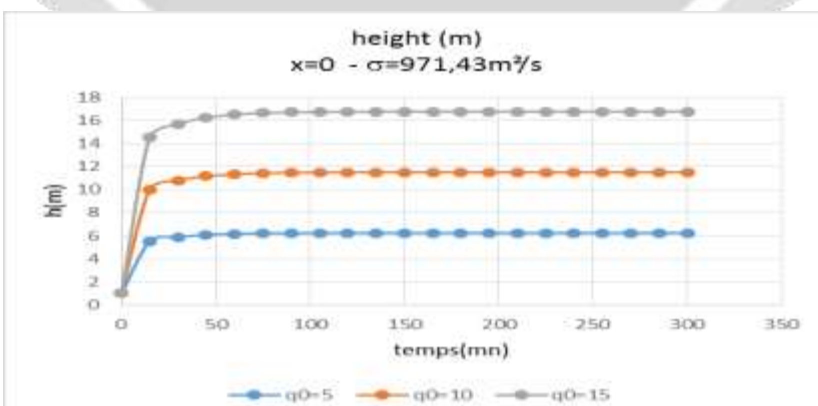

Chart-8: Evolution of height upstream (x=0) as a function of time for σ=971.43m²/s

In chart-9, at the distance x=400m from the origin, always for σ=971.43m²/s, and for h=1m at the initial moment, the curves stabilize after 1h45mn at:

- h=1,78m for q_0=5m^3/s

- h=2,56m for q_0=10m^3/s
- h=3,35m for q_0=15m^3/s

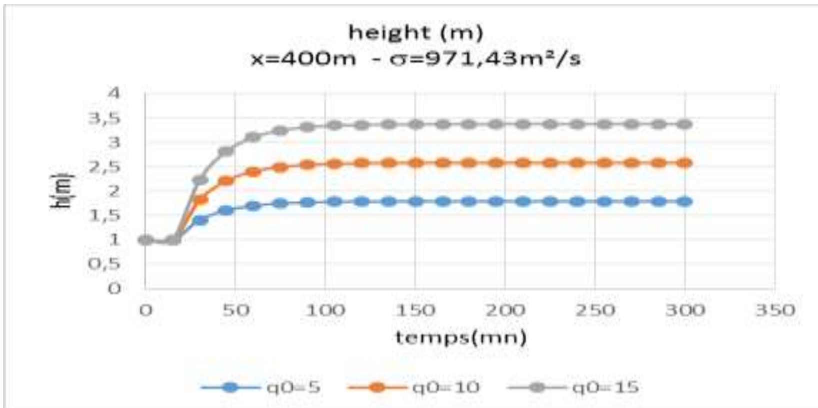

Chart-9: Height at point x=400m as a function of for different q_0, with σ =971.43m²/s

In chart-10, at the distance x=1000m from the origin, always for σ=971.43m²/s, and for h=1m at the initial moment, the curves stabilize after 2h15mn at:

- h=1,64m for q_0=5m^3/s
- h=2,29m for q_0=10m^3/s
- h=2,93m for q_0=15m^3/s

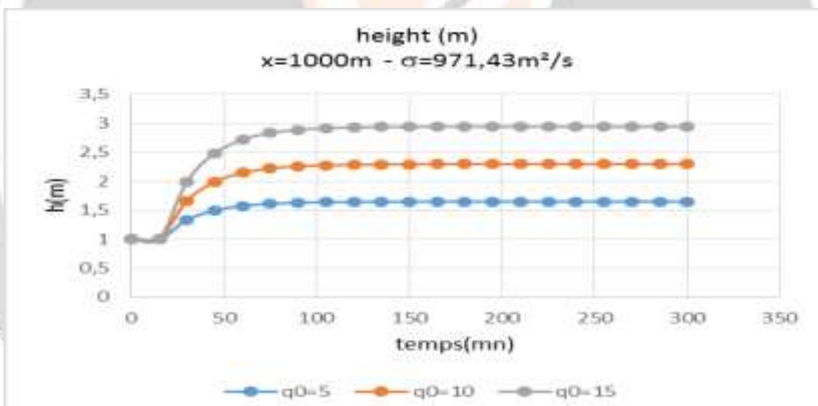

Chart-10: Height at point x=1000m as a function of time for different q_0, with σ =971.43m²/s

As we move away from the origin, the gap between the 3 curves of the river height as a function of time decreases. In chart-11, for x=1600m, these curves stabilize only after 2h15mn at:

