STATIC ANALYSIS OF SIGMOID FGM PLATES

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ABSTRACT

The static characteristics of Sigmoid Functionally Graded Material (S-FGM) plates of double curvature motivates the author to perform this research study. Principle of virtual work are utilised to derive the governing equation of motion and associated natural boundry conditions. Analytical solutions are obtained using Navier's technique for simply supported boundry conditions. Some numerical results are compared with published results and found to be in excellent agreement. Both the effect of shear strain and displacement are included in the theory. Based on a nonlocal elasticity theory, a model for sigmoid functionally graded material (S-FGM) nanoscale plate with first-order shear deformation is studied. The material properties of S-FGM nanoscale plate are assumed to vary according to sigmoid function (two power law distribution) of the volume fraction of the constituents. The solutions of S-FGM nanoscale plate are presented to illustrate the effect of nonlocal theory on bending response of the S-FGM nanoscale plates. The effects of nonlocal parameters, power law index, aspect ratio, elastic modulus ratio, side-to-thickness ratio, and loading type on bending response are investigated.

Keyword: - *Sigmoid Plate theory, Functionally graded material plates, Static Analysis*

1. INTRODUCTION

Functionally graded materials (FGMs) are multifunctional materials, which contain a spatial variation in composition and/or microstructure for the specific purpose of controlling variations in thermal, structural or functional properties. FGMs produced using ceramic and metal has the property of metallic tenacity and yet it is heat proof and anti-corrosive like ceramic. It can also be used as a material to withstand thermal stress. The solutions of S-FGM nanoscale plate are presented to illustrate the effect of nonlocal theory on bending and vibration response of the S-FGM nanoscale plates. The effects of nonlocal parameters, power law index, aspect ratio, elastic modulus ratio, side-to-thickness ratio, and loading type on bending and vibration response are investigated. Results of the present theory show a good agreement with the reference solutions. These results can be used for evaluating the reliability of size-dependent SFGM nanoscale plate models developed in the future.

The FGM plate has been modeled using a three-dimensional (3D) theory of linear elasticity or by two-dimensional (2D) plate theory of plane stress and plane strain. Mindlin [1] developed the transverse shear effect was, based on the linear variation of transverse shear shape function, which is known as "First-order Shear Deformation Theory (FSDT). Due to the linear variation of transverse shear shape function, transverse shear strain and hence the transverse shear stress results in a constant value which violates the parabolic variation of transverse shear stress along with the thickness.

Reddy [2] presented the most widely used polynomial based "Higher-order Shear Deformation Theory" (HSDT) is "Third-order Shear Deformation Theory" (TSDT) by expanding the m-plane displacements up to the third order of thickness coordinate and satisfying the zero transverse shear stresses at the top and the bottom surface of the plate. Thai and Vo [3] proposed a theory that contained four unknowns and also had a strong similarity with CPT Neves et al [4] proposed a Quasi 3D SSDT theory in which polynomial function is considered in the transverse direction.

Mantarı et al. [5] proposed mixed sinusoidal and exponential based shear strain function Thai et al and Nguyen et al [6] proposed inverse tangential shear shape function for composite laminated plates and FGM plates respectively. The inverse co-tangent shear shape function is proposed by Grover et al [7] for static buckling and free vibration analysis of composite laminated plates.

The objective of this article is to present the static behaviour of sigmoid FGM plates. The plate may be either perfectly porous homogenous or has a perfect homogeneity shape depending on the values of the volume fraction of voids or of the graded factors.

A Navier solution is used to obtain closed form solutions for simple supported FG plates. Several important aspects, i.e. aspect ratios, exponent graded factor as well as porosity volume fraction, which affect deflections and stresse, are investigated.

2. BASIC ASSUMPTIONS OF THE PLATE THEORY

Consider a FG thick rectangular plate of length a, width b, and thickness. The coordinate system is taken such that the x-y plane coincides with midplane of the plate, Let the FG plate be subjected to a transverse load q(x ,y) The plate is composed of a functionally graded material across the thickness direction. The assumptions of the present plate theory areas follows:-

- the displacements are small in comparison with the plate thickness, therefore, the strains involved are infinitesimal;
- the transverse displacement w includes two components of bending w, and shear w, these components are functions of the coordinates x,y only.

