Sum of cubes of digits of any double-digit number

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ABSTRACT

In this paper, we computed the sum of cubes of digits of any double-digit number and have obtained cycles for the same. All the numbers with cycles of sum of cubes of digits are classified into different tables depending upon the last number obtained. Most of these numbers satisfy specific property. Tables containing these cycles along with the cross-digit sum of the cycles are categorically presented and analyzed.

Keyword: - *Armstrong number, Sum of cubes.*

1. INTRODUCTION

The mathematical exploration of numbers and their properties has fascinated humanity for millennia. Among various investigations into numbers, the study of their digits and the operations performed on them holds a special place. The origins of studying numbers and their digit properties can be traced back to ancient civilizations. Early mathematicians in Babylon, Egypt, and India were deeply engaged in the analysis of numbers, although their focus was more on practical computations than on recreational or abstract number properties. The fascination with digit-based operations likely emerged alongside the evolution of positional numeral systems, such as the Hindu-Arabic system, which provided a natural framework for exploring the individual components of numbers. In India, Vedic Mathematics [3] is an ancient system of mathematical techniques and principles derived from the Vedas, the ancient Indian scriptures. It is based on 16 sutras (aphorisms) and 13 sub-sutras, which provide shortcuts and patterns for performing various arithmetic and algebraic calculations. This system is highly efficient for mental calculations, offering simple methods to solve complex problems, including multiplication, division, and quadratic equations.

One of the earliest known explorations of the sum of cubes of digits comes from the field of recreational mathematics. Mathematicians and enthusiasts began experimenting with numbers, seeking patterns, relationships, and special properties. The sum of the cubes of digits was found to be connected to the concept of Armstrong numbers. A number is considered an Armstrong number [1,4] if it is equal to the sum of its digits each raised to the power of the number of digits. For three-digit numbers, this concept ovelaps with the sum of cubes of digits. For example, 153 is a three-digit number such that $1^3 + 5^3 + 3^3 = 153$. The sum of cubes of digits is also plays a role in mathematical puzzles and number sequences. In the modern era, the study of the sum of cubes of digits intersects with computational mathematics and digital systems. Algorithms for digit-based operations have practical applications in error detection, cryptography, and digital root calculations. These operations also appear in recreational mathematics competitions, inspiring students and enthusiasts to delve deeper into number theory. The sum of the cubes of the digits of a number, while seemingly simple, represents a rich interplay of historical curiosity, recreational mathematics, and theoretical inquiry. From ancient numeration systems to modern computational explorations, this concept continues to intrigue and inspire, reminding us of the endless wonders within the world of numbers.

Definition 1.1. *The cross-digit sum* [2] *of a number is calculated by summing its digits. If the resulting sum is greater than a single digit, the digits of this sum are added together repeatedly until a single-digit number is obtained. Alternatively, the cross-digit sum of a number can be found by taking the remainder when the number is divided by 9.*

In this article, we have calculated the cycles of sum of cubes of digits of numbers up to 99, organized in several tables. The cross- digit sum of all these cycles are also evaluated. Most of the cycles end in an Armstrong number except few. These cycles are categorically represented.

2. SUM OF CUBES OF DIGITS OF ANY DOUBLE-DIGIT NUMBER

In the Tables 1 −4 below the cycles of sum of cubes of numbers up to 99.

Serial Number	**Number**					**Steps**					
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	**01**	**1**									
2	**02**	08	512	134	92	737	713	**371**			
3	**03**	27	351	**153**							
4	**04**	64	280	520	**133**						
5	**05**	125	134	92	737	713	**371**				
6	**06**	216	225	141	66	432	99	1458	702	351	**153**
7	**07**	343	118	514	190	730	**370**				
8	**08**	512	134	92	737	713	**371**				
9	**09**	729	1080	513	**153**						
10	**10**	**1**									

