Studies of energy contribution input by electromagnetic wave radiation by a circular black body, heated using a Solar paraboloid concentrator.

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ABSTRACT

This work is based on the field of energy physics and particularly affects solar energy which is new and renewable to heat a circular black body placed at the focus of a dish concentrator. The principle chosen to study the energy input of the heated body is limited to the analysis of the radiation, without taking into account the thermal conduction in the air and the thermal convection in the materials. To ensure the reliability of numerical calculations and graphical representations, the MATLAB software was chosen for all operations and programming. In these cases, the results obtained are analyzed and interpreted according to the magnitudes of the constituent elements of the system (paraboloid, blackbody) and discussed according to the effect of the presence of the atmosphere on the solar constant or in its absence. This study allows researchers to determine the color of a heated body at a given temperature by studying Planck's function. For the two black body shapes studied, the wavelength calculations correspond to the maximum values of the luminances making it possible to prove that this color belongs to the Infrared (IR). For this study, we set the dimensions of the solar concentrator (height, focus), the black body and that of gray, taking into account that the latter is smaller in order to neglect radiation losses. We find that the black body heated in our study emits Infrared.

Keyword : - *Paraboloid, Solar, blackbody,* focus

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1. Introduction

Solar energy is characterized by the constant called power flow, emitted per unit area. This power is inexhaustible and can be reunited at the focus of a dish concentrator. However, a black body placed at this point, much smaller in size than that of the concentrator opening, heats up by absorbing all the solar radiation reflected by this system. It follows that, at a given equilibrium temperature, this body emits radiation to another body. This program, which follows Planck's law, allows researchers to determine the color of the heated body by studying the luminance that is a function of this equilibrium temperature. For this study, we set the dimensions of the solar concentrator (height, focus), the black body and the gray one, making sure that the latter is as small as possible, in order to be able to neglect the losses by radiation.

2. Theoretical methods

In general, our method requires integrating Bougouer's formula [1], [2], which calculates the power radiated by a surface element (dS1) to another surface element (dS2) which is respectively an angle θ1 and θ2 with respect to the normal.

$$d^2P_{1\to 2} = \frac{\epsilon_r I_S}{\pi} dS_2 \frac{dS_1 \cos\theta_2}{(r_{12})^2} \cos\theta_1 \qquad (1)$$

I_S is the solar constant that we discuss according to the effect of the atmosphere and is the emissivity of the body considered. The integration takes place twice, the first of which is relative to the surface (S1) to calculate the power received at a point of the surface (S2) and the second is relative to the surface (S2) to calculate the power total received on the surface (S2); Three steps are followed to reach our goal.

2.1 Calculation of power received at the surface surrounding the concentrator focal point

Equation (1) can be written as:

$$d^2P_{1\to 2} = \frac{\epsilon_r I_S}{\pi} dS \frac{dS_1}{(r_{12})^2} \left(\frac{1}{\sqrt{1+\left(\frac{r}{2f}\right)^2}}\right)\left(\frac{1}{2f} r\right) \qquad (2)$$

With dS and dS1 are respectively surface elements of the paraboloid [3] and the black body in its focus.
r_{12}:the distance between dS and dS_1.
f: focal length of the parabolize
r: distance obtained by the change of variables
Formula of change of the useful variables:

$$\vec{r}_1(r,\varphi) = \begin{pmatrix} r\cos\varphi \\ r\sin\varphi \\ \frac{1}{4f} r^2 \end{pmatrix} \qquad (3)$$

$\vec{r}_1$: vector position of the surface element dS by report on origin O (summit of the paraboloïd) of the reference mark $(O,\vec{x},\vec{y},\vec{z})$ considered.

With:

$$\begin{cases} r \in [0, R] \\ \varphi \in [0, 2\pi] \end{cases}$$

Where R is the ray of the superior opening of the parabolize.

