SECOND ORDER ANALYSIS OF RCC CHIMNEY USING BEAM-COLUMN THEORY

Dr. S.S Patil[1] , Mr. Mundewadi A.A.R[2]

[1] *Professor , Civil Engg Dept, Walchand Institute of Technology, Solapur, Maharashtra,India*

[2] *Student, Civil Engg Dept, Walchand Institute of Technology, Solapur, Maharashtra,India*

ABSTRACT

RCC Chimneys are normally designed for critical loads produced by earthquake or wind. Chimneys being tall and flexible structure, it is expected that wind loads will be more critical than earthquake loads. Looking at the height at which chimneys are being designed it is necessary to analyse the second order effect produced due to the self weight of the structure itself and the wind load acting on it. In this paper , equations will be developed considering a standard case of tall RCC chimney with one end fixed and other end free. Loads considered will be a combination wind load and dead load of the structure. Wind load will be calculated as per IS 4998 (Part 1): 1992. Across wind loads will be neglected as it is a part of dynamic analysis. Finally the values obtained by carrying out the analysis by first order and second order will be compared and the effect of second order analysis with the increase in height will be analysed.Beam column theory will be used for the analysis. Using beam-column theory slender chimney can be analyzed for secondary moment with lateral deformation.

Keyword:- *RCC Chimney, Beam-column theory, Second order analysis, P-Delta analysis*

---

1. INTRODUCTION

Chimney as we know them today, are tall slender structures which fulfil an important function. They had a humble beginning as household vents and over the years, as vents grew larger and taller; they came to be known as chimneys.

A chimney is a system for venting hot flue gases or smoke from a boiler, stove, furnace or fireplace to the outside atmosphere. Tall RC chimneys are commonly used to discharge pollutants at higher elevation. They are typically almost vertical to ensure that the hot gases flow smoothly, drawing air into the combustion through the chimney effect. Chimneys are tall to increase their draw of air for combustion and to disperse the pollutants in flue gases over a greater area in order to reduce the pollutant concentrations in compliance with regulatory or other limits. Chimneys with height exceeding 150 m are considered as tall chimneys. However it is not only a matter of height but also the aspect ratio when it comes to classifying a chimney as tall. Today, Reinforced Concrete is the dominant material used for the construction of tall chimneys and for short chimneys precast concrete with or without pre stressing, Modern industrial chimneys consists of a concrete windshield with a number of steel stacks on the inside. Chimneys can be classified as one of those structures for which use of concrete is eminently used. Tall reinforced concrete (RC) chimneys form an important component of major industries and power plants.

2. DESCRIPTION OF LOADING

Details of the parameters are as follows

1. Height of the chimney - 270 m
2. Outer diameter of bottom - 36.6 m
3. Outer diameter at top - 19.6 m
4. Thickness of shell - 0.8 m
5. Grade of concrete - M30

6. Seismic zone - III
7. Basic wind speed - 33 m/sec (for Solapur)
8. Foundation type - RCC circular mat
9. Density of concrete - 1 kN/m^3
10. Design life of structure - 100 yrs
11. Terrain category - 4
12. Probability factor - 1.06
13. Topography factor - 1
14. Class of structure - Class C

3. WIND ANALYSIS

Along wind loads are caused by the 'drag' component of the wind force on the chimney. This is accompanied by 'gust buffeting' causing a dynamic responsein the direction of the mean flow.In the present analysis the across wind effect which is due to the dynamic response of chimney is neglected.Along wind effect is due to the direct buffeting action, when the wind acts in the face of the structure. For the porpose of estimation if these loads the chimney is modelled to act on the exposed face of the chimney causing predominent moments in the chimney.