TABLE 1. Sum of cubes of digits from 0 to 10

Serial Number	**Number**					**Steps**					
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
11	**11**	02	08	512	134	92	737	713	**371**		
12	**12**	09	729	1080	513	**153**					
13	**13**	28	520	**133**							
14	**14**	65	341	92	737	713	**371**				
15	**15**	126	225	141	66	432	99	1458	702	351	**153**
16	**16**	**217**									
17	**17**	344	155	251	134	92	737	713	**371**		
18	**18**	513	**153**								
19	**19**	730	**370**								
20	**20**	08	512	134	92	737	713	**371**			
21	**21**	09	729	1080	513	**153**					
22	**22**	16	**217**								
23	**23**	35	152	134	92	737	713	**371**			

24	**24**	72	351	**153**							
25	**25**	**133**									
26	**26**	224	80	512	134	92	737	713	**371**		
27	**27**	351	**153**								
28	**28**	520	**133**								
29	**29**	737	713	**371**							
30	**30**	27	351	**153**							
31	**31**	28	520	**133**							
32	**32**	35	152	134	92	737	713	**371**			
33	**33**	54	189	1242	81	513	**153**				
34	**34**	91	730	**370**							
35	**35**	152	134	92	737	713	**371**				
36	**36**	243	99	1458	702	351	**153**				
37	**37**	**370**									
38	**38**	539	881	1025	134	92	737	713	**371**		
39	**39**	756	684	792	1080	513	**153**				
40	**40**	64	280	520	**133**						

TABLE 2. Sum of cubes of digits from 11 to 40

Serial Number	**Number**					**Steps**					
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
41	**41**	65	341	92	737	713	**371**				
42	**42**	72	351	**153**							
43	**43**	91	730	**370**							
44	**44**	128	521	134	92	737	713	**371**			
45	**45**	189	1242	81	513	**153**					
46	**46**	280	520	**133**							
47	**47**	**407**									
48	**48**	576	684	792	1080	513	**153**				
49	**49**	793	1099	**1459**							
50	**50**	125	134	92	737	713	**371**				
51	**51**	126	225	141	66	432	99	1458	702	351	**153**
52	**52**	**133**									
53	**53**	152	134	92	737	713	**371**				
54	**54**	189	1242	81	513	**153**					

55	**55**	250	**133**								
56	**56**	341	92	737	713	**371**					
57	**57**	468	792	1080	513	**153**					
58	**58**	637	586	853	664	496	1009	730	**370**		
59	**59**	854	701	344	155	251	134	92	737	713	**371**
60	**60**	216	225	141	66	432	99	1458	702	351	**153**
61	**61**	**217**									
62	**62**	224	80	512	134	92	737	713	**371**		
63	**63**	243	99	1458	702	351	**153**				
64	**64**	280	520	**133**							
65	**65**	341	92	737	713	**371**					
66	**66**	432	99	1458	702	351	**153**				
67	**67**	559	979	1801	514	190	730	**370**			

TABLE 3. Sum of cubes of digits from 41 to 67

Serial Number	**Number**					**Steps**						
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
68	**68**	728	863	755	593	881	1025	134	92	737	713	**371**
69	**69**	945	918	1242	81	513	**153**					
70	**70**	343	118	514	190	730	**370**					
71	**71**	344	155	251	134	92	737	713	**371**			
72	**72**	351	**153**									
73	**73**	**370**										
74	**74**	**407**										
75	**75**	468	792	1080	513	**153**						
76	**76**	559	979	1801	514	190	730	**370**				
77	**77**	686	944	857	980	1241	74	**407**				
78	**78**	855	762	567	684	792	1080	513	**153**			
79	**79**	1072	352	160	**217**							
80	**80**	512	134	92	737	713	**371**					
81	**81**	513	**153**									
82	**82**	520	**133**									
83	**83**	539	881	1025	134	92	737	713	**371**			
84	**84**	576	684	792	1080	513	**153**					

85	**85**	637	586	853	664	496	1009	730	**370**			
86	**86**	728	863	755	593	881	1025	134	92	737	713	**371**
87	**87**	855	762	567	684	792	1080	513	153			
88	**88**	1024	73	**370**								
89	**89**	1241	74	**407**								
90	**90**	729	1080	513	**153**							
91	**91**	730	**370**									
92	**92**	737	713	**371**								
93	**93**	756	684	792	1080	513	**153**					
94	**94**	793	1099	**1459**								
95	**95**	854	701	344	155	251	134	92	737	713	**371**	
96	**96**	945	918	1242	81	513	**153**					
97	**97**	1072	352	160	**217**							
98	**98**	1241	74	**407**								
99	**99**	1458	702	351	**153**							

TABLE 4. Sum of cubes of digits from 67 to 99

Sum of cube of digits of any number ending at $371 = 53 \times 7$ has the form $3n + 2$, where $n \in \{0, 1, 2, \ldots\}$ except for 47, 74, 77, 89, 98. These are 28 such numbers.