Furthermore

$$\frac{\partial \vec{r}_1}{\partial r} = \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ \frac{1}{2f} r \end{pmatrix} \qquad (4)$$

$$\frac{\partial \vec{r}_1}{\partial \varphi} = \begin{pmatrix} -r\sin\varphi \\ r\cos\varphi \\ 0 \end{pmatrix} \qquad (5)$$

From where

$$\overrightarrow{dS} = \left(\frac{\partial \vec{r}_1}{\partial r} \wedge \frac{\partial \vec{r}_1}{\partial \varphi}\right) dr d\varphi = r \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ \frac{1}{2f} r \end{pmatrix} \wedge \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix} = r \begin{pmatrix} -\frac{1}{2f} r\cos\varphi \\ -\frac{1}{2f} r\sin\varphi \\ 1 \end{pmatrix} dr d\varphi \quad (6)$$

Finally:

$$dS = \left|\overrightarrow{dS}\right| = r\left(\sqrt{1 + \frac{r^2}{4f^2}}\right) dr d\varphi \quad (7)$$

$$\int \frac{dx}{x^2 + 1} = \mathrm{Arctan}(x) \quad (8)$$

2.2 Calculation of the maximum temperature at the focal point of the concentrator

The maximal temperature to the foyer [3], a concentrator parabolize solar can be gotten by:

$$C = \frac{S_c}{S_r} \quad (9)$$

With:

SC: surface collector

Sr: receiving surface

Or:

$$C = \frac{I_{max}}{I_S} \quad (10)$$

With: I_{max}: the maximal intensity in the focal volume.

And by applying the law of Stephan Boltzmann, we can associate the temperature T of a black or gray body to the maximum illumination received. That is to say :

$$\sigma T^4 = \epsilon_r I_{max} = \epsilon_r C I_S \quad (11)$$

With:

I_{max}: the maximal intensity in the focal volume.

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$$\sigma T^4 = \epsilon_r I_{max} = \epsilon_r C I_S \quad (11)$$

With:

ϵ_r: Emissivity of the shape (reflector).

And while supposing that the incidental radiances are normal to the surface collector, it allows us to pull the temperature maximal attainable in the focus of a solar cooker.

$$T = \sqrt[4]{\frac{\epsilon_r C I_S}{\sigma}} \quad (12)$$

And with (9), the formula (12) becomes:

$$T = \sqrt[4]{\frac{\epsilon_r 8H\sqrt{fH} I_S}{3R_1^2\sigma}} \quad (13)$$

- without attenuation of the atmosphere:

Without considering the attenuation due to the presence of the atmosphere, the calculation of the solar record is obtained by:

$$I(r) = 3{,}02\ 10^{25}\left(\frac{1}{r^2}\right)\ wm^{-2} \quad (14)$$

- With attenuation of the atmosphere:

Considering the attenuation due to the presence of the atmosphere, the calculation of the solar record [4] is obtained by :

$$I'_s(\theta) = 1353 e^{-[0{,}16+0{,}22\,m(\theta)]}\cos\theta \quad (15)$$

With:

$$m(\theta) = \begin{cases} \dfrac{1}{\cos\theta} & \text{si } 0 \le \theta \le 80^\circ \\ [\cos\theta + 0{,}15\ (93{,}885 - \theta)^{-1{,}253}]^{-1} & \text{si } 80^\circ \le \theta \le 90^\circ \end{cases} \quad (16)$$

2.3 Calculation of the power radiated by the black body toward a gray body:

The power radiated by the black body towards a gray body is thus obtained by integrating (1) and using the Residue theorem:

$$I = \int_0^{2\pi} \frac{P(\cos t, \sin t)}{Q(\cos t, \sin t)} dt = 2i\pi \sum_k \text{Rés}\,[f, z_{0k}] \quad (17)$$

With z_{ok} a kineme pole is as $|z_{ok}|<1$

and:

$$\text{Rés}[f, z_0] = \frac{1}{(p-1)!} h^{(p-1)}(z_0) \quad (18)$$

Where:

$$h(z)=(z-z_0)^p f(z) \quad (19)$$

With:

$$f(z) = \frac{1}{g(z)} \quad (20)$$

The pole z_0 of f(z) is a root of the equation $g(z_0)=0$ and the p order of the pole z_0 of f(z) is the first power of the factor $(z-z_0)$ of no hopeless coefficient in the development in series of Taylor of g(z) around z_0.

