

# Spur Gear Theory and Design

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## ABSTRACT

*Gears are commonly used for transmitting motion and power. In precision machines, in which a definite velocity ratio is of importance, the only positive drive is by means of gears or toothed wheels. They develop high stress concentration at the root and the point of contact. The repeated stressing on the fillets causes the fatigue failure of gear tooth. The main objective of this study is to add different shaped holes to reduce stress concentration. A finite element model of spur gear with a segment of three teeth is considered for analysis and stress concentration reducing holes of various sizes are introduced on gear teeth at various locations. Analysis revealed that aero-fin shaped hole introduced along the stress flow direction yielded better results...*

## Introduction :-

Gears are used for a wide range of industrial applications. They have varied application starting from textile looms to aviation industries. They are the most common means of transmitting power. They change the rate of rotation of machinery shaft and also the axis of rotation. For high speed machinery, such as an automobile transmission, they are the optimal medium for low energy loss and high accuracy. Their function is to convert input provided by prime mover into an output with lower speed and corresponding higher torque. Toothed gears are used to transmit the power with high velocity ratio, high efficiency, reliability in service, compact layout... During this phase, they encounter high stress at the point of contact. A pair of teeth in action is generally subjected to two types of cyclic stresses:

i) Bending stresses inducing bending fatigue

ii) Contact stress causing contact fatigue. Both these types of stresses may not attain their maximum values at the same point of contact. However, combined action of both of them is the reason of failure of gear tooth leading to fracture at the root of a tooth under bending fatigue and surface failure, due to contact fatigue. When loads are applied to the bodies, their surfaces deform elastically near the point of contact. Stresses developed by Normal force in a photo-elastic model of gear tooth. The highest stresses exist at regions where the lines are bunched closest together. The highest stress occurs at two locations:

1. At contact point where the force  $F$  acts

2. At the fillet region near the base of the tooth. 3. The surface failures occurring mainly due to contact fatigue are pitting and scoring. It is a phenomenon in which small particles are removed from the surface of the tooth due to the high contact stresses that are present between mating teeth. Pitting is actually the fatigue failure of the tooth surface. Hardness is the primary property of the gear tooth that provides resistance to pitting. In other words, pitting is a surface fatigue failure due to many repetitions of high contact stress, which occurs on gear tooth surfaces when a pair of teeth is transmitting power. Gear teeth failure due to contact. Fatigue is a common phenomenon observed. Even a slight reduction in the stress at root results in great increase in the fatigue life of a gear. For many years, gear design has been improved by using improved material, hardening surfaces with heat treatment and carburization, and shot peening to improve surface finish etc.

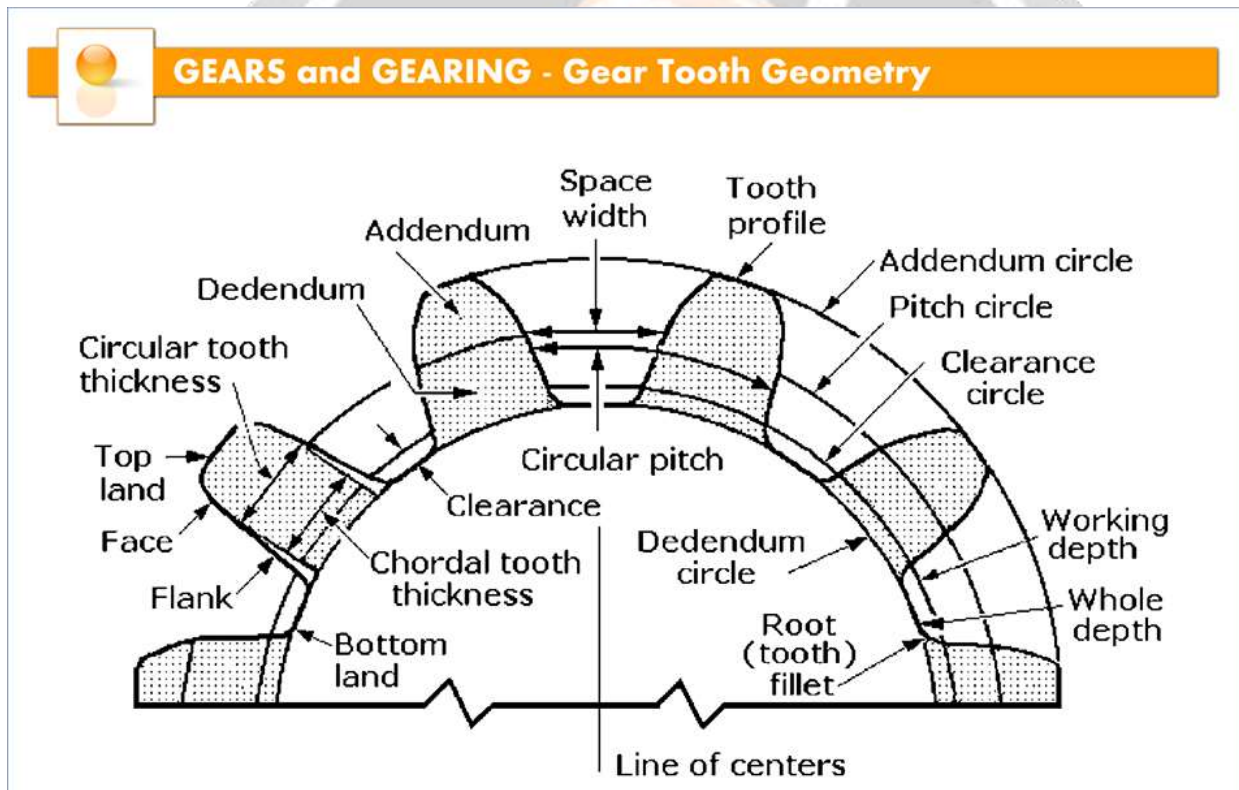
Few more efforts have been made to improve the durability and strength by altering the pressure angle, using the asymmetric teeth, altering the geometry of root fillet curve and so on. Some research work is also done using the stress redistribution techniques by introducing the stress relieving features in the stressed zone to the advantage of reduction of root fillet stress in spur gear. This also ensures interchange ability of existing gear systems. The studies in which combination of circular and elliptical stress relieving features are used obtained better results than using circular stress

relieving features alone which are used by earlier researchers. In this research work, an aero-fin shaped stress relieving feature is tried. A finite element model with a segment of three teeth is considered for analysis and a stress relieving feature of various sizes are introduced on gear teeth at various locations.