$$w(x,y,z) = w_b(x,y) + w_s(x,y) \tag{1}$$

- the transverse normal stress σz, is negligible in companson with in-plane stresses σx and σy.
- the displacements u in the x-direction and v in the y-direction consist of extension, bending and shear components,

$$U = u_0 + u_b + u_s\,, \quad V = v_0 + v_b + v_s \tag{2}$$

The bending components u_b and v_b are assumed to be similar to the displacements given by the classical plate theory. Therefore, the expression for u_b and v_b can be given as

$$u_b = -z\frac{\partial w_b}{\partial_x}\ , \quad v_b = -z\frac{\partial w_b}{\partial_y} \tag{3}$$

The shear components $\mathbf{u_s}$ and $\mathbf{v_s}$ give rise, in conjunction with $\mathbf{w_s}$ to the parabolic variations of the shear strains $\boldsymbol{\gamma}_{xz}$ and $\boldsymbol{\gamma}_{yz}$ and hence to shear stresses $\boldsymbol{\tau}_{xz}$ and $\boldsymbol{\tau}_{yz}$ through the thickness of the plate in such a way that shear stresses $\boldsymbol{\tau}_{xz}$ and $\boldsymbol{\tau}_{yz}$ are zero at the top and bottom faces of the plate. Consequently, the expression for u_s and v_s can be given as

$$\mathrm{u_s} = \mathrm{f(z)}\frac{\partial \mathrm{w_s}}{\partial_x}\ , \quad \mathrm{v_s} = \mathrm{f(z)}\frac{\partial \mathrm{w_s}}{\partial_y} \tag{4}$$

where

$$f(z) = \frac{h}{\Pi}\left(sin\frac{\Pi z}{h}\right) - z \tag{5}$$

3. KINEMATICS

3.1 Displacement field

The displacement field of the FG plate under consideration is given below:

$$u = u_0 - z\frac{\partial w_b}{\partial x} + f(z)\frac{\partial w_s}{\partial x}$$
$$v = v_0 - z\frac{\partial w_b}{\partial y} + f(z)\frac{\partial w_s}{\partial y}$$
$$w = w_b + w_s \qquad (6)$$

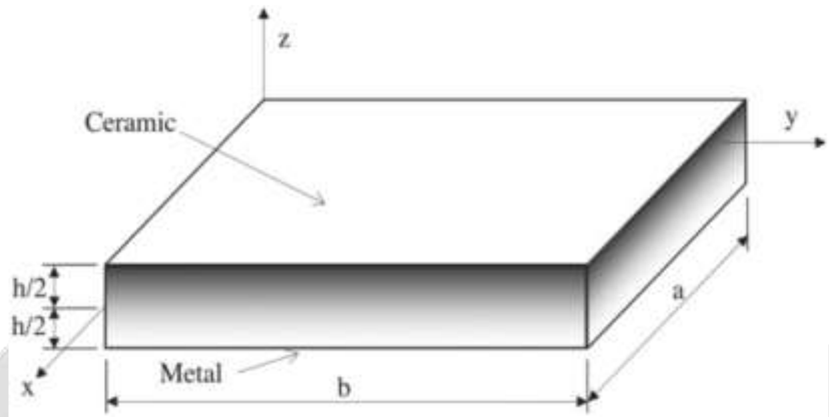

Fig -1: Geometry and coordinates of the FG plate

3.2 Strain- Displacement relationship

For the small plate deformation, the six strain components

($\boldsymbol{\varepsilon}_{x}$, $\boldsymbol{\varepsilon}_{y}$, $\boldsymbol{\varepsilon}_{z}$, $\boldsymbol{\gamma}_{xy}$, $\boldsymbol{\gamma}_{xz}$, $\boldsymbol{\gamma}_{yz}$) and three displacement components (u,v,w) are related according to the well-known linear kinematic relations.

$$_x = \frac{\partial u_0}{\partial x} - z\,\frac{\partial^2 w_b}{\partial x^2} + f(z)\frac{\partial^2 w_s}{\partial x^2} \quad _y = \frac{\partial v_0}{\partial y} - z\,\frac{\partial^2 w_b}{\partial y^2} + f(z)\frac{\partial^2 w_s}{\partial y^2}$$

$$_z = 0$$

$$\gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial y} - 2z\,\frac{\partial^2 w_b}{\partial x \partial y} + 2f(z)\frac{\partial^2 w_s}{\partial x \partial y}$$

$$\gamma_{xz} = \frac{\partial u_0}{\partial z} - z\,\frac{\partial^2 w_b}{\partial x \partial z} + f(z)\frac{\partial^2 w_s}{\partial x \partial z} + \frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}$$

$$\gamma_{yz} = \frac{\partial v_0}{\partial z} - z\,\frac{\partial^2 w_b}{\partial y \partial z} + f(z)\frac{\partial^2 w_s}{\partial y \partial z} + \frac{\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y} \qquad (7)$$