- h=1,26m for q_0=5m^3/s
- h=1,52m for q_0=10m^3/s
- h=1,78m for q_0=15m^3/s

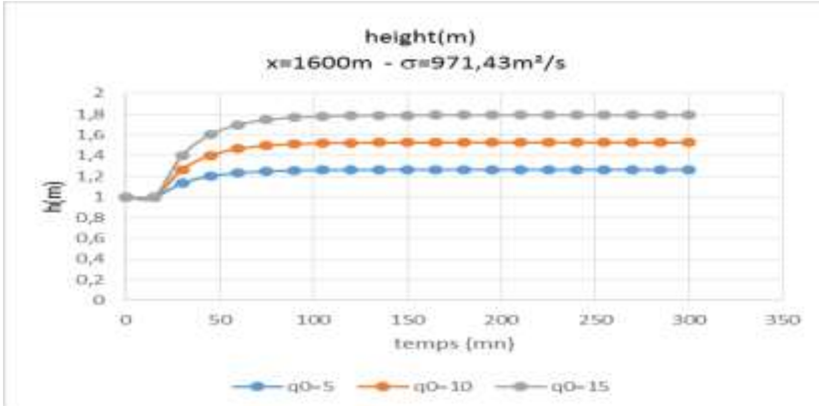

Chart-11 : Height at point x=1600m as a function of time for different q_0, with σ =971.43m²/s

In chart-12, at point x=2000m of the origin, the heights stabilize a little forward, at the moment t=2h, with:

- h=1,27m for $q_0=15m^3/s$
- h=1,18m for $q_0=10m^3/s$
- h=1,09m for $q_0=5m^3/s$

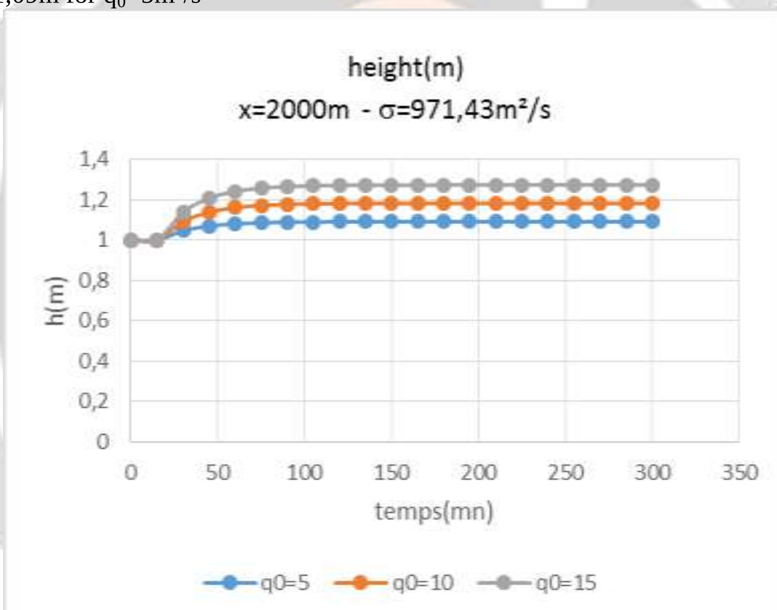

Chart-12: Height at point x=2000m as a function of time for different q_0, with σ =971.43m²/s

Chart- 13 and chart-14 show the river height for q_0=5, 10 and 15m3/s, σ=971.43m²/s, depending on x, corresponding to t=1h and t=5h. We see that the heights stabilize around x=400m.

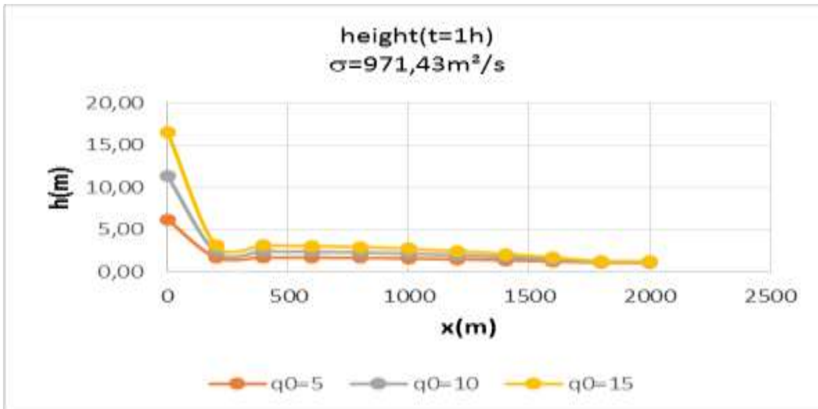

Chart-13: Evolution of the height along the river for different values of q_0, with σ =971.43m²/s, at time t=1h