Two methods of estimating of wind loads are given in IS 4998 (part 1)

1. Simplified Method
2. Random Risponse Method

Simplified method will be used for the estimation of wind loads

SIMPLIFIED METHOD

The along wind load or the draf force per unit height of the Chimney at any level shall be calculated from the equation:

$$F_z = p_z \, . C_D . d_z$$

Where ,

p_z = design wind pressure obtained in accordance with IS 875 (Part 3): 1987

z =height of any section of the chimney in m measured from the top of foundation

C_D = drag coefficient of the chimney to be taken as 0.8

d_z = diameter of chimney at height z in m

∴ Wind load at top of chimney = 15.633 kN/m

Wind load at bottom of chimney = 6.644 kN/m

4. DEAD LOAD CALCULATIONS

Dead load calculations are carried out by Frustum of cone method.Dead load for each 20 m interval of distance of chimney is calculated by developing a programme in excel.The results for the same are displayed below

Height(m)	Dead Load (kN m)
0	463046.2
20	418695.3
40	376097.1
60	335081.9
80	295649.8

100	257800.7
120	221629
140	186851.8
160	116035
180	122291.7
200	92345.4
220	63982.17
240	37202
260	12004.9
270	0

5. BEAM COLUMN THEORY

Timoshenko (1961) described theory of beam-column. Using this theory of Beam column, general equation is developed for analysis of chimney subjected to vertical & lateral load and moment at top.

In the elementary theory of bending, it is found that stresses and deflections in beam are directly proportional to the applied loads. This condition requires that the change in shape of the beam due to bending must not affect the action of the applied loads. For example, the beam in fig (2.la) is subjected only lateral loads, such as Q1 and Q2, the presence of small deflections δ_1 and δ_2 and slight changes in the vertical lines of action of the loads W_1 have only an insignificant effect on the moments and shear forces. Thus it is possible to make calculation for deflections, stresses, moments on the basis of initial configuration of the beam. Under this conditions, and also if Hook's law holds for the material, the deflections are proportional to the acting forces and the principle of superposition is valid i.e. the final deformations produced by the individual forces.

Conditions are entirely different when both uniaxial and lateral forces act simultaneously on the beam, as in fig (2.1b). The bending moments, shear forces, stresses and deflections in the beam will not be the proportional to the magnitude of the axial load. Furthermore there values wi11 be dependent upon the magnitude of the deflection produced and will be sensitive to the even slight eccentricities in the application of the axial load. Beams subjected to the axial compression and simultaneously supporting lateral loads are known as beam-columns.

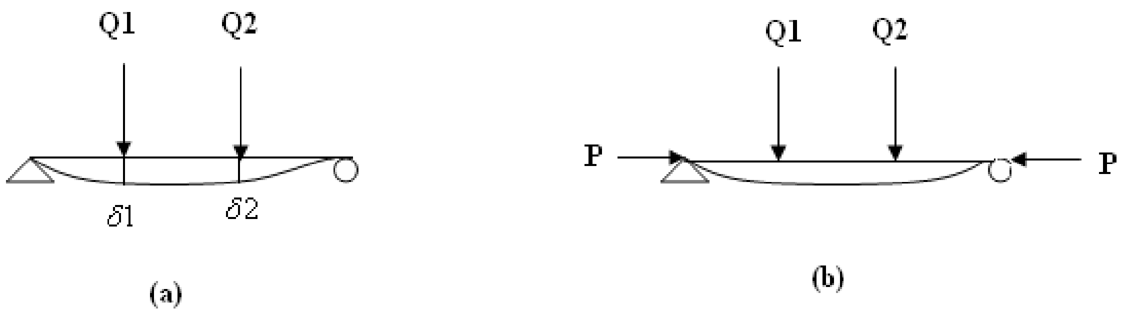
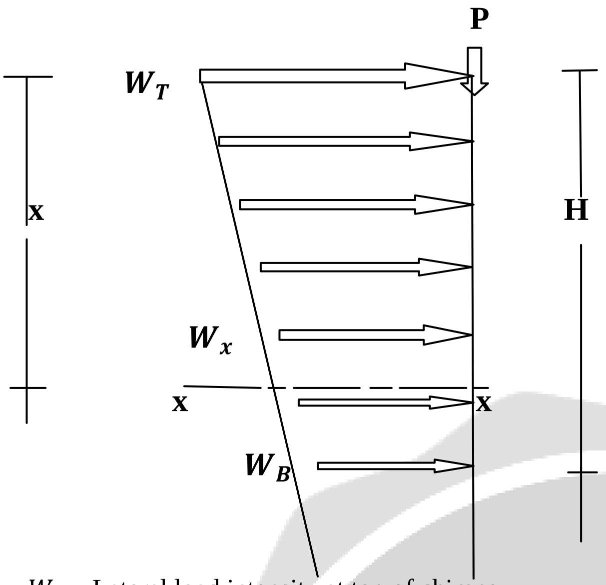