3.3 Stress Strain relationship

The stress strain relationship of FG plate is given as follows:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \qquad (8)$$

Where,

$$Q_{11} = Q_{22} = \frac{E(z)}{(1-\mu^2)}$$

$$Q_{12} = \frac{\mu E(z)}{(1-\mu^2)}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+\mu)} \quad (9)$$

Where,

$$E(z) = E_m + (E_c - E_m)\left(\frac{z}{h} + 0.5\right)^P \quad (10)$$

Where Ec and Em are the corresponding properties of the ceramic and the metal respectively, and p is the volume fraction exponent, which takes value greater than or equal to zero.

3.4 Governing equations and boundary conditions

For the case of static analysis, According to Principle of virtual work,

$$\int_0^a \int_0^b \int_{-h/2}^{+h/2} [\sigma_x.\delta\varepsilon_x + \sigma_y.\delta\varepsilon_y + \tau_{xy}.\delta\gamma_{xy} + \tau_{xz}.\delta\gamma_{xz} + \tau_{yz}.\delta x\gamma_{yz}]\, dx.dy.dz = \int_0^a \int_0^b q.\delta w.dx.dy \quad (11)$$

Where, δ is strain energy variation of the plate.

Substituting values of stresses and strains from Eq (7) in Eq (11), then integrating it by parts and equating it to zero we get six governing equations as follows:

$$\delta u_{\circ} = \left(-A_{11}\frac{\partial^2 u_{\circ}}{\partial x^2}\right) - \left(A_{66}\frac{\partial^2 u_{\circ}}{\partial y^2}\right) - \left(A_{12}\frac{\partial^2 v_{\circ}}{\partial x \partial y}\right) - \left(A_{66}\frac{\partial^2 v_{\circ}}{\partial x \partial y}\right) + \left(B_{11}\frac{\partial^3 wb}{\partial x^3}\right) + \left(B_{12}\frac{\partial^3 wb}{\partial x \partial y^2}\right) + \left(2B_{66}\frac{\partial^3 wb}{\partial x \partial y^2}\right) - \left(C_{11}\frac{\partial^3 ws}{\partial x^3}\right) - \left(C_{12}\frac{\partial^3 ws}{\partial x \partial y^2}\right) - \left(2C_{66}\frac{\partial^3 ws}{\partial x \partial y^2}\right) = 0 \quad (12)$$

$$\delta v_{\circ} = \left(-A_{66}\frac{\partial^2 u_{\circ}}{\partial x \partial y}\right) - \left(A_{21}\frac{\partial^2 u_{\circ}}{\partial x \partial y}\right) - \left(A_{66}\frac{\partial^2 v_{\circ}}{\partial x^2}\right) - \left(A_{22}\frac{\partial^2 v_{\circ}}{\partial y^2}\right) + \left(B_{22}\frac{\partial^3 wb}{\partial y^3}\right) + \left(B_{21}\frac{\partial^3 wb}{\partial x^2 \partial y}\right) + \left(2B_{66}\frac{\partial^3 wb}{\partial x^2 \partial y}\right) - \left(C_{22}\frac{\partial^3 ws}{\partial y^3}\right) - \left(C_{21}\frac{\partial^3 ws}{\partial x^2 \partial y}\right) - \left(2C_{66}\frac{\partial^3 ws}{\partial x^2 \partial y}\right) = 0 \quad (13)$$