## II. Purpose:

Gearing is one of the most critical components in mechanical power transmission systems. The transfer of power between gears takes place at the contact between the mating teeth. During operation, meshed gears' teeth flanks are submitted to high contact pressures and due to the repeated stresses, damage on the teeth flanks, in addition to tooth breakage at the root of the tooth are one of the most frequent causes of gear failure. This fatigue failure of the tooth decides the reliability of the gear. However, by introducing stress relieving features to the gear, the points of stress concentration can be decreased which enhances life of gear. A study is done on spur gear with involute profile by adding stress relieving features of different shapes and best among them is proposed....Involute Gear Profiles are used as they are easy to manufacture, compared to Cycloidal tooth profiles..

## III. Gear Nomenclature:





- 1) Pitch surface: The surface of the imaginary rolling cylinder that replaces the toothed gear. Pitch circle: A normal section of the pitch surface.
- 2) Addendum circle: A circle bounding the ends of the teeth, in a normal section of the gear. 3) Dedendum circle or Root circle: The circle bounding the spaces between the teeth, in a normal section of the gear.
- 3) Addendum: The radial distance between the pitch circle and the addendum circle.
- 4) Dedendum: The radial distance between the pitch circle and the root circle.
- 5) Clearance: The difference between the Dedendum of one gear and the Addendum of the mating gear.
- 6) Face of a tooth: That part of the tooth surface lying outside the pitch surface. Flank of a tooth: The part of the tooth surface lying inside the pitch surface. Top land: The top surface of a gear tooth. 7) Bottom land: The bottom surface of the tooth space.
- 8) Circular thickness: The thickness of the tooth measured on the pitch circle. It is the length of an arc and not the length of a straight line. 9
- 9) Tooth space: The space between successive teeth.
- 10) Width of space: The distance between adjacent teeth measured on the pitch circle.
- 11) Backlash: The difference between the tooth thickness of one gear and the tooth space of the mating gear.

12) Circular pitch (P): The width of a tooth and a space, measured on the pitch circle. It is equal to the pitch circumference divided by the number of teeth.

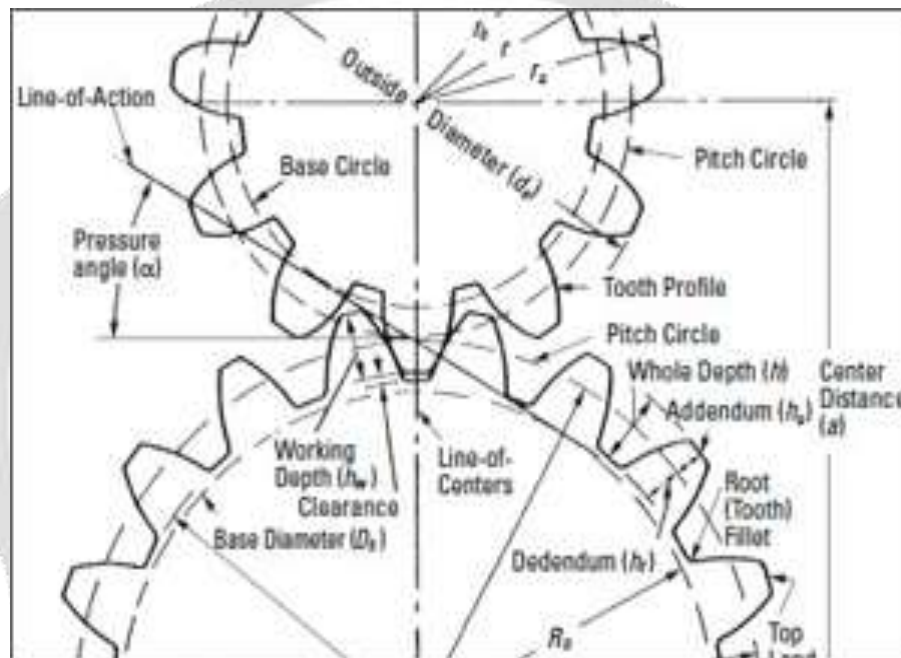
13) Diametrical pitch (P): The number of teeth of a gear per unit pitch diameter. The diametric pitch is hence the number of teeth divided by the pitch diameter.

14) Module (M): Pitch diameter divided by number of teeth. (PCD/NOT). The pitch diameter is usually specified in millimeters.

15) Fillet Radius: The small radius that connects the profile of a tooth to the root circle.

16) Base circle: An imaginary circle used in involute gearing to generate the involutes that form the tooth profiles.

17) Contact Ratio: The average number of gear tooth pairs in contact on a pair of meshing gears.



18) Pitch point: The point of tangency of the pitch circles of a pair of mating gears. Common tangent: The line tangent to the pitch circle at the pitch point.

19) Line of action: A line normal to a pair of mating tooth profiles at their point of contact.

20) Path of contact: The path traced by the contact point of a pair of tooth profiles.

21) Pressure angle: The angle between the common normal at the point of tooth contact and the common tangent to the pitch circles. Pressure angle is also the angle between the line of action and the common tangent.

22) Crowning: Grinding of tooth edges to prevent edge loading is known as crowning.