Serial Number	**Number**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1	**68**	728	863	755	593	881	1025	134	92	737	713	**371**
2	**86**	728	863	755	593	881	1025	134	92	737	713	**371**
Cross-digit sum	**5**	**8**	**8**	**8**	**8**	**8**	**8**	**8**	**2**	**8**	**2**	**2**
3	**59**	854	701	344	155	251	134	92	737	713	**371**	
4	**95**	854	701	344	155	251	134	92	737	713	**371**	
Cross-digit sum	**5**	**8**	**8**	**2**	**2**	**8**	**8**	**2**	**8**	**2**	**2**	
5	**38**	539	881	1025	134	92	737	713	**371**			
6	**83**	539	881	1025	134	92	737	713	**371**			
Cross-digit sum	**2**	**8**	**8**	**8**	**8**	**2**	**8**	**2**	**2**			
7	**17**	344	155	251	134	92	737	713	**371**			
8	**71**	344	155	251	134	92	737	713	**371**			
Cross-digit sum	**8**	**2**	**2**	**8**	**8**	**2**	**8**	**2**	**2**			
9	**11**	02	08	512	134	92	737	713	**371**			
Cross-digit sum	**2**	**2**	**8**	**8**	**8**	**2**	**8**	**2**	**2**			
10	**26**	224	80	512	134	92	737	713	**371**			
11	**62**	224	80	512	134	92	737	713	**371**			

Cross-digit sum	**8**	**8**	**8**	**8**	**8**	**2**	**8**	**2**	**2**			
12	**02**	08	512	134	92	737	713	**371**				
13	**20**	08	512	134	92	737	713	**371**				
Cross-digit sum	**2**	**8**	**8**	**8**	**2**	**8**	**2**	**2**				
14	**44**	128	521	134	92	737	713	**371**				
Cross-digit sum	**8**	**2**	**8**	**8**	**2**	**8**	**2**	**2**				
15	**23**	35	152	134	92	737	713	**371**				
16	**32**	35	152	134	92	737	713	**371**				
Cross-digit sum	**5**	**8**	**8**	**8**	**2**	**8**	**2**	**2**				
17	**08**	512	134	92	737	713	**371**					
18	**80**	512	134	92	737	713	**371**					
Cross-digit sum	**8**	**8**	**8**	**2**	**8**	**2**	**2**					

TABLE 5. Sum of cubes of digits of numbers ending in 371 and their cross-digit sum.

Serial Number	**Number**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
19	**05**	125	134	92	737	713	**371**					
20	**50**	125	134	92	737	713	**371**					
Cross-digit sum	**5**	**8**	**8**	**2**	**8**	**2**	**2**					
21	**35**	152	134	92	737	713	**371**					
22	**53**	152	134	92	737	713	**371**					
Cross-digit sum	**8**	**8**	**8**	**2**	**8**	**2**	**2**					
23	**14**	65	341	92	737	713	**371**					
24	**41**	65	341	92	737	713	**371**					
Cross-digit sum	**5**	**2**	**8**	**2**	**8**	**2**	**2**					
25	**56**	341	92	737	713	**371**						
26	**65**	341	92	737	713	**371**						
Cross-digit sum	**2**	**8**	**2**	**8**	**2**	**2**						
27	**29**	737	713	**371**								
28	**92**	737	713	**371**								
Cross-digit sum	**2**	**8**	**2**	**2**								

TABLE 6. Sum of cubes of digits of numbers ending in 371 and their cross-digit sum.