$$\delta w_b = \left(-B_{11}\frac{\partial^3 u_{\circ}}{\partial x^3}\right) - \left(B_{21}\frac{\partial^3 u_{\circ}}{\partial x \partial y^2}\right) - \left(2B_{66}\frac{\partial^3 u_{\circ}}{\partial x \partial y^2}\right) - \left(B_{12}\frac{\partial^3 v_{\circ}}{\partial x^2 \partial y}\right) - \left(B_{22}\frac{\partial^3 v_{\circ}}{\partial y^3}\right) - \left(2B_{66}\frac{\partial^3 v_{\circ}}{\partial x^2 \partial y}\right) + \left(D_{11}\frac{\partial^4 wb}{\partial x^4}\right) + \left(D_{12}\frac{\partial^4 wb}{\partial x^2 \partial y^2}\right) + \left(D_{21}\frac{\partial^4 wb}{\partial x^2 \partial y^2}\right) + \left(4D_{66}\frac{\partial^4 wb}{\partial x^2 \partial y^2}\right) + \left(D_{22}\frac{\partial^4 wb}{\partial y^4}\right) - \left(E_{11}\frac{\partial^4 ws}{\partial x^4}\right) - \left(E_{12}\frac{\partial^4 ws}{\partial x^2 \partial y^2}\right) - \left(E_{21}\frac{\partial^4 ws}{\partial x^2 \partial y^2}\right) - \left(4E_{66}\frac{\partial^4 ws}{\partial x^2 \partial y^2}\right) - \left(E_{22}\frac{\partial^4 ws}{\partial y^4}\right) = -q \quad (14)$$

$$\delta w_s = \left(C_{11}\frac{\partial^3 u_{\circ}}{\partial x^3}\right) + \left(C_{21}\frac{\partial^3 u_{\circ}}{\partial x \partial y^2}\right) + \left(2C_{66}\frac{\partial^3 u_{\circ}}{\partial x \partial y^2}\right) + \left(C_{12}\frac{\partial^3 v_{\circ}}{\partial x^2 \partial y}\right) + \left(C_{22}\frac{\partial^3 v_{\circ}}{\partial y^3}\right) + \left(2C_{66}\frac{\partial^3 v_{\circ}}{\partial x^2 \partial y}\right) - \left(E_{11}\frac{\partial^4 wb}{\partial x^4}\right) - \left(E_{12}\frac{\partial^4 wb}{\partial x^2 \partial y^2}\right) - \left(E_{21}\frac{\partial^4 wb}{\partial x^2 \partial y^2}\right) - \left(4E_{66}\frac{\partial^4 wb}{\partial x^2 \partial y^2}\right) - \left(E_{22}\frac{\partial^4 wb}{\partial y^4}\right) + \left(F_{11}\frac{\partial^4 ws}{\partial x^4}\right) + \left(F_{12}\frac{\partial^4 ws}{\partial x^2 \partial y^2}\right) + \left(F_{21}\frac{\partial^4 ws}{\partial x^2 \partial y^2}\right) + \left(4F_{66}\frac{\partial^4 ws}{\partial x^2 \partial y^2}\right) + \left(F_{22}\frac{\partial^4 ws}{\partial y^4}\right) + \left(G_{44}\frac{\partial^2 ws}{\partial x^2}\right) + \left(G_{55}\frac{\partial^2 ws}{\partial x^2}\right) = -q \quad (15)$$

Solving Eq. 6,7,8,9,and 10 we get stiffness constants.

Where stiffness constants are,

$A_{ij} = Q_{ij} \int_{-h/2}^{h/2} dz$

$B_{ij} = Q_{ij} \int_{-h/2}^{h/2} z\, dz$

$C_{ij} = Q_{ij} \int_{-h/2}^{h/2} f(z) dz$

$D_{ij} = Q_{ij} \int_{-h/2}^{h/2} z^2 dz$

$E_{ij} = Q_{ij} \int_{-h/2}^{h/2} z\, f(z) dz$

$F_{ij} = Q_{ij} \int_{-h/2}^{h/2} f(z)^2 dz$

$G_{ij} = Q_{ij} \int_{-h/2}^{h/2} f'(z) dz$ (16)

3.5. Navier's Solution

To prove the efficient and validity of presented theory, the Navier's solution technique is employed to determine numerical solution for simply supported FG plate. Following are boundary conditions at simply supported edge of plate.

At x=0, x=a and y=0, y=b

The following solution form is aasumed for unknown variables in displacement fields which satisfies simply supported boundry conditions exactly mentioned in equations.

$$u_0 = u_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$v_0 = v_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b}$$

$$wb = wb_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$ws = ws_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \quad (17)$$

Where, $\alpha = \frac{m\pi}{a}$ and $\beta = \frac{n\pi}{b}$ and m and n are mode numbers. For the case of a sinusoidally distributed load, we have m = n = 1 and $q_{11} = q_\circ$ where $q_\circ$ represents the intensity of the load at the plate's center.