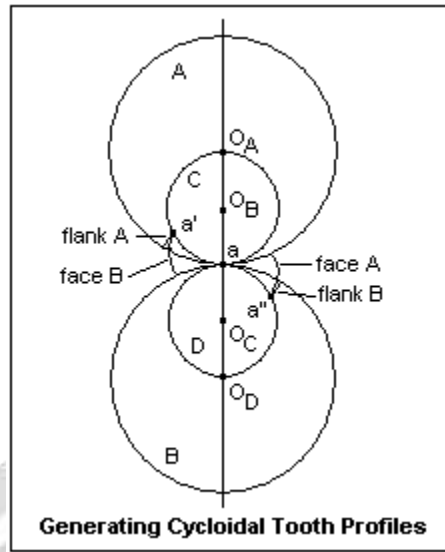
## SPUR GEAR:



Spur gears are the most common type of gears. They are used to transmit rotary motion between parallel shafts i.e., they are usually cylindrical in shape, and the teeth are straight and parallel to the axis of rotation. Sometimes many spur gears are used at once to create very large gear reductions. Spur gears are used in many devices but not in cars as they produce large noises. ..The tooth profiles are designed so as to produce a constant angular-velocity ratio during meshing, they are said to have conjugate action. A geometric relationship can be derived for the form of tooth profiles to provide conjugate action which is summarized as Law of Gearing as follows: "A common normal to the tooth profiles at their point of contact must, in all positions of the contacting teeth, pass through a fixed point on the line-of-centres called the pitch point." Any two curves or profiles engaging each other and satisfying the law of gearing are conjugate curves. The angular-velocity ratio between the two arms is inversely proportional to their radii to the point P. Circles drawn through point P from each centre are called pitch circles, and the radius of each circle is called the pitch radius. Point P is called the pitch point...

### Tooth profile:

**Cycloid AL :-** The cycloid AL gear profile is a form of toothed gear used in mechanical clocks. A Cycloid is the curve traced by a point on the circumference of a circle which rolls without slipping on the fixed straight line..The gear tooth profile is based on the epicycloids and hypocycloid curves, which are the curves generated by a circle rolling around the outside and inside of another circle, respectively. An advantage of the cycloid AL teeth over the involute one is that wear of Cycloid AL tooth is not as fast as with involute tooth. For this reason, gears transmitting very large amount of power are sometimes cut with cycloid AL teeth.



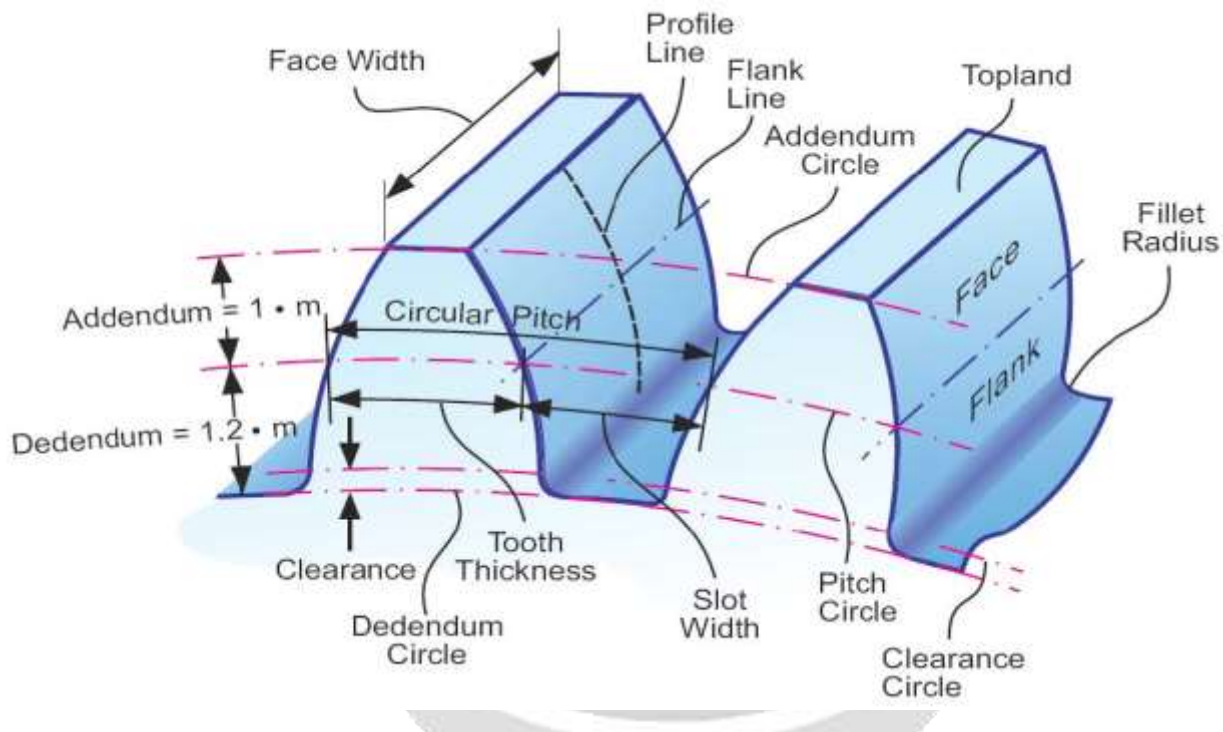
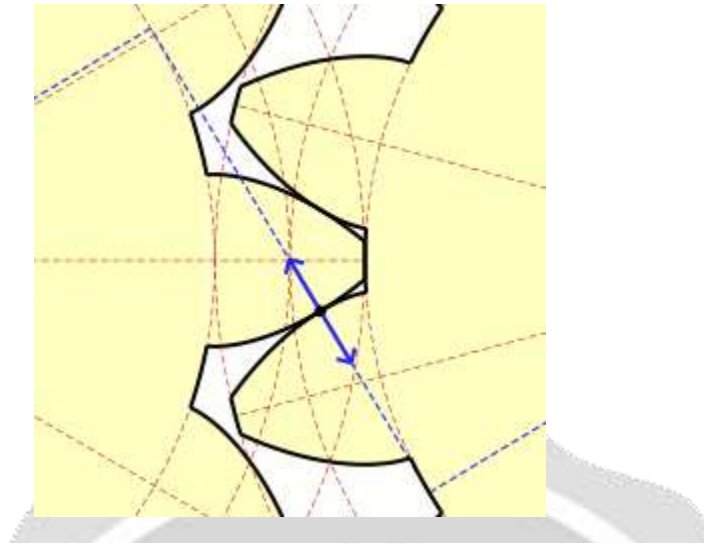
Nearly all practical gears now have involute tooth profiles. The involute tooth has great advantages in ease of manufacture, interchangeability, and variability of centre-to-centre distances. Originally, however, cycloidal profiles were used. Cycloidal profiles are as technically suitable as involute profiles, perhaps even slightly superior in some respects. They were satisfactory when gears were specially made in mating pairs, and were expensive. Accurate involute teeth became much easier and cheaper to produce; together with their other advantages, they have completely supplanted cycloidal gears. Many modern engineering texts and handbooks at most only mention cycloidal gears in passing. The inventor of cycloidal gears does not appear to be known; there are claims for Desargues, de la Hire, Roemer and Camus. They probably came into use early in the 17th century. The *common cycloid* is the curve traced by a point on the circumference of a circle as the circle rolls without slipping on a straight line. Parametric equations are  $x = r(t - \sin t)$ ,  $y = r(1 - \cos t)$ , where  $t$  is the angle of rotation. This graceful curve has several interesting properties. The one of most interest with respect to cycloidal gears is that the normal to the curve at any point passes through the point where the describing circle passing through the given point is in contact with the straight line. It is easy to see why this is the case, since the point of contact with the straight line is instantaneously at rest, and the circle is instantaneously rotating about this point. Therefore, the velocity of the point must be normal to a line from this instantaneous centre to the point. It is very important to grasp this notion, since it is the very basis of the use of cycloidal curves as gear tooth profiles.