There are ten categories of cycle of numbers for which sum of cubes of digits of a double-digit number ending at 371. The number 371 is an Armstrong number as $371 = 3^3 + 7^3 + 1^3$.

Serial Number	Category	Steps	Numbers	Total
1	**A1**	11	68,86: 92,29	04
2	**A2**	08	38,83	02
3	**A3**	06	05,50	02
4	**A4**	10	59,95	02
5	**A5**	08	17,71	02
6	**A6**	08	11: 02, 20: 08, 80	05
7	**A7**	08	26, 62	02
8	**A8**	07	44	01
9	**A9**	07	23, 32: 35, 53	04
10	**A10**	06	14, 41: 65, 56	04
			Grand Total	28

Following are all the 10 categories of cycles:

A1 $68 \rightarrow 728 \rightarrow 863 \rightarrow 755 \rightarrow 593 \rightarrow 881 \rightarrow 1025 \rightarrow 134 \rightarrow 92 \rightarrow 737 \rightarrow 713 \rightarrow 371$

A2 $38 \rightarrow 593 \rightarrow 881 \rightarrow 1025 \rightarrow 134 \rightarrow 92 \rightarrow 737 \rightarrow 713 \rightarrow 371$

A3 $05 \rightarrow 125 \rightarrow 134 \rightarrow 92 \rightarrow 737 \rightarrow 713 \rightarrow 371$

A4 $59 \rightarrow 854 \rightarrow 701 \rightarrow 344 \rightarrow 155 \rightarrow 251 \rightarrow 134 \rightarrow 92 \rightarrow 737 \rightarrow 713 \rightarrow 371$

A5 $17 \rightarrow 344 \rightarrow 155 \rightarrow 251 \rightarrow 134 \rightarrow 92 \rightarrow 737 \rightarrow 713 \rightarrow 371$

A6 $11 \rightarrow 02 \rightarrow 08 \rightarrow 512 \rightarrow 134 \rightarrow 92 \rightarrow 737 \rightarrow 713 \rightarrow 371$

A7 $26 \rightarrow 224 \rightarrow 80 \rightarrow 512 \rightarrow 134 \rightarrow 92 \rightarrow 737 \rightarrow 713 \rightarrow 371$

A8 $44 \rightarrow 128 \rightarrow 521 \rightarrow 134 \rightarrow 92 \rightarrow 737 \rightarrow 713 \rightarrow 371$

A9 $23 \rightarrow 35 \rightarrow 152 \rightarrow 134 \rightarrow 92 \rightarrow 737 \rightarrow 713 \rightarrow 371$

A10 $14 \rightarrow 65 \rightarrow 341 \rightarrow 92 \rightarrow 737 \rightarrow 713 \rightarrow 371$

Subsequently, we have table containing numbers ending in $217 = 31 \times 7$, there are total five such numbers up to 99.

Serial Number	Number	1	2	3	4
1	**79**	1072	352	160	**217**
2	**97**	1072	352	160	**217**
Cross-digit sum	**7**	**1**	**1**	**7**	**1**
3	**22**	16	**217**		

Cross-digit sum	**4**	**7**	**1**		
4	**16**	**217**			
5	**61**	**217**			
Cross-digit sum	**7**	**1**			

TABLE 7. Sum of cubes of digits of numbers ending in 217 and their cross-digit sum.

There are two categories of numbers of which sum of cube of digits of a double-digit number ends at 217, which is not an Armstrong number since $2^3 + 1^3 + 7^3 = 352 \neq 217$.

Serial Number	Category	Steps	Numbers	Total
1	**B1**	04	79, 97	02
2	**B2**	02	22: 16, 61	03
			Grand Total	05