Substituting Navier's solution form in Eq (12) to (15) we get following four equations, $\delta u_0 = (A_{11}\alpha_2 + A_{66}\beta_2)u_{mn} + (A_{12} + A_{66})\alpha\beta\, v_{mn} - [(B_{12} + 2B_{66})\alpha\beta_2 +$

$B_{11}\beta^3]\, w_{bmn} + [(C_{12} + 2C_{66})\alpha\beta^2 + C_{11}\alpha^3]\, w_{smn} = 0$ (18)

$\delta v_0 = (A_{66}\alpha_2 + A_{22}\beta_2)u_{mn} + (A_{12} + A_{66})\alpha\beta\, v_{mn} - [(B_{12} + 2B_{66})\alpha\beta_2 +$

$B_{11}\beta^3]\, w_{bmn} + [(C_{12} + 2C_{66})\alpha\beta^2 + C_{22}\alpha^3]\, w_{smn} = 0$ (19)

$\delta w_b = -[(B_{21} + 2B_{66})\alpha\beta_2 + B_{11}\alpha_3]u_{mn} - [(B_{12} + 2B_{66})\alpha_2\beta + B_{22}\beta_3]v_{mn} +$

$[(D_{11}\alpha_4) + (D_{12} + D_{21} + 4D_{66})\alpha_2\beta_2 + (D_{22}\beta_4)]\, w_{bmn} - [(E_{11}\alpha_4) + (E_{12} + E_{21} +$

$4E_{66})\alpha^2\beta^2 + (E_{22}\beta^4)]\, w_{smn} = -q_{mn}$ (20)

$$\delta w_s = [(C_{21} + 2C_{66})\alpha\beta^2 + C_{11}\alpha^3]u_{mn} + [(C_{12} + 2C_{66})\alpha^2\beta + C_{22}]v_{mn} - [(E_{11}\alpha^4) + (E_{12} + E_{21} + 4E_{66})\alpha^2\beta^2 + (E_{22}\beta^4)]\, w_{bmn} + [(F_{11}\alpha^4) + (F_{12} + F_{21} + 4F_{66})\alpha^2\beta^2 + (F_{22}\beta^4) + (G_{55}\alpha^2 + G_{44}\beta^2)]\, w_{smn} = -q_{mn} \quad (21)$$

One can write equations (18), (19), (20), (21) in following matrix form.

Matrix form,

[K] {Δ} = {F}

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \begin{Bmatrix} u_{mn} \\ v_{mn} \\ w_{bmn} \\ w_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -q_{mn} \\ -q_{mn} \end{Bmatrix} \quad (22)$$

Where [K] = Stiffness Matrix

The elements of stiffness matrix are:

$K_{11} = (A_{11}\alpha^2 + A_{66}\beta^2)$

$K_{12} = (A_{12} + A_{66})\alpha\beta$

$K_{13} = -[(B_{12} + 2B_{66})\alpha\beta^2 + B_{11}\alpha^3]$

$K_{14} = [(C_{12} + 2C_{66})\alpha\beta^2 + C_{11}\alpha^3]$

$K_{21} = (A_{21} + A_{66})\alpha\beta$

$K_{22} = (A_{66}\alpha^2 + A_{22}\beta^2)$

$K_{23} = -[(B_{21} + 2B_{66})\alpha^2\beta + B_{22}\beta^3]$

$K_{24} = [(C_{21} + 2C_{66})\alpha^2\beta + C_{22}\beta^3]$

$K_{31} = -[(B_{21} + 2B_{66})\alpha\beta^2 + B_{11}\alpha^3]$

$K_{32} = -[(B_{12} + 2B_{66})\alpha^2\beta + B_{22}\alpha^3]$

$K_{33} = [(D_{11}\alpha^4) + (D_{12} + D_{21} + 4D_{66})\alpha^2\beta^2 + (D_{22}\beta^4)]$

$K_{34} = -[(E_{11}\alpha^4) + (E_{12} + E_{21} + 4E_{66})\alpha^2\beta^2 + (E_{22}\beta^4)]$

$K_{41} = [(C_{11}\alpha^3) + (C_{21} + 2C_{66})\alpha\beta^2]$ $K_{42} = [(C_{22}\beta^3) + (C_{12} + 2C_{66})\alpha^2\beta]$