Figure 2.1 bending of Beam-Column

The difference between the behavior of short and slender columns is that, when slender columns are loaded even with axial loads, the lateral deflection (measured from the original centre line along its length) becomes appreciable where as in short columns this lateral deflection is very small and can be neglected. Hence slender columns, have to be designed for not only the external axial forces acting on them but also for the secondary moment produced by the lateral deflection.

6. EVALUATION OF EQUATIONS USING BEAM-COLUMN THEORY

6.1 FIRST ORDER ANALYSIS OF CHIMNEY



W_T = Lateral load intensity at top of chimney

W_B= Lateral load intensity at bottom of chimney

H = height of chimney

W_x= Lateral load intensity at general section 'x' on chimney

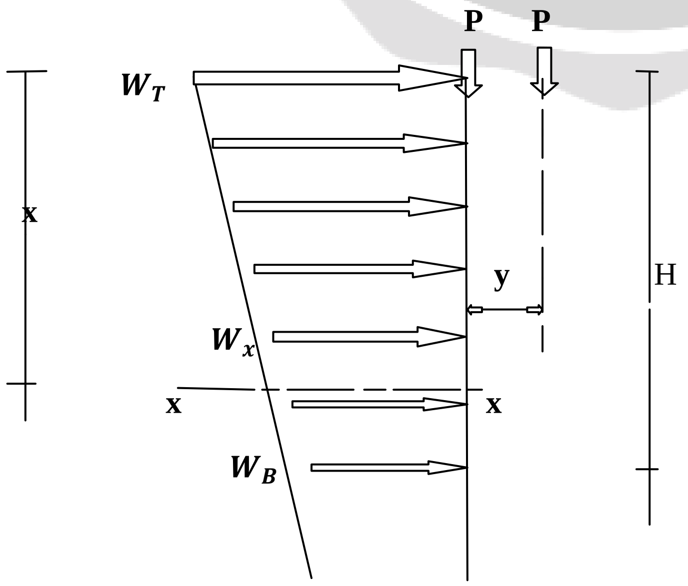
$$= W_T - \left[\frac{(W_T - W_B)x}{H}\right]$$

Let ' M_x ' be the bending moment at a general section 'x' from top of chimney

$$\therefore M_x = -\frac{W_T x^2}{2} + \left(\frac{W_T - W_B x^3}{6H}\right) \quad \text{…………………………………………..…..6.1.1}$$

This is the equation for Bending Moment of Chimney at any height 'x'

6.2. SECOND ORDER ANALYSIS OF CHIMNEY



W_T = Lateral load intensity at top of chimney

W_B= Lateral load intensity at bottom of chimney

H = height of chimney

W_x= Lateral load intensity at general section 'x' on chimney

$$= W_T - \left[\frac{(W_T - W_B)x}{H}\right]$$

Put, $k_w = \frac{(W_T - W_B)}{H}$

$$\therefore W_x = W_T - k_w x$$

∴Bending moment at a general section 'x' is given by

$$M_x = Py - \frac{W_T x^2}{2} + \frac{k_w x^3}{6} \quad \text{...............6.2.1}$$

Now we have, $EI_x \frac{\partial^2 y}{\partial x^2} = -M$

$$\therefore EI_x \frac{\partial^2 y}{\partial x^2} = -Py + \frac{W_T x^2}{2} - \frac{k_w x^3}{6}$$

$$\therefore EI_x \frac{\partial^2 y}{\partial x^2} + Py = \frac{W_T x^2}{2} - \frac{k_w x^3}{6} \quad \text{...............6.2.2}$$

For a circular hollow section the moment of inertia is given by,

$$I_x = \frac{\pi}{64}(d_1^4 - d_2^4)$$

And for a chimney the moment of inertia varies according to the height and is different for different section and depends upon cross section