1) Since the cycloid AL teeth have wider flanks, therefore the cyclical gears are stronger than the involute gears, for the same pitch. These are preferred for cast teeth.

2) In cycloid AL gears, the contact takes place between a convex flank and concave surface, whereas in involute gears, the convex surfaces are in contact. This condition results in less wear in cycloid AL wear and however the difference in wear is negligible.

3) The interference in cycloid AL gears does not occur at all. Though there are advantages of cycloid AL gears they are outweighed by the greater simplicity and flexibility of the involute gears.

**Involute:** The involute gear profile is the most normally used system for gearing. In an involute gear, the profiles of the teeth are involutes of a circle. The involute of a circle is the spiralling curve traced by the end of an imaginary taut string unwinding itself from that stationary circle called the base circle. In involute gear design, contact between a pair of teeth occurs at a single instantaneous point. Rotation of the gears causes the location of this contact point to move across the respective tooth surfaces.



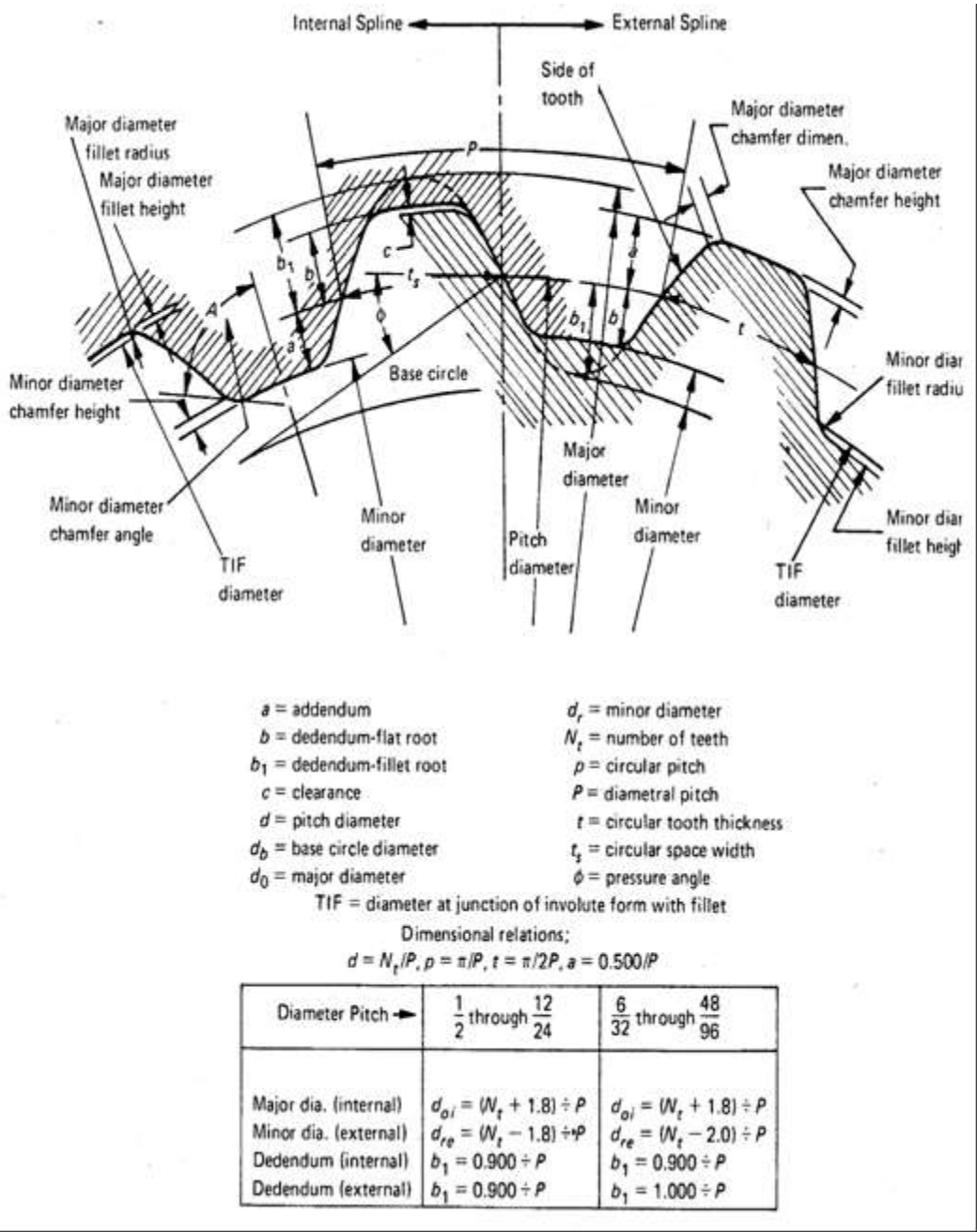
Two involute gears, the left driving the right: Blue arrows show the contact forces between them. The force line (or [line of action](#)) runs along a tangent common to both base circles. (In this situation, there is no force, and no contact needed, along the opposite common tangent not shown). The involutes here are traced out in converse fashion: points (of contact) move along the *stationary* force-vector "string" as if it was being unwound from the left *rotating* base circle, and wound onto the right *rotating* base circle.