B1 $79 \rightarrow 1072 \rightarrow 352 \rightarrow 160 \rightarrow 217$

B2 $22 \rightarrow 16 \rightarrow 217$

Serial Number	Number	1	2	3	4	5	6	7	8
1	**58**	637	586	853	664	496	1009	730	**370**
2	**85**	637	586	853	664	496	1009	730	**370**
Cross-digit sum	**4**	**7**	**1**	**7**	**7**	**1**	**1**	**1**	**1**
3	**67**	559	979	1801	514	190	730	**370**	
4	**76**	559	979	1801	514	190	730	**370**	
Cross-digit sum	**4**	**1**	**7**	**1**	**1**	**1**	**1**	**1**	
5	**07**	343	118	514	190	730	**370**		
6	**70**	343	118	514	190	730	**370**		
Cross-digit sum	**7**	**1**	**1**	**1**	**1**	**1**	**1**		
7	**34**	91	730	**370**					
8	**43**	91	730	**370**					
Cross-digit sum	**7**	**1**	**1**	**1**					
9	**88**	1024	73	**370**					
Cross-digit sum	**7**	**7**	**1**	**1**					
10	**19**	730	**370**						
11	**91**	730	**370**						
Cross-digit sum	**1**	**1**	**1**						

TABLE 8. Sum of cubes of digits of numbers ending in 370 and their cross-digit

sum.

Serial Number	Number	1	2	3	4	5	6	7	8
12	37	370							
13	73	370							
Cross-digit sum	1	1							

TABLE 9. Sum of cubes of digits of numbers ending in 370 and their cross-digit sum.

There are total thirteen numbers up to 99 such that their sum of cubes of digits ends in $370 = 37 \times 10$. Table 8 above gives detailed cycles of all these numbers. There are five categories of cycle of numbers for which sum of cube of digits of a double-digit numbers end at 370. The number 370 is an Armstrong number since $370 = 3^3 + 7^3 + 0^3$.

Serial Number	Category	Steps	Numbers	Total
1	C1	07	67, 76	02
2	C2	06	07, 70	02
3	C3	03	34, 43: 91, 19	04
4	C4	08	58, 85	02
5	C5	03	88: 73, 37	03
			Grand Total	13

C1 $67 \rightarrow 559 \rightarrow 979 \rightarrow 1801 \rightarrow 514 \rightarrow 190 \rightarrow 730 \rightarrow 370$

C2 $07 \rightarrow 343 \rightarrow 118 \rightarrow 514 \rightarrow 190 \rightarrow 730 \rightarrow 370$

C3 $34 \rightarrow 91 \rightarrow 730 \rightarrow 370$

C4 $58 \rightarrow 637 \rightarrow 586 \rightarrow 853 \rightarrow 664 \rightarrow 496 \rightarrow 1009 \rightarrow 730 \rightarrow 370$

C5 $88 \rightarrow 1024 \rightarrow 73 \rightarrow 370$

There are eleven numbers up to 99 whose sum of cube of digits ends in $133 = 19 \times 7$ given in Table 10 below:

Serial Number	Number	1	2	3	4
1	**04**	64	280	520	133
2	**40**	64	280	520	133
Cross-digit sum	**4**	**1**	**1**	**7**	**7**
3	**46**	280	520	133	
4	**64**	280	520	133	
Cross-digit sum	**1**	**1**	**7**	**7**	
5	**13**	28	520	133	
6	**31**	28	520	133	
Cross-digit sum	**4**	**1**	**7**	**7**	
7	**28**	520	133		
8	**82**	520	133		
Cross-digit sum	**1**	**7**	**7**		
9	**55**	250	133		
Cross-digit sum	**1**	**7**	**7**		
10	**25**	133			
11	**52**	133			
Cross-digit sum	**7**	**7**			

TABLE 10. Sum of cubes of digits of numbers ending in 133 and their cross-digit sum.

The are four categories of cycle of numbers for which sum of cubes of digits of double-digit numbers end at 133. The number 133 is not an Armstrong number as $1^3 + 3^3 + 3^3 = 55 \neq 133$.

Serial Number	Category	Steps	Numbers	Total
1	**D1**	04	04, 40:64, 46	04
2	**D2**	03	13, 31:28, 82	04
3	**D3**	02	55	01
4	**D4**	01	25, 52	02
			Grand Total	11

D1 $04 \rightarrow 64 \rightarrow 280 \rightarrow 520 \rightarrow 133$

D2 $13 \rightarrow 28 \rightarrow 520 \rightarrow 133$

D3 $55 \rightarrow 250 \rightarrow 133$

D4 $25 \rightarrow 133$

There are two numbers up to 99 whose sum of cube of digits ends in 1459.