On solving above equation we get,

Y= complimentary solution + particular solution

Where,

$$y_c = A\sin(\alpha x) + B\cos(\alpha x)$$

Where,

$$\alpha = \sqrt{\frac{P}{EI_x}}$$

And, $y_p = ax^3 + bx^2 + cx = d$

On solving above equation we get,

$$y_p = -\frac{k_w x^3}{6P} + \frac{W_T x^2}{2P} + 0x = ax^3 + bx^2 + cx$$

Now equating according to power of 'x', we get complete solution

$$y = y_c + y_p$$

$\therefore$ $y = A\sin(\alpha x) + B\cos(\alpha x) - \frac{k_w x^3}{6P} + \frac{W_T x^2}{2P}$ ………………………………….……. 6.2.3

Taking derivative with respect to ‘x’, we get

$\frac{\partial y}{\partial x} = A\alpha cos\alpha x - B\alpha sin\alpha x - \frac{k_w x^2}{2P} + \frac{W_T x}{P}$ ……………………………...……..…....... 6.2.4

Taking derivative of equation 2.3 with respect to ‘x’, we get

$\frac{\partial^2 y}{\partial x^2} = -A\alpha^2 sin\alpha x - B\alpha^2 cos\alpha x - \frac{k_w x}{P} + \frac{W_T}{P}$ ……………………………...….………6.2.5

Now using boundary condition, when x=0, $\frac{\partial^2 y}{\partial x^2} = 0$

Equation becomes

$$0 = -B\alpha^2 cos\alpha x - \frac{k_w x}{P} + \frac{W_T}{P}$$

$\therefore B = \frac{-\frac{k_w x}{P} + \frac{W_T}{P}}{\alpha^2 cos\alpha x}$ ……………………………………………………….…………..6.2.6

Now using boundary condition, when x=H, $\frac{\partial y}{\partial x} = 0$

We get, $0 = A\alpha\ cos\alpha x - B\alpha\ sin\alpha H - \frac{K_w H^2}{2P} + \frac{W_T H}{P}$

$$\therefore A = \frac{B sin\alpha H + \frac{K_w H^2}{2P} - \frac{W_T H}{P}}{\alpha\ cos\alpha H}$$

Now substitute the values of A and B in equation 2.3

$\therefore$ We get,

$$y = \left\{\frac{B sin\alpha H + \frac{K_w H^2}{2P} - \frac{W_T H}{P}}{\alpha\ cos\alpha H} sin\alpha x\right\} + \left\{\left[\frac{-\frac{k_w x}{P} + \frac{W_T}{P}}{\alpha^2 cos\alpha x}\right] cos\alpha x\right\} - \frac{K_w x^3}{6P} + \frac{W_T x^2}{2P}$$

This is the equation of Deflection for Second Order Analysis of Chimney at any height ‘x’

Now substituting the value of ‘y’ in equation 2.1

We get,

$$M_x = \{P\left(\left\{\frac{B sin\alpha H + \frac{K_w H^2}{2P} - \frac{W_T H}{P}}{\alpha\ cos\alpha H} sin\alpha x\right\} + \left\{\left[\frac{-\frac{k_w x}{P} + \frac{W_T}{P}}{\alpha^2 cos\alpha x}\right] cos\alpha x\right\} - \frac{K_w x^3}{6P} + \frac{W_T x^2}{2P}\right)\} - \frac{W_T x^2}{2} - \frac{k_w x^3}{6}$$

This is the equation of Bending Moment for Second Order Analysis of Chimney at any height ‘x’

$\therefore$ By using the derived equations a programme is developed in excel for calculation of moments and displacement at every 20 m height of chimney.The results of the same are displayed below

Height (m)	BM (kN m) (First order)	BM2 (kN m) (Second order)	Disp (y)(mm)
0	497056.5	497056.5	0
20	430767	432405.1	3.912
40	368512.5	370624.2	5.614
60	310470.5	313155.4	8.012
80	256818.5	260108.3	11.127
100	207733.9	211455.4	14.535
120	163394.2	167605.4	19
140	123976.7	128275.5	23.14
160	89659.06	92665.93	25.913
180	60618.6	64280.74	29.946
200	37032.81	40081.74	33.016
220	19079.14	21382.81	36.005
240	6935.033	8395.312	39.252
260	777.953	1314.219	44.67
270	0	0	46.294