1. Involute teeth are very easy to manufacture and the actual distance between the centres may deviate slightly from the theoretical distance without affecting the velocity ratio or general performance. Because of this distinct advantage, gears with involute profile teeth are used more than those with cycloid AL teeth.

2. In involute gears, the pressure angle, from the start of the engagement of teeth to its end remains constant. It is necessary for smooth running and less wear of gears. But in cycloid AL gears, the pressure angle is maximum at the beginning of engagement, reduces to zero at pitch point, starts increasing again and becomes maximum at the end of engagement. This does not yield smooth running of gears.

3. The face and flank of involute teeth are generated by a single curve where as in cycloid AL gears, double curves are required for the face and flank respectively.

4. Thus the involute teeth are easy to manufacture than cycloid AL teeth. The only disadvantage of involute teeth is that the interference occurs with pinions having smaller number of teeth.



**System of Gear Teeth:-** The Following Gear systems of gear teeth are commonly used in practice:

- 1) 14 1/2° Composite system
- 2) 14 1/2° Full depth involute system
- 3) 20° Full depth involute system
- and 4) 20° Stub involute system...

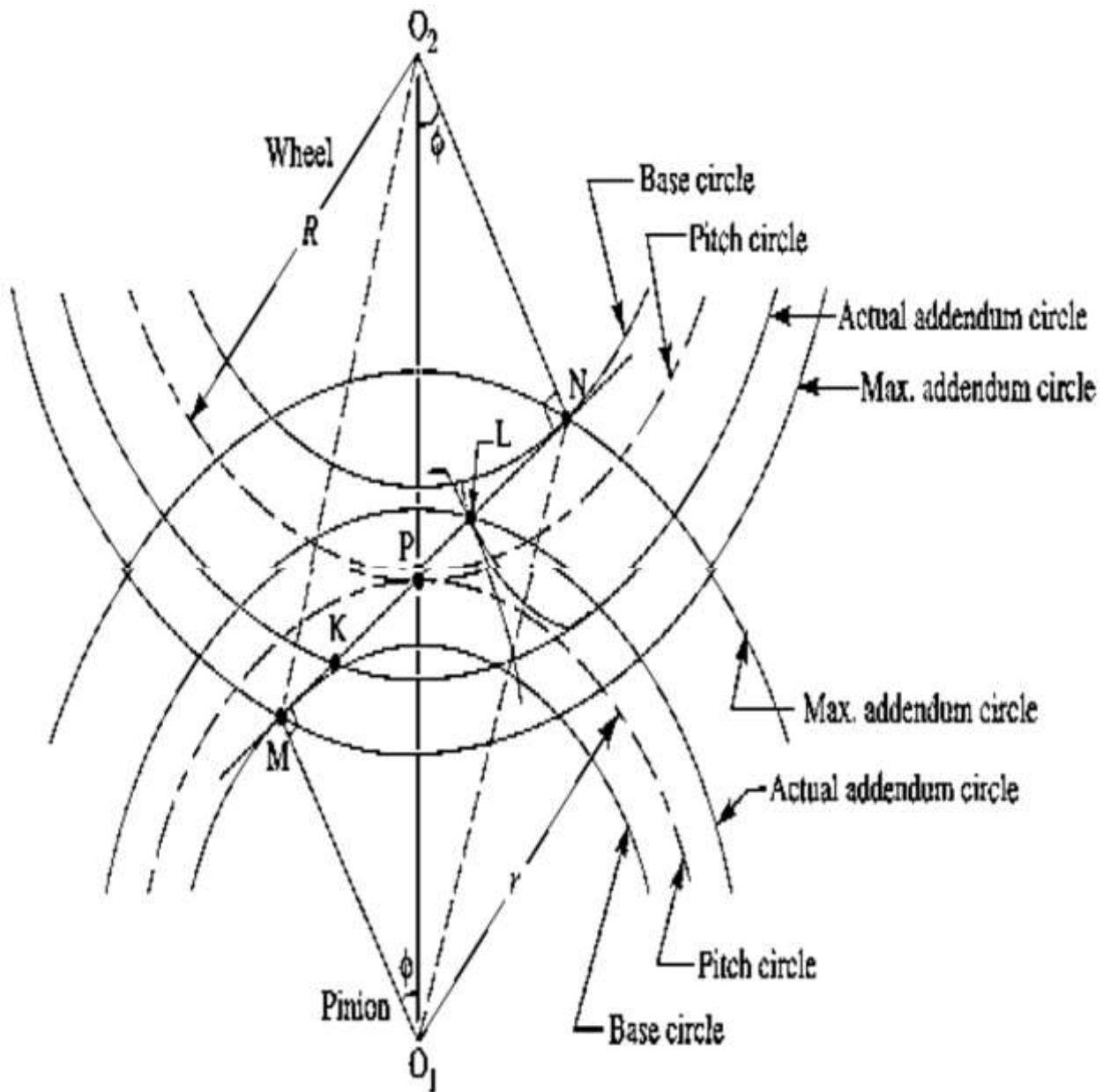
The 14 1/2° Composite system is used for general purpose gears. It is stronger but had no interchangeability. The tooth profile of this system has cycloidal curve at the top and bottom and involute curve at the middle portion. The teeth are produced by form milling cutters or hobs.