The calculation of this power requires to use the formulas of following integrals:

$$\int \frac{1}{(x^2+b)^{\frac{3}{2}}}dx = \frac{1}{b}\frac{x}{\sqrt{x^2+b}} \tag{21}$$

$$\int \frac{X}{\sqrt{X^2+b}}dX = \int \frac{2X}{2\sqrt{X^2+b}}dX = \sqrt{X^2+b} \tag{22}$$

2.4 Calculation of luminance as a function of wavelength:

The luminance [5], [6], [7] as a function of the wavelength is obtained by Planck's law:

$$L_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{\left(e^{\frac{hc}{\lambda k_B T}}-1\right)} \tag{23}$$

3. Results:

3.1 The power received at the surface surrounding the concentrator's focus:

The equation (1), allows us to get:

$$P_{1\to 2} = 4\pi f^2 \epsilon_r I_S R_1^2 \left\{ \mathrm{Arctan}\left(\sqrt{\frac{H}{f}}\right) - \frac{\sqrt{\frac{H}{f}}}{\left(\frac{H}{f}+1\right)} \right\} \tag{24}$$

By taking the following numeric values:

$$\begin{cases} f = 0{,}25\mathrm{cm} \\ \epsilon_r = 0{,}96 \end{cases}$$

- without attenuation:

The maximal value of the solar constant is

$$I_{Smax}(\theta) = 1353\mathrm{Wm}^{-2} \tag{25}$$

So we have:

$$P_{1\to 2} = 1019{,}62\ R_1^2 \left\{ \mathrm{Arctan}\left(\sqrt{4H}\right) - \frac{\sqrt{4H}}{(4H+1)} \right\} \tag{26}$$

And at various values of R1 given in the following Table 1:

Table 1: Values of ray of the circular black body.

R_1 (en m)	0,04	0,05	0,06	0,07

While making vary the height HS of 0 to 0,5m, we have the following curves:

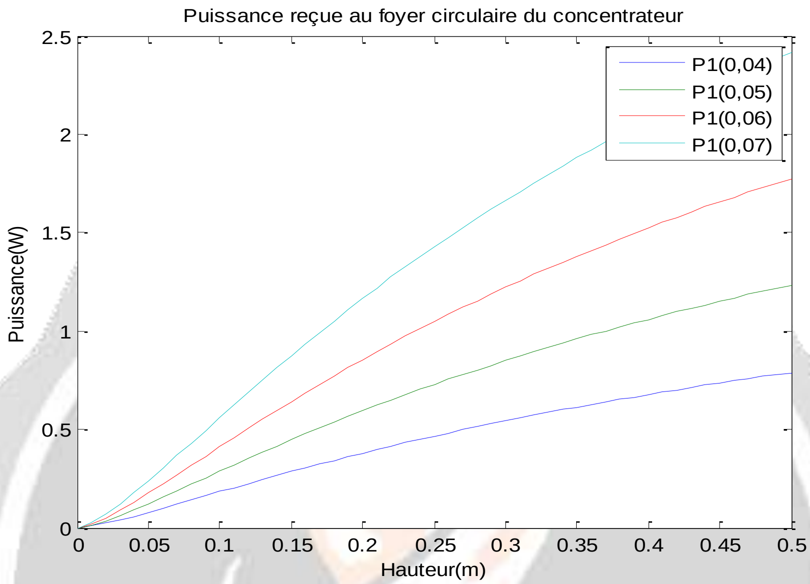

Fig 1: Representative curve of the power received in a circular surface without attenuation, according to the height.

- With attenuation of the atmosphere:

The maximal value of the solar constant is:

$$I'_{Smax}(\theta) = 925{,}26\ W.m^{-2} \tag{27}$$

Then we have:

$$P_{1\to 2} = 697{,}27\ R_1^2 \left\{ \mathrm{Arctan}\left(\sqrt{4H}\right) - \frac{\sqrt{4H}}{(4H+1)} \right\} \tag{28}$$

and with previous values, we have the following curves:

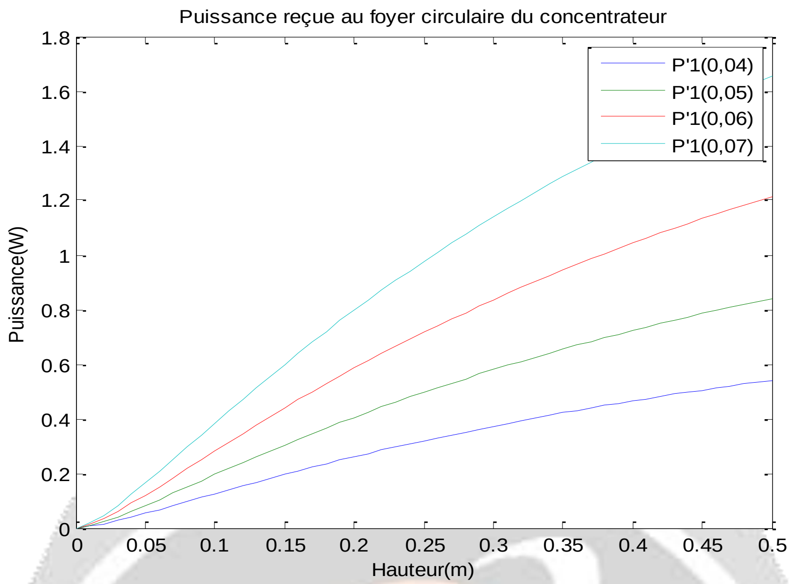

Fig. 2: Representative curve of the power received in circular surface with attenuation, according to the height

3.2 The maximal temperature to the concentrator's focus:

While fixing the value of f and for the following cases:

- Without attenuation of the atmosphere:

In this case we get:

$$T = \frac{418{,}23}{\sqrt{R_1}} H^{\frac{3}{8}} \tag{29}$$

For the previous values, we have the following curves:

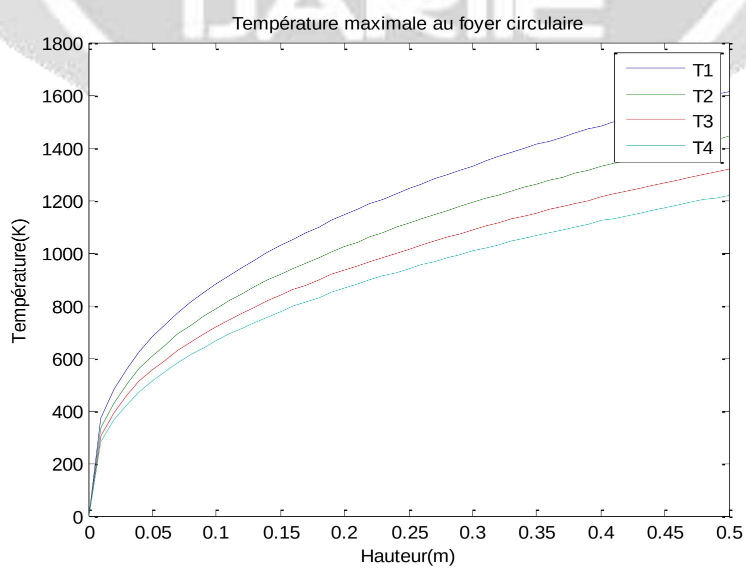

Fig 3: Representative curve of the maximal temperature to the circular focus according to the height.

- With the attenuation of the atmosphere:

In this case, we get:

$$T = \frac{380{,}33}{\sqrt{R_1}} H^{\frac{3}{8}} \qquad (30)$$

These curves are:

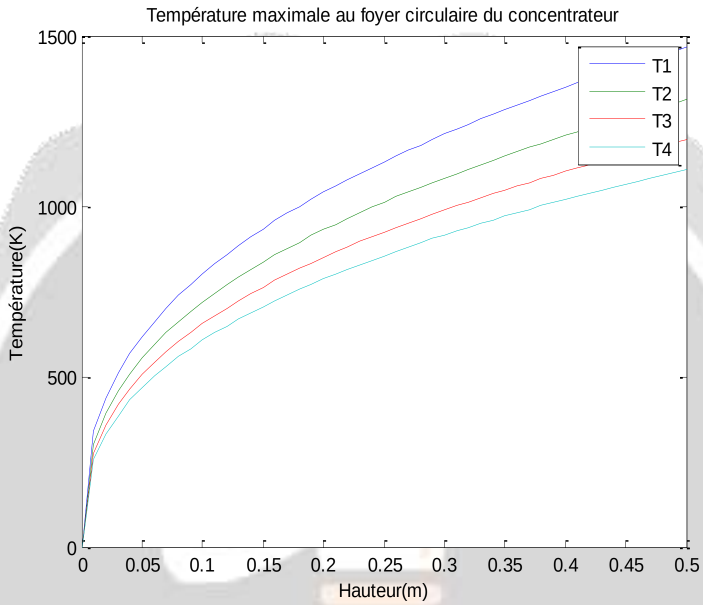

Fig 4: Curve representative of the maximum temperature at the rectangular focus according to the height of the dish