7. EFFECT OF SECOND ORDER ANALYSIS

With reference oto the results obtained, the effect of second order analysis on chimney needs to be shown.Hence, the above results are analysed and the second order effect with increasing height is studied.The percentage increase in moments due to second order with increasing height is shown below

Height (m)	BM (kN/m) (1st order)	BM(kN/m) (2nd order)	y (m)	y(mm)	BM2-BM1	Percentage Increase in BM
0	497056.5	506569.9	0	0	9513.41	1.8780053
10	463418.4	472579.5	0.001198	1.198	9161.053	1.9385211
20	430767	439575.7	0.002397	2.397	8808.705	2.0039108
30	399124.3	407580.6	0.003595	3.595	8456.363	2.0747707
40	368512.5	376616.5	0.004793	4.793	8104.008	2.1517929
50	338953.9	346705.5	0.005991	5.991	7751.668	2.2358073
60	310470.5	317869.9	0.00719	7.19	7399.317	2.3277819
70	283084.7	290131.7	0.008388	8.388	7046.969	2.4288865
80	256818.5	263513.1	0.009586	9.586	6694.62	2.5405261
90	231694.2	238036.5	0.010784	10.784	6342.27	2.664411
100	207733.9	213723.8	0.011983	11.983	5989.916	2.8026432
110	184959.8	190597.4	0.01318	13.18	5637.57	2.9578419
120	163394.2	168679.4	0.014379	14.379	5285.223	3.1332952
130	143059.1	147991.9	0.015577	15.577	4932.871	3.3332024

140	123976.7	128557.3	0.016776	16.776	4580.519	3.5630189
150	106169.1	110397.5	0.017974	17.974	4228.452	3.830206
160	89659.06	93534.89	0.019172	19.172	3875.832	4.1437286
170	74468.09	77991.56	0.02037	20.37	3523.474	4.5177632
180	60618.6	63789.73	0.021569	21.569	3171.13	4.9712234
190	48132.78	50951.56	0.022767	22.767	2818.78	5.5322737
200	37032.81	39499.24	0.023965	23.965	2466.431	6.2442502
210	27340.87	29454.95	0.025163	25.163	2114.083	7.1773448
220	19079.14	20840.87	0.026362	26.362	1761.734	8.4532661
230	12269.8	13679.19	0.02756	27.56	1409.392	10.303187
240	6935.033	7992	0.028758	28.758	1056.967	13.225309
250	3097.025	3801.72	0.029957	29.957	704.6953	18.536223
260	777.9531	1130.3	0.031155	31.155	352.3469	31.172867

8. CONCLUSIONS

On analysing the effect of second order analysis on a tall chimney, the following conclusions are drawn

- Second order effect is neglegible at the height where its self weight is zero
- Second order effect increases with increase in height
- Moments of first order and second order are maximum at the bottom of the chimney

9. REFERENCES

1) Manohar, S.N., "Tall Chimneys", Tata McGraw-Hill Publishing Company Limited, New Delhi, 1981.

2) I.S 4998 (Part1) – 1992, Indian Standard Code of Practice for Criteria for Design of Reinforced Concrete Chimneys. Part 1: Assessment of Loads. (Bureau of Indian Standard, New Delhi)

3) IS 1893(Part1) – 2002, Indian Standard Code of Practice for Criteria for Earthquake Resistant Design of Structures. Bureau of Indian Standard, (New Delhi)

4) IS 875 (Part3) – 1987, Indian Standard Code of Practice for Criteria for Design Loads (Other than Earthquake) For Buildings and Structures. (Bureau of Indian Standard, New Delhi)

5) B. J. Davidson " P Delta effects in multi-storey structural design "

BIOGRAPHIES

Dr. S.S Patil ,

M.E in Civil – Structures (W.I.T Sangli), Ph.D.(U.V.C.E Bangalore),

Professor and Head of Civil Engg Dept, Walchand Institute of Technology, Solapur,Maharashtra

patilss1962@gmail.com

B.E. (Civil Engineering), M.E.
(Civil - Structures),
aatifmundewadi@yahoo.in