$K_{43} = -[(E_{11}\alpha^4) + (E_{12} + E_{21} + 4E_{66})\alpha^2\beta^2 + (E_{22}\beta^4)]$

$K_{44} = [(F_{11}\alpha^4) + (F_{12} + F_{21} + 4F_{66})\alpha^2\beta^2 + (F_{22}\beta^4) + (G_{55}\alpha^2 + G_{44}\beta^2)]$ (23)

Where, $\alpha = \frac{m\pi}{a}$ and $\beta = \frac{n\pi}{b}$

3. NUMERICAL RESULTS AND DISCUSSION

The present theory is applied to the static analysis of FG plates. The FG plate is supposed to be aluminum and alumina with the following material properties:

Metal (aluminum, Al) : E_m =70 x 10^9 N/m^2 ; μ = 0.3

Ceramic (Alumina, Al_2O_3) : $E_c = 380x10^9$ N/m^2 ; $\mu = 0.3$

The various non-dimensionless parameters used are :

$$w = \frac{10hEo}{a2q0},\ w\left(\frac{a}{2}, \frac{b}{2}\right)$$

$$\sigma_x = \frac{10h^2}{a2q0},\ \sigma_x\left(\frac{a}{2}, \frac{b}{2}, \frac{h}{2}\right)$$

$$\tau_{xz} = \frac{h}{aq0},\ \tau_{xz}\left(0, \frac{b}{2}, 0\right)$$

Table -1: Comparison of deflections and dimensionless axial stress of FG plate for different volume fraction values

Theory	**p**	**W**	**σ_x**	**τ_{xz}**
S. Merdaci	Ceramic	0.08122	1.99550	0.24618
FSDPT [62]		0.07791	1.97576	0.15915
PSDPT[49]		0.07791	1.99432	0.23857
SSDPT[61]		0.07790	1.999550	0.24618
Present		0.08808	2.70969	0.14941
S. Merdaci	1	0.19703	0.94407	0.34103
FSDPT [62]		0.19609	0.93765	0.26880
PSDPT[49]		0.19604	0.94370	0.33433
SSDPT[61]		0.19604	0.94407	0.34103
Present		0.17684	0.87023	0.35334
S. Merdaci	2	0.28479	1.37702	0.41426
FSDPT [62]		0.28661	1.36934	0.34892
PSDPT[49]		0.28490	1.37662	0.40919
SSDPT[61]		0.28479	1.37702	0.41426
Present		0.30864	1.03557	0.41125
S. Merdaci	5	0.33606	1.62591	0.47502
FSDPT [62]		0.33851	1.61758	0.41003
PSDPT[49]		0.33624	1.62552	0.47133
SSDPT[61]		0.33606	1.62591	0.47502
Present		0.32396	1.25657	0.39746

S. Merdaci	10	0.38090	1.84026	0.57591
FSDPT [62]		0.38402	1.83097	1.83097
PSDPT[49]		0.38116	1.83989	0.57591
SSDPT[61]		0.38090	1.84026	0.57337
Present		0.37276	1.46967	0.38419

Table -2: Deflections of S-FG plate

W				
P=0	P=1	P=2	P=5	P=10
0.08808	0.176836	0.308641	0.323958	0.372755
0.08808	0.176836	0.308641	0.323958	0.372755
0.08808	0.176836	0.308641	0.323958	0.372755
0.08808	0.176836	0.308641	0.323958	0.372755
0.08808	0.176836	0.308641	0.323958	0.372755
0.08808	0.176836	0.308641	0.323958	0.372755
0.08808	0.176836	0.308641	0.323958	0.372755
0.08808	0.176836	0.308641	0.323958	0.372755
0.08808	0.176836	0.308641	0.323958	0.372755
0.08808	0.176836	0.308641	0.323958	0.372755

Table -3: Axial stress of first layer S-FG plate

σ_x					
	P=0	P=1	P=2	P=5	P=10
LAYER 1	-2.709688	-0.870229	-1.035571	-1.256572	-1.469672
	-1.757538	-0.825161	-0.720446	-0.836684	-0.969539

	-1.058202	-0.664131	-0.510557	-0.525398	-0.598086
	-0.573921	-0.46426	-0.354815	-0.307753	-0.336104
	-0.245889	-0.26372	-0.220389	-0.157851	-0.153705
	0	0	0	0	0

Table -4: Axial stress of second layer S-FG plate

σx					
	P=0	P=1	P=2	P=5	P=10
LAYER 2	0	0	0	0	0
	0.245889	0.26372	0.220389	0.157851	0.153705
	0.573921	0.46426	0.354815	0.307753	0.336104
	1.058202	0.664131	0.510557	0.525398	0.598086
	1.757538	0.825161	0.720446	0.836684	0.969539
	2.709688	0.870229	1.035571	1.256572	1.469672

Table -5: Transverse shear stress of first layer S-FG plate.