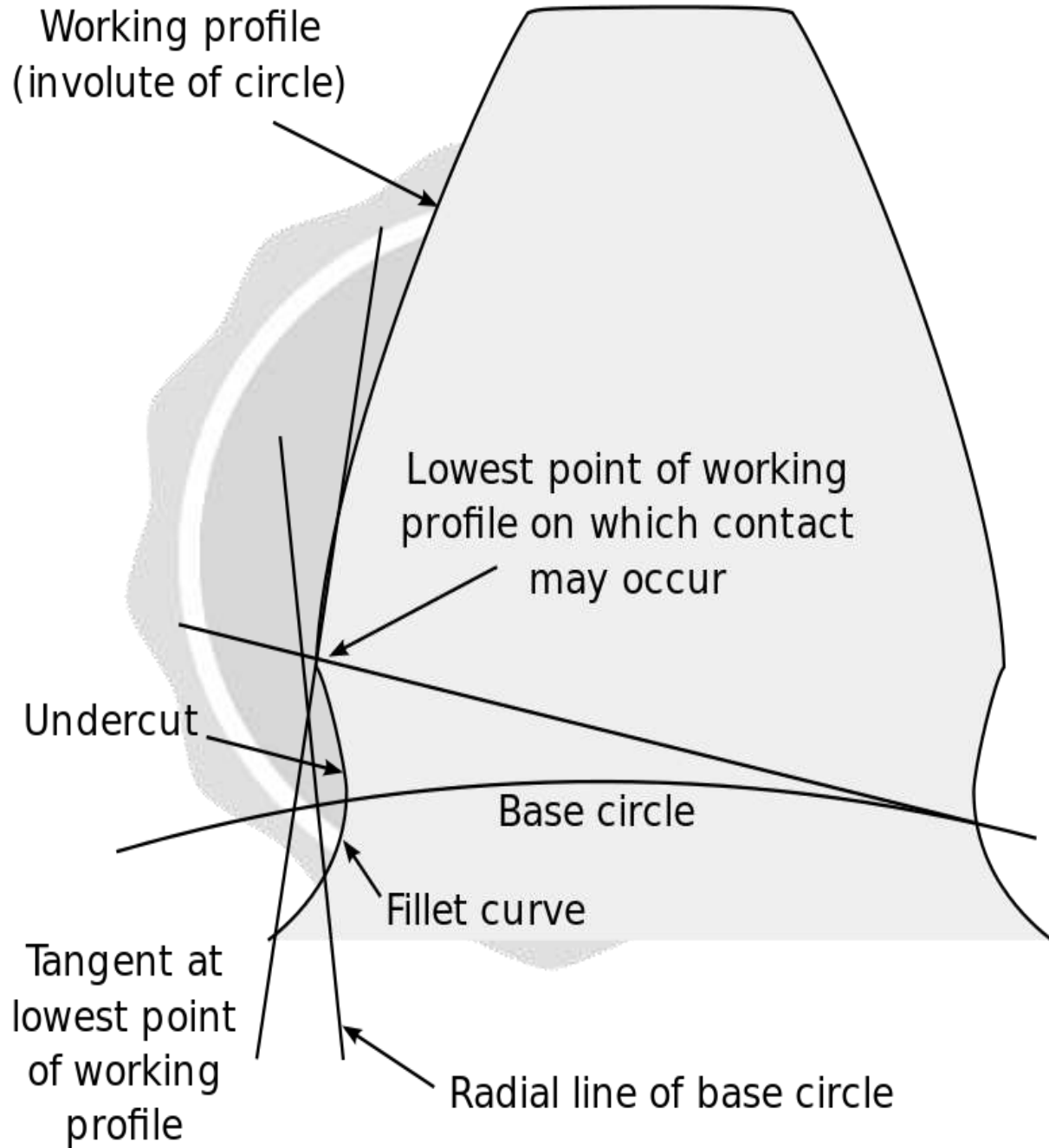
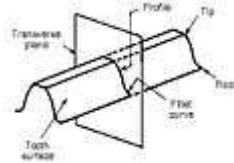


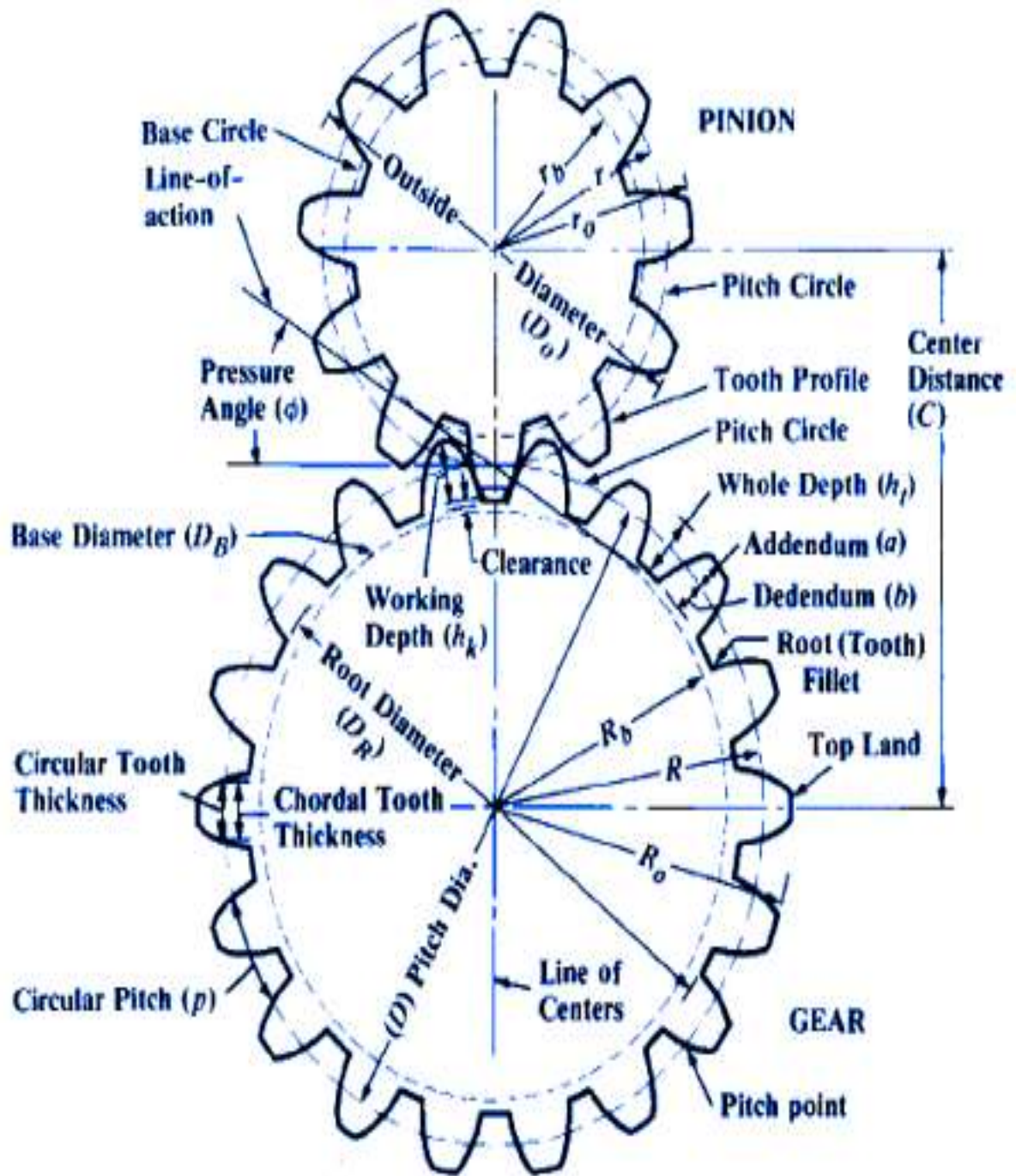
The tooth profile of 14 1/2° Full depth involute system was developed for use with gear hobs for spur and helical gears.