Serial Number	Number	1	2	3
1	**49**	793	1099	1459
2	**94**	793	1099	1459
Cross-digit sum	**4**	**1**	**1**	**1**

TABLE 11. Sum of cubes of digits of numbers ending in 1459 and their cross-digit sum.

There is only one category of cycle of numbers for which sum of cubes of digits of double-digit numbers end at 1459. The number 1459 is a prime number but it is not an Armstrong number as $1^4 + 4^4 + 5^4 + 9^4 = 7251 \neq 1459$.

Serial Number	Category	Steps	Numbers	Total
1	**E**	03	49, 94	02

E $49 \rightarrow 793 \rightarrow 1099 \rightarrow 117 \rightarrow 1459$

There are five numbers up to 99 whose sum of cubes of digits ends in $407 = 37 \times 11$.

Serial Number	Number	1	2	3	4	5	6	7
1	**77**	686	944	857	980	1241	74	407
Cross-digit sum	**5**	**2**	**8**	**2**	**8**	**8**	**2**	**2**
2	**89**	1241	74	407				
3	**98**	1241	74	407				
Cross-digit sum	**8**	**8**	**2**	**2**				
4	**47**	407						
5	**74**	407						
Cross-digit sum	**2**	**2**						

TABLE 12. Sum of cubes of digits of numbers ending in 407 and their cross-digit sum.

There are two categories of cycle of numbers for which sum of cubes of digits of double-digit numbers end in 407. The number 407 is an Armstrong number as $407 = 4^3 + 0^3 + 7^3$.

Serial Number	Category	Steps	Numbers	Total

1	**F1**	07	77:74, 47	03
2	**F2**	03	89, 98	02
			Grand Total	05

F1 $77 \rightarrow 686 \rightarrow 944 \rightarrow 857 \rightarrow 980 \rightarrow 1241 \rightarrow 74$ (or 47) $\rightarrow 407$

F2 89 (or 98) $\rightarrow 1241 \rightarrow 74$ (or 47) $\rightarrow 407$

There are 33 numbers up to 99 whose sum of cube of digits ends in 153. In fact, any such number has the form $3n$ where $n \in \mathrm{N}$, $3n \leq 99$.

Serial Number	**Number**					**Steps**					
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	**06**	216	225	141	66	432	99	1458	702	351	153
2	**60**	216	225	141	66	432	99	1458	702	351	153
Cross-digit sum	**6**	**9**	**9**	**6**	**3**	**9**	**9**	**9**	**9**	**9**	**9**
3	**15**	126	228	141	66	432	99	1458	702	351	153
4	**51**	126	228	141	66	432	99	1458	702	351	153
Cross-digit sum	**6**	**9**	**9**	**6**	**3**	**9**	**9**	**9**	**9**	**9**	**9**
5	**78**	855	762	567	684	792	1080	513	153		
6	**87**	855	762	567	684	792	1080	513	153		
Cross-digit sum	**6**	**9**	**6**	**9**	**9**	**9**	**9**	**9**	**9**		
7	**36**	243	99	1458	702	351	153				
8	**63**	243	99	1458	702	351	153				
Cross-digit sum	**9**	**9**	**9**	**9**	**9**	**9**	**9**				
9	**39**	756	684	792	1080	513	153				
10	**93**	756	684	792	1080	513	153				
Cross-digit sum	**3**	**9**	**9**	**9**	**9**	**9**	**9**				

TABLE 13. Sum of cubes of digits of a double-digit number ending in 153

Serial Number	**Number**					**Steps**					
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
11	**48**	576	684	792	1080	513	153				
12	**84**	576	684	792	1080	513	153				
Cross-digit sum	**3**	**9**	**9**	**9**	**9**	**9**	**9**				
13	**66**	432	99	1458	702	351	153				
Cross-digit sum	**3**	**9**	**9**	**9**	**9**	**9**	**9**				
14	**33**	54	189	1242	81	513	153				
Cross-digit sum	**6**	**9**	**9**	**9**	**9**	**9**	**9**				
15	**69**	945	918	1242	81	513	153				
16	**96**	945	918	1242	81	513	153				
Cross-digit sum	**6**	**9**	**9**	**9**	**9**	**9**	**9**				
17	**12**	09	729	1080	513	153					