3.3 The power radiated by the black body toward a gray body:

Is $d^2\Phi_{1\to2}$ the elementary energy flux emitted by the surface element $d\Sigma_1$ de la surface (Σ_1) at the temperature T_1 which will be received by the surface element $d\Sigma_2$ from the surface (Σ_2). The total (hemispheric) flux emitted by the surface (Σ_1) is:

Using the Formula Bougouer (1), the infinitesimal flux emitted by the surface element dS_1 et intercepted by the surface element $d\Sigma_2$ is :

$$d^2\,\Phi_{1\to2} = \frac{\sigma T_1^4}{\pi} dS_1 \frac{dS_2 \cos\theta_2}{(r_{12})^2} \cos\theta_1 \qquad (31)$$

The two surface elements dS1 and dS2 are separated by the distance $|\vec{r}_2 - \vec{r}_1|$ that we note by r_{12}.

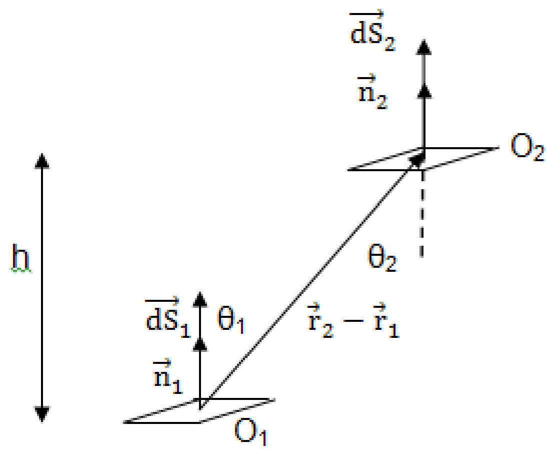

Fig. 1：Simple presentation of figure 31, with parameters in polar coordinates.

The two surface elements dS1 and dS2 are separated by h.

The black surface (S1) emits electromagnetic radiation to the gray surface (S2). As the two surfaces are circular then note respectively their radii by R1 and R2. They are elongated and of the same axis.

Since the two surfaces are parallel, then: θ1 = θ2 = θ, and that:

$$\cos\theta = \frac{h}{r_{12}}$$

From where equation (31) becomes:

$$d^2\Phi_{1\to 2} = h^2 \frac{\sigma T_1^4}{\pi} dS_1 \frac{dS_2}{(r_{12})^4} \tag{32}$$

First, by integrating on the surface (S1) one can write:

$$d\Phi_{1\to 2} = h^2 \frac{\sigma T_1^4}{\pi} dS_2 \iint_{(S_1)} \frac{dS_1}{(r_{12})^4} \tag{33}$$

Parameters the surface (S1):

$$\vec{r}_1(r_1, \varphi) = r_1 \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{pmatrix}$$

with

$$\begin{cases} r_1 \in [0, R_1] \\ \varphi \in [0, 2\pi] \end{cases}$$

From where:

$$\overrightarrow{dS_1} = \left(\frac{\partial \vec{r}_1}{\partial r_1} \wedge \frac{\partial \vec{r}_1}{\partial \varphi}\right) dr_1 d\varphi = r_1 \begin{pmatrix}\cos\varphi \\ \sin\varphi \\ 0\end{pmatrix} \wedge \begin{pmatrix}-\sin\varphi \\ \cos\varphi \\ 0\end{pmatrix} = r_1 \begin{pmatrix}0 \\ 0 \\ 1\end{pmatrix} dr_1 d\varphi \qquad (34)$$

$$r_{12}{}^2 = r_1^2 - 2(x_2 \cos\varphi + y_2 \sin\varphi) r_1 + r^2 + h^2$$

$$d\Phi_{1\to 2} = h^2 \frac{\sigma T_1^4}{\pi} dS_2 \iint_{(S_1)} \frac{r_1 dr_1 d\varphi}{[r_1^2 - 2(x_2 \cos\varphi + y_2 \sin\varphi) r_1 + r^2 + h^2]^2} \qquad (35)$$

$$d\Phi_{1\to 2} = \frac{1}{2}\frac{\sigma T_1^4}{\pi}\left[1 - \frac{(\rho_2^2 - R_1^2 + h^2)}{\sqrt{(\rho_2^2 - R_1^2 + h^2)^2 + 4R_1^2 h^2}}\right] dS_2 \qquad (36)$$