The tooth profile of 20° Full depth involute system may be cut by Hobs. The increase of the pressure angle from 14 1/2° to 20° results in a stronger tooth, because the tooth acting as the beam is wider at the base. The 20° Stub involute system has a strong tooth to take heavy loads....

Interference in Involute Gear:-



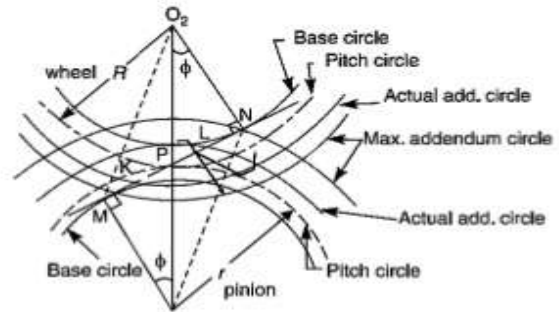




## Minimum Number of Teeth on the Pinion in Order to Avoid Interference

- We have already discussed that in order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency. The limiting condition reaches, when the addendum circles of pinion and wheel pass through points *N* and *M*

- t* = Number of teeth on the pinion,,
- T* = Number of teeth on the wheel,
- m* = Module of the teeth,
- r* = Pitch circle radius of pinion =  $m \cdot t / 2$
- G* = Gear ratio =  $T / t = R / r$
- $\phi$  = Pressure angle or angle of obliquity.



From triangle  $O_1NP$ ,

$$\begin{aligned} (O_1N)^2 &= (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN \\ &= r^2 + R^2 \sin^2 \phi - 2r \cdot R \sin \phi \cos (90^\circ + \phi) \quad \dots(\because PN = O_2P \sin \phi = R \sin \phi) \end{aligned}$$

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## MINIMUM NUMBER OF TEETH TO AVOID INTERFERENCE

On pinion

$$\therefore t = \frac{2A_p}{\sqrt{1 + \frac{T}{t} \left( \frac{T}{t} + 2 \right) \sin^2 \phi} - 1} = \frac{2A_p}{\sqrt{1 + G(G + 2) \sin^2 \phi} - 1}$$

On wheel

$$T = \frac{2A_w}{\sqrt{1 + \frac{t}{T} \left( \frac{t}{T} + 2 \right) \sin^2 \phi} - 1} = \frac{2A_w}{\sqrt{1 + \frac{1}{G} \left( \frac{1}{G} + 2 \right) \sin^2 \phi} - 1}$$



### Force Analysis:-

To transmit power from one gear to the other, force is applied by the tooth of the driving gear on the mating tooth of the driven gear. This force, called as Normal Force ( ) acts along the pressure line and is always normal to the tooth surface. This normal force can be resolved into two components:

1. Tangential component ( $F_t$ )– helps in transmission of torque and determines its magnitude.
  2. Radial component ( $F_r$ )– tends to push the gears apart, has no contribution in power transmission.
- If torque to be transmitted, T is known, tangential component of force can be calculated as,

$$F_t = \frac{T}{d/2}$$

Where d is pitch circle diameter

$F_n$  acts along the pressure line and  $F_t$  along the common tangent, therefore angle between  $F_n$  and  $F_t$  is  $\phi$ , the pressure angle. Referring to Figure 23.1, following relations can be written:

$$F_r = F_t \tan \phi \text{ and } F_n = F_t / \cos \phi$$

Above analysis is based on the following assumptions:

1.  $F_n$  remains constant in the power transmission ( $F_n$  changes with change in position of the point of contact).
2. Only one pair of teeth takes the entire load (Load is often shared by more than one pair).
3. Loads are static i.e. gears run at a low speed (There are dynamic loads in actual practice).

### Gear Material

As for any other component, selection of material depends upon performance requirements, material properties, manufacturing aspects & cost and availability. Gear material should have good strength, high endurance limit, good wear resistance, low coefficient of friction and good manufacturability. Gears are subjected to dynamic loads and very high bending and contact stresses.

Cast iron can be used for lighter stress conditions. It has good castability, machinability and wear properties. Also its vibration damping property makes the operation quieter. For light to medium duty gears steel castings and structural steels may be used. Hardened and tempered steel or case-hardened steels are used for heavy duty applications. For working conditions where special properties, like resistance to heat, corrosion or wear, are desired, alloy steels can be used bronzes, aluminium and zinc alloys are some other material that are used for manufacturing gears, due to their high strength and good sliding properties. Non-metallic materials like acetal, nylon and other plastics are also used for manufacturing gears. These are quieter, durable, cheaper and don't require lubrication also but their load carrying capacity is low. Gears can be manufactured with the help of casting, forging or machining.