τxz					
	P=0	P=1	P=2	P=5	P=10
LAYER 1	0	0	0	0	0
	-0.149410	-0.083629	-0.071743	-0.083464	-0.101150
	-0.284194	-0.207896	-0.173823	-0.198976	-0.192398
	-0.391160	-0.353344	-0.291546	-0.270853	-0.264820
	-0.459836	-0.494380	-0.411255	-0.378512	-0.345140
	-0.483510	-0.502886	-0.448456	-0.397460	-0.384191

Table -6: Transverse shear stress of second layer S-FG plate.

τ_{xz}					
	P=0	P=1	P=2	P=5	P=10
LAYER 2	0.483510	0.502886	0.448456	0.397460	0.384191
	0.459836	0.494380	0.411255	0.378512	0.345140
	0.391160	0.353344	0.291546	0.270853	0.264820
	0.284194	0.207896	0.173823	0.198976	0.192398
	0.149410	0.083629	0.071743	0.083464	0.101150
	0	0	0	0	0

Comparison of deflections and dimensionless axial stresses and transverse shear stresses of FG plate for different volume fractions is shown in Table 1. The present predictions are compared with first order, parabolic and sinusoidal shear deformation theories. It is observed that, the value of deflection and transverse shear stresses increases and decreases the axial stress for different volume fraction values. Deflections, axial stresses and transverse shear stresses of S-FG plate for different volume fractions is shown in Table 2, 3, 4, 5, 6. It is observed that, the value of deflection increases with increase in volume fraction values.