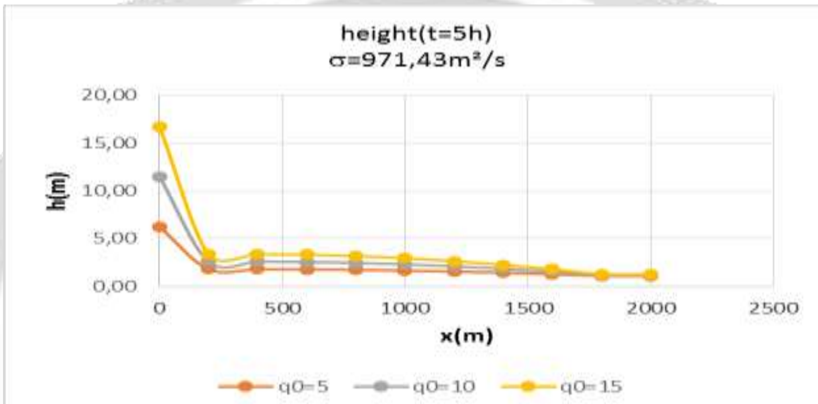

Chart-14: Evolution of the height along the river for different values of q_0, with σ =971.43m²/s, at time t=5h

Chart-15 and chart-16 represent the variation of river height h based on the initial q0 values at the origin and x=2000m from the origin, always setting the value of σ =971.43m²/s, for different time values.
For t=0, the height is always 1m. Indeed, for t=1, 2, 3, 4 and 5h, originally x=0, the different curves are superimposed according to equation h (m) = 1,0503q0 + 1,000 (chart-15). And at x=2000m of the origin, the equation of this curve is h(m) = 0.0182q0 + 1.000 (chart-16).

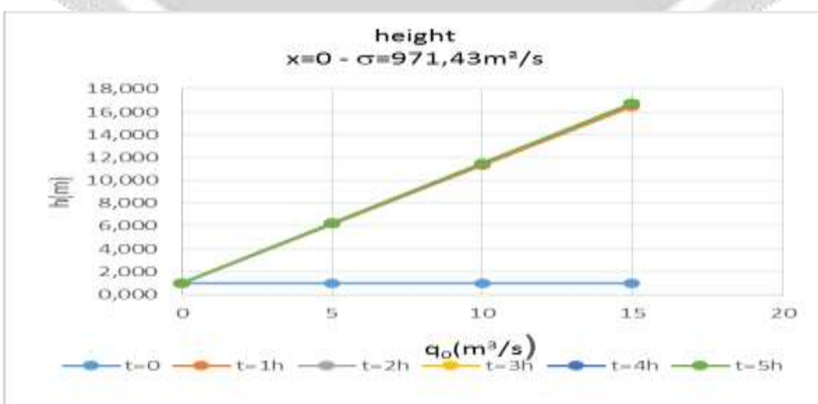

Chart-15: Evolution of height in upstream as a function of q0, for different values of time

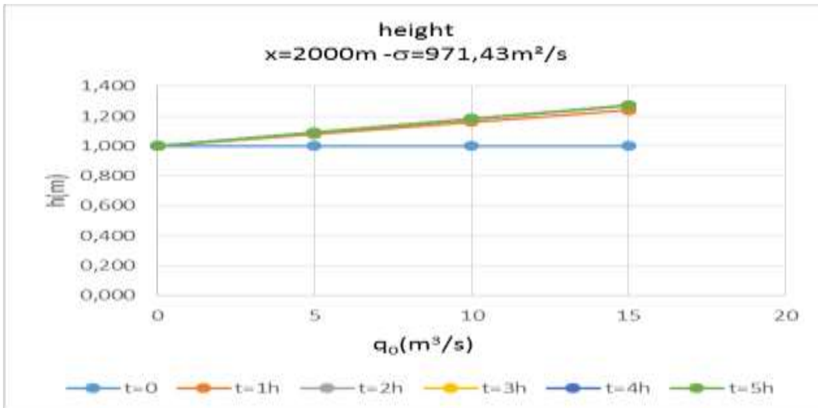

Chart-16: Height evolution in downstream, x=2000m, as a function of q0, for different values of time

In chart-17, the representative curves of height h as a function of q0, at x=2000m from the origin, at the time t=5h, according to the values of σ=309.09, 485.71, 680 and 971.43m²/s are straight lines, whose equations are:

- for =971.43m²/s, $h(m) = 0,0182q_0 + 1$, $R^2 = 1$;
- for =680m²/s, $h(m) = 0,0275q_0 + 1$, $R^2 = 1$;
- for =485.71m²/s, $h(m) = 0,0388q_0 + 1$, $R^2 = 1$;
- for =309.09m²/s, $h(m) = 0,0582q_0 + 1$, $R^2 = 1$;

The slopes of these lines decrease with σ.