## **Gear Tooth Failure**

Gear tooth may fail in two ways – breakage of tooth due to overloading or fatigue or Surface damage of the tooth due to wear, pitting or scoring. These are discussed below:

### **Breakage of Tooth**

Each tooth of gear is subjected to bending stress, which is maximum at the fillet of its base. Gear tooth may fail, if this stress reaches the yield strength of the material. Also as the load on the tooth is dynamic in nature, it leads to variable stresses and the tooth may fail due to fatigue.

### **23.3.2. Surface Damage of the Tooth**

Surface of gear tooth is subjected to very high contact stresses at the point of contact between two mating gears, where the normal force acts. Surface of the tooth may get damaged due to these very high contact stresses. Gear-tooth surface damage is a very complex phenomenon as excessive loading and lubrication breakdown may lead to various combinations of abrasion, pitting, and scoring. These three basic types of surface deterioration are briefly discussed below:

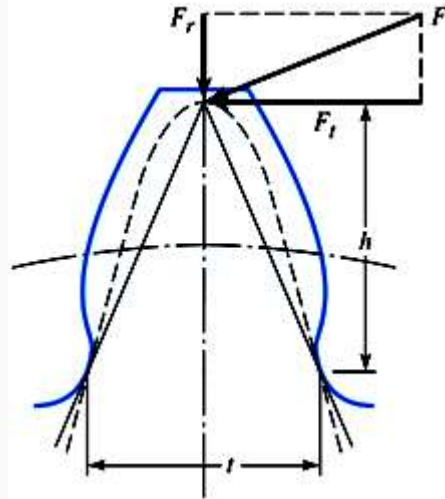
Abrasion (Abrasive wear) is a type of wear caused by the presence of foreign particles on the surfaces of contact between two mating gears. These foreign particles may be present at the time of assembly or may enter along with the lubrication oil if it is not filtered properly.

Scoring is a form of adhesive wear, which is generally caused due to the failure of lubrication film between the two surfaces i.e. due to inadequate lubrication and occurs in gears rotating at very high speeds. Inadequate lubrication leads to increase in coefficient of sliding friction that combines with high sliding velocity and very high tooth loads to produce a high rate of localized heat generation. This high temperatures and pressure condition causes welding and tearing apart of the tooth surface, which is called scoring. It can be prevented by providing good surface finish and using an appropriate lubricant.

Pitting and Spalling are types of surface fatigue failures that are characterized by the separation of small bits of material from the surface of the tooth. Due to very high localized stresses, cracks are initiated on and under the contacting surfaces of the teeth, which start propagating, finally resulting in loss of material from the surface in the form of bits. If this phenomenon starts with surface cracks resulting in pits of relatively smaller size, it is called pitting. And if it originates with subsurface cracks resulting in thin flakes of surface material, it is called Spalling.

### **Beam Strength of Gear Tooth**

Wilfred Lewis presented first recognized analysis of bending stress in gear tooth in 1892, which serves as the basis of bending stress analysis of gear teeth, even today. Lewis considered the gear tooth as a cantilever beam, subjected to bending moment due the tangential force acting on it, as shown in Figure



Lewis Analysis is based on following assumptions:

1. Only one pair of teeth takes the entire load (Load is often shared by more than one pair).
2. The effect of radial component,  $F_r$ , is negligible ( $F_r$  induces compressive stress).
3. The load is uniformly distributed across the full face width. (Gear should be rigid and accurately machined for this).
4. Tooth sliding friction forces are negligible.
5. Effect of stress concentration is negligible.

Cross-section of the gear tooth, considered as a cantilever beam, varies from the free end to the fixed end. A parabola can be constructed within the tooth profile, as shown in Figure 23.1, giving the outline of a beam of uniform strength. Cross-section at XX is critical, where this parabola is tangent to the tooth profile. If  $b$ ,  $t$  and  $h$  are width, thickness and height of the gear tooth, Bending Stress at section XX can be given by,

$$\sigma_b = \frac{My}{I} = \frac{F_t h(t/2)}{bt^3/12} = \frac{6F_t h}{bt^2}$$

Multiplying nominator and denominator by 'm' and rearranging,

$$F_t = mb\sigma_b \left( \frac{t^2}{6hm} \right) = mb\sigma_b Y \quad \text{where } Y = t^2 / 6hm = \text{Lewis Form Factor}$$

Above equation gives the relation between tangential force and bending stress in the tooth. increases with increase in . The maximum tangential load that can be transmitted by the tooth without bending failure is called its Beam Strength . Therefore in the above relation, if  $\sigma_b$  is replaced by allowable bending stress,  $[\sigma_b]$  of the tooth, corresponding value of  $F_t$  is the beam strength.

$$\therefore S_b = mb[\sigma_b] Y$$

Value of  $Y$  for different systems can be taken as given below:

$$Y = \pi \left[ 0.124 - \frac{0.684}{Z} \right] \text{ for } 14.5^\circ \text{ involute teeth}$$

$$= \pi \left[ 0.154 - \frac{0.912}{Z} \right] \text{ for } 20^\circ \text{ full depth teeth}$$

$$= \pi \left[ 0.175 - \frac{0.950}{Z} \right] \text{ for } 20^\circ \text{ stub teeth}$$

For the gears to be safe in bending,  $\sigma_b$  should be less than  $[\sigma_b]$  or  $F_t$  should be less than  $S_b$ . If gear and pinion are made of same material, pinion is the weaker member and design is based on its strength. But if those are made of different materials, it is evident from the Lewis Equation that the gear or pinion having lesser value of product  $[\sigma_b] \times Y$  is weaker, as  $m$  and  $b$  are same for pinion and gear. Also as the gears are subjected to variable stresses, design is based on the endurance limit and allowable bending stress can be taken as,  $[\sigma_b] = S_e/f_{os}$

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