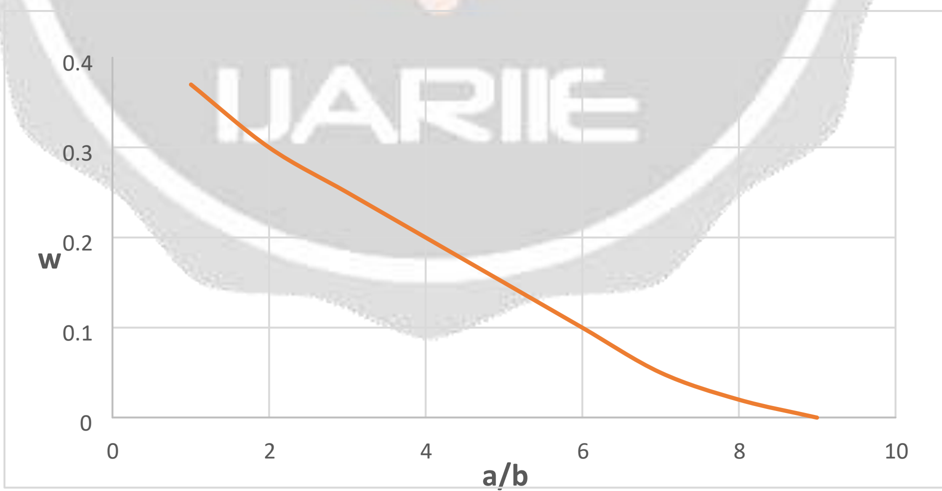

Fig -2: Displacement variation of S-FGM plate

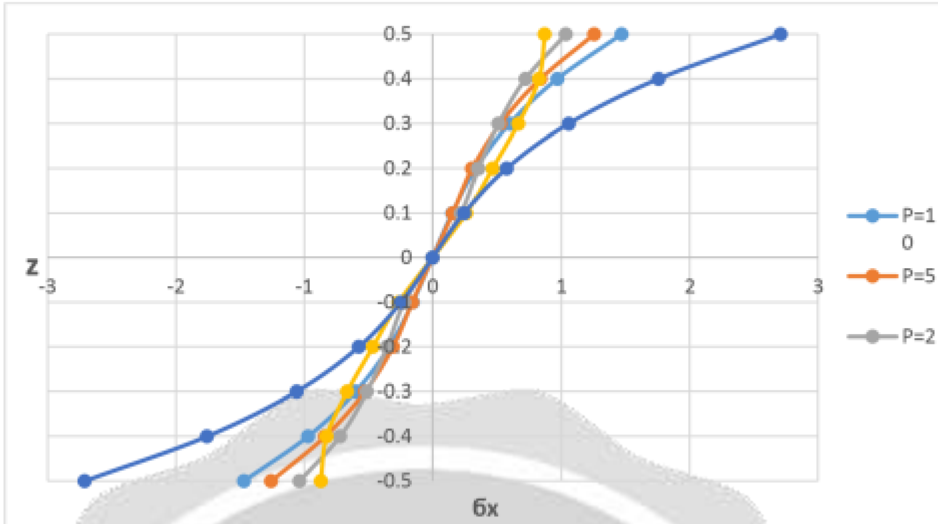

Fig -3: Distribution of axial stress of FGM plate

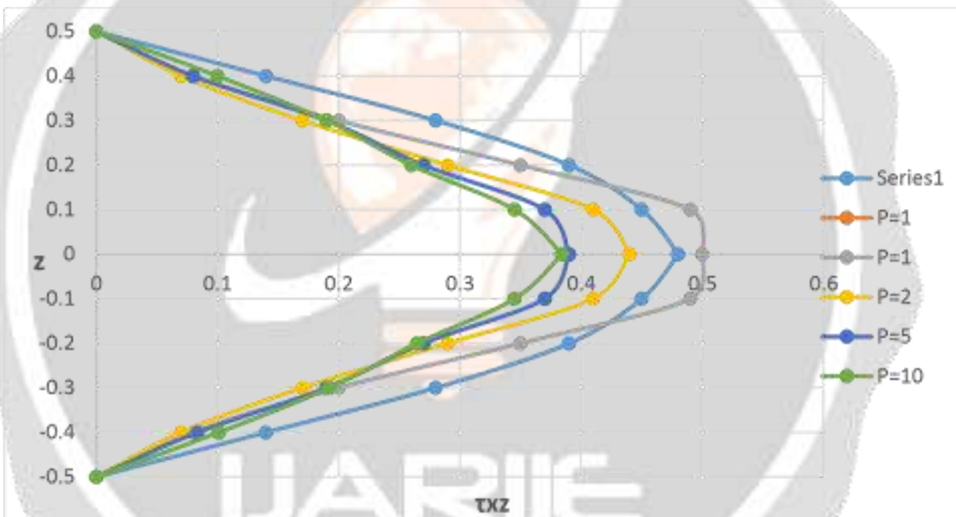

Fig -4: Distribution of transverse shear stress of FGM plate.

Variation of the dimensional displacement as a function of geometric ratio (a/b) for a ratio of equal thickness (a/h=10) and a material index p=0 to 10 is shown in Fig. 2.It is observed that the deflection of the FG plate decreases as the geometric ratio increases. Variation of the axial stress across the plate thickness in FGM is shown in Fig. 3. It is seen that the stresses are tensile above the meridian plane and compressive below the meridian plane. The maximum stress depends on the value of the exponent of the volume fraction p.
Shear stresses are plotted through the transverse thickness distribution in Fig. 4. It is observed that the transverse shear stress decreases at a point on the meridian plane of FG plate.

5. CONCLUSIONS

The theory is evaluated for simply supported sigmoidal FGM plate subjected to the static conditions. This theory satisfies the nullity of the stresses at the upper and lower surfaces of the plate without using the shear correction factor, contrary to other theories. The effect of various parameters, such as thickness ration, gradient index and

volume fraction of ceramic-metal plates are discussed. From the numerical results and discussion various conclusions are drawn that is the value of deflection and transverse shear stresses increases for different volume fraction values and decreases the axial stresses for different volume fraction values. The value of deflection increases with increase in volume fraction values.
The deflection of the FG plate decreases as the geometric ratio increases. The stresses are tensile above the meridian plane and compressive below the meridian plane. The maximum stress depends on the value of the exponent of the volume fraction p. The transverse shear stress decreases at a point on the meridian plane of FG plate.

5.1 Scope for future work

This theory is applicable to static and vibrational analysis of plates and shells.

This theory is applicable to dynamic analysis of plates and shells.

This theory can be applied to thermal vibrational analysis of FG plates.

This theory can be applied to bending analysis of FG plates.

This theory can be applied to buckling analysis of FG plates.

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