18	**21**	09	729	1080	513	153					
Cross-digit sum	**3**	**9**	**9**	**9**	**9**	**9**					
19	**57**	468	792	1080	513	153					
20	**75**	468	792	1080	513	153					
Cross-digit sum	**3**	**9**	**9**	**9**	**9**	**9**					
21	**45**	189	1242	81	513	153					
22	**54**	189	1242	81	513	153					
Cross-digit sum	**9**	**9**	**9**	**9**	**9**	**9**					
23	**09**	729	1080	513	153						
24	**90**	729	1080	513	153						
Cross-digit sum	**9**	**9**	**9**	**9**	**9**						
25	**99**	1458	702	351	153						
Cross-digit sum	**9**	**9**	**9**	**9**	**9**						
26 **27**	**03** **30**	27 27	351 351	153 153							
Cross-digit sum	**3**	**9**	**9**	**9**							
28	**24**	72	351	153							
29	**42**	72	351	153							
Cross-digit sum	**6**	**9**	**9**	**9**							
30	**27**	351	153								
31	**72**	351	153								
Cross-digit sum	**9**	**9**	**9**								
32	**18**	513	153								
33	**81**	513	153								
Cross-digit sum	**9**	**9**	**9**								

TABLE 14. Sum of cubes of digits of a double-digit number ending in 153

There are twelve categories of cycle of numbers for which sum of cube of digits of a double-digit number end at 153. The number 159 is an Armstrong number because $153 = 1^3 + 5^3 + 3^3$.

Summarization:

Serial Number	Category	Steps	Numbers	Total
1	**G1**	10	66:99:06, 60	04
2	**G2**	10	15, 51	02
3	**G3**	06	36, 63	02
4	**G4**	08	78, 87	02
5	**G5**	06	39, 93	02
6	**G6**	06	48, 84	02
7	**G7**	05	57, 75	02
8	**G8**	05	12, 21:09, 90	04
9	**G9**	06	33:54, 45:81, 18	05
10	**G10**	06	69, 96	02
11	**G11**	03	03, 30:27,72	04

12	**G12**	02	24, 42	02
			Grand Total	33

G1 06 → 216 → 225 → 141 → 66 → 432 → 99 → 1458 → 702 → 351 → 153

G2 15 → 126 → 225 → 141 → 66 → 432 → 99 → 1458 → 702 → 351 → 153

G3 36 → 243 → 99 → 1458 → 702 → 351 → 153

G4 78 → 855 → 762 → 567 → 684 → 792 → 1080 → 513 → 153

G5 39 → 756 → 684 → 792 → 1080 → 513 → 153

G6 48 → 576 → 684 → 792 → 1080 → 513 → 153

G7 57 → 468 → 792 → 1080 → 513 → 153

G8 12 → 09 → 729 → 1080 → 513 → 153

G9 33 → 54 → 189 → 1242 → 81 → 513 → 153

G10 $69 \rightarrow 945 \rightarrow 918 \rightarrow 1242 \rightarrow 81 \rightarrow 513 \rightarrow 153$

G11 $03 \rightarrow 27 \rightarrow 351 \rightarrow 153$

G12 $24 \rightarrow 72 \rightarrow 351 \rightarrow 153$

Sum of cubes of digits of the number 01 and 10 is 1. The number 01 is an Armstrong number since $1^1 = 1$ but 10 is not an Armstrong numbers since $1^2 + 0^2 = 1 \neq 10$.

Serial Number	Type	Sum	Frequency	Equation
1	**A**	371	28	53×07
2	**B**	217	05	31×07
3	**C**	370	13	37×10
4	**D**	133	11	19×07
5	**E**	1459	02	01×1459
6	**F**	407	05	37×11
7	**G**	153	33	17×09
8	**H**	01	02	01×01
Grand Total			99	

There are only 80 numbers which have sum of cubes of double-digit numbers ending in an Armstrong number. The remaining 19 double digit numbers do not end in an Armstrong number.

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