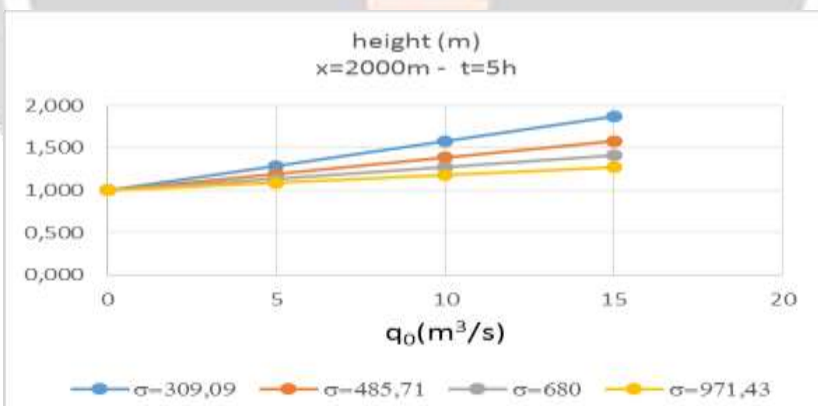

Chart-17: Height evolution in downstream, x=2000m, as a function of q0, for different values after 5h

In chart-18, we represent these same curves at the x=2000m x-point, at the time t=5h, but according to the attenuation coefficient σ . The different curves are almost hyperbolic. If we look for the equation of these curves in third-degree polynomial form, we obtain the following results:

- ✓ for q_0=5m³/s, $h=-6.10^{-10}\sigma^3 + 2.10^{-6}\sigma^2 - 0,0015\sigma + 1,6238$, $R^2 = 1$
- ✓ for q_0=10m³/s, $h = -10^{-9}\sigma^3 + 3.10^{-6}\sigma^2 - 0,0031\sigma + 2,2692$, $R^2 = 1$
- ✓ for q_0=15m³/s, $h = -2.10^{-9}\sigma^3 + 510^{-6}\sigma^2 - 0,0047\sigma + 2,8962$, $R^2 = 1$

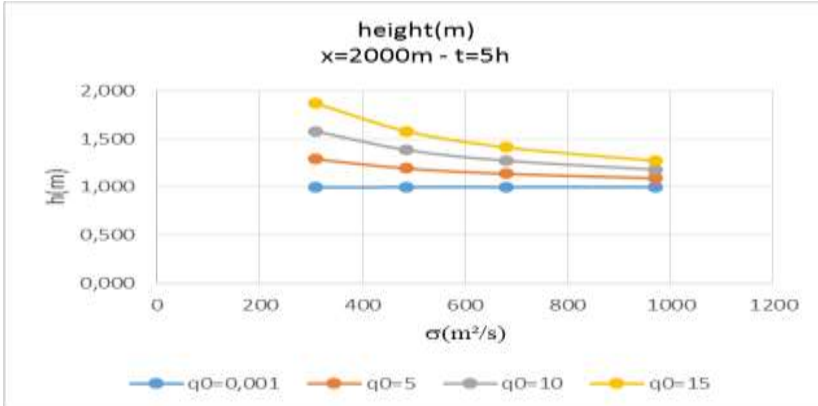

Chart-18: Height evolution in downstream, x=2000m, as a function of σ for different values of q_0 after 5h

4. CONCLUSIONS

We solved the equation of shallow water equation, in the hypothesis of a one-dimensional and quasi-horizontal flow, taking from the conservation of mass, the preservation of the quantity of movement, in which the terms of inertia are negligible before the terms of gravity. In the calculation, the B-mirror was fixed to 5 meters, which we assumed was the frequent case, but on the other hand, initial values of the constant flood rate over time and varied. We limited our study to a length of 2 000 metres for five hours. Also, the variation in the mitigation coefficient, corresponding to the variation in the slope of the river, was taken into account. Several results concerning the evolution of floods over time and in space.

The results obtained showed us that for a fixed value of the attenuation coefficient, the flood height increases linearly with the initial value of the flood flow, but decreases when one moves away from the origin. This justifies that the flood waves diffuse, and the maximum of this wave decreases when moving from upstream to downstream. On the other hand, by fixing the initial flow rate, the height decreases hyperbolically according to the attenuation coefficientσ. The polynomial approximation of order 3 allowed us to interpret the results obtained in the interval of studies.

Then we'll try to resume that work, but assuming that the initial flow varies over time.

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