Now integrate with dS2

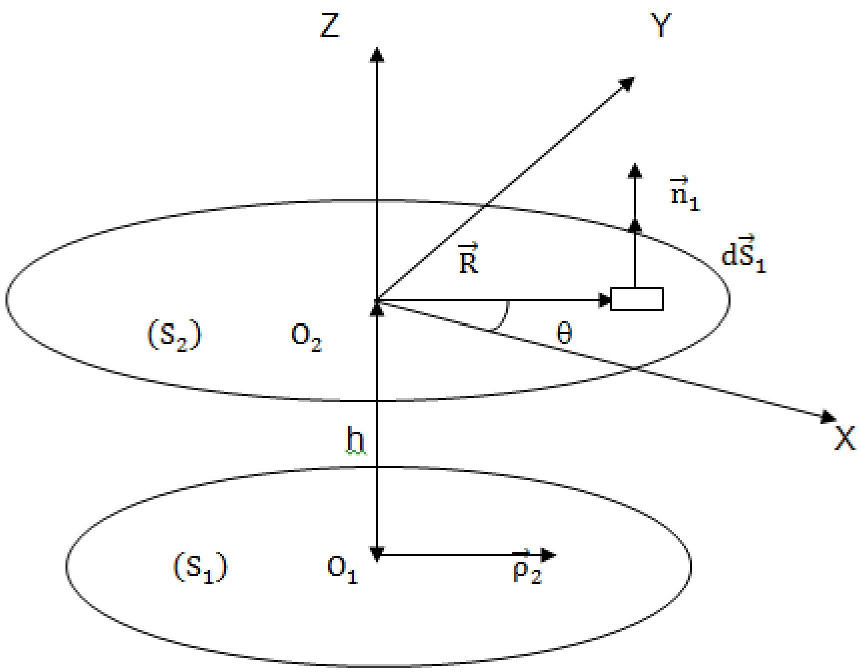

Fig. 2 : Positioning of the receiving surface element of the radiations emitted by the black body in polar coordinates.

$$\Phi_{1\to 2} = \frac{1}{2}\frac{\sigma T_1^4}{\pi}\left[S_2 - \pi \int_0^{R_2} \frac{(R^2 - R_1^2 + h^2)}{\sqrt{(R^2 - R_1^2 + h^2)^2 + 4R_1^2 h^2}} 2R\, dR\right] \qquad (38)$$

$dS_2 = RdRd\theta$

With:

$R \in [0, R_2]$ et $\theta \in [0, 2\pi]$

$$\Phi_{1\to 2} = \frac{\pi}{2}\left(\frac{\sigma T_1^4}{\pi}\right)\left[R_1^2 + R_2^2 + h^2 - \sqrt{(R_1^2 - R_2^2 - h^2)^2 + 4R_1^2 h^2}\right] \qquad (39)$$

We find that this power is:

$$\Phi_{1\to2} = \frac{\pi}{2}\left(\frac{\sigma T_1^4}{\pi}\right)R_1^2\left\{1 + \left(\frac{R_2}{R_1}\right)^2 + \left(\frac{h}{R_1}\right)^2 - \sqrt{[1 - \left(\frac{R_2}{R_1}\right)^2 - \left(\frac{h}{R_1}\right)^2]^2 + 4\left(\frac{h}{R_1}\right)^2}\right\} \quad (40)$$

3.4 The luminance according to the length of wave:

The luminance according to the length of wave is given by the law of Planck:

$$L_\lambda(T) = \frac{1{,}19\ 10^{-16}}{\lambda^5}\frac{1}{\left(e^{\frac{1{,}44\ 10^{-2}}{\lambda T}} - 1\right)} \quad (41)$$

- Without attenuation of the atmosphere:

For the two heights of the dish, and the values of the maximal temperatures found, we have the following curves:

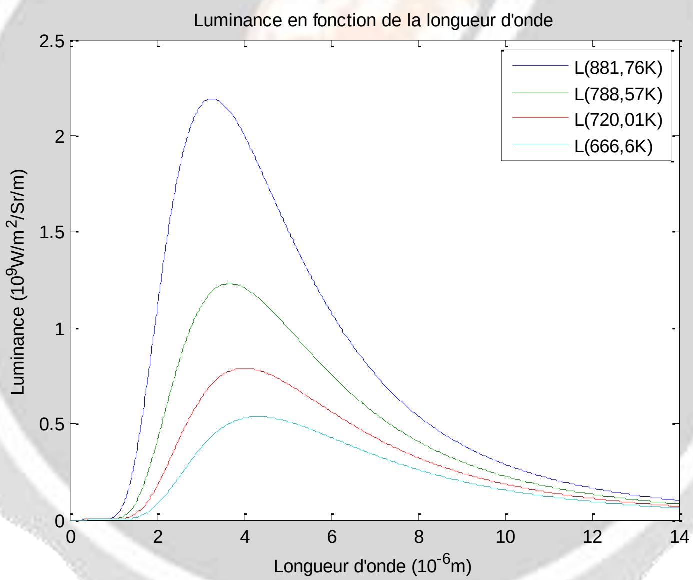

Fig 5: Representative curve of the luminance as a function of the wavelength for the height of 0.1m.

and :

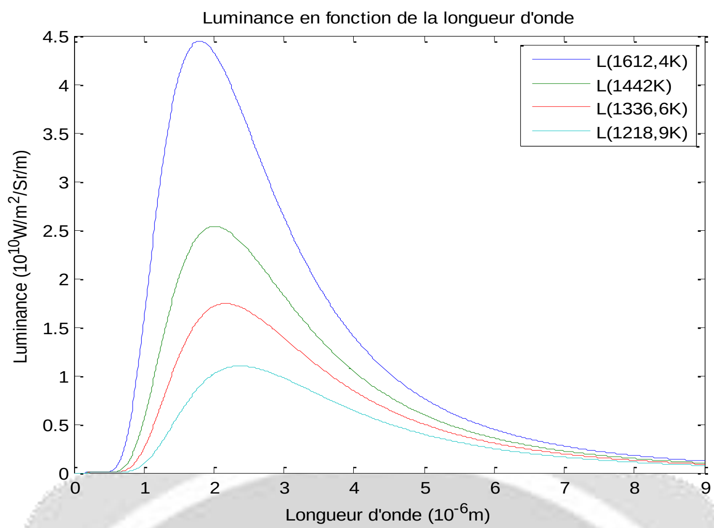

Fig. 6: Representative curve of the luminance according to the length of wave for the height of 0.5m.

- **With attenuation of the atmosphere:**

For this case and for the two heights of the paraboloid, we have the two following curve groups:

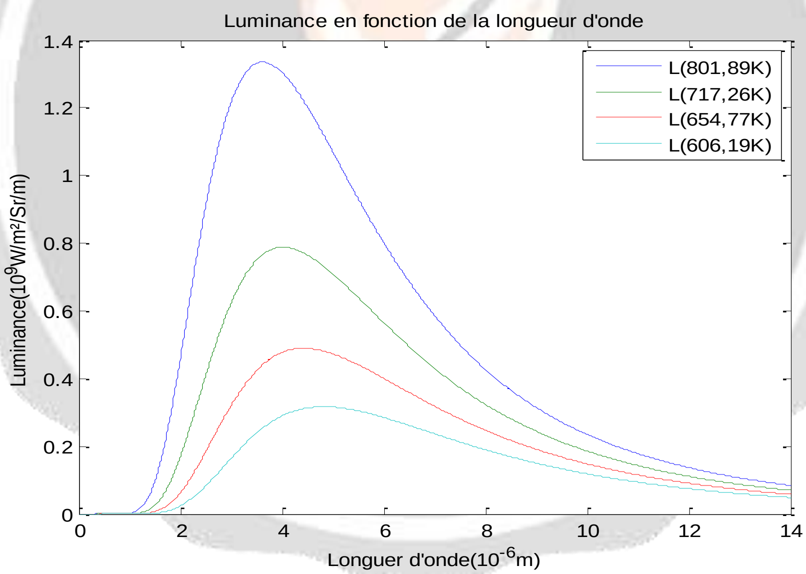

Fig 7: Curve representative of luminance, a circular black body as a function of the wavelength for the height of 0.1m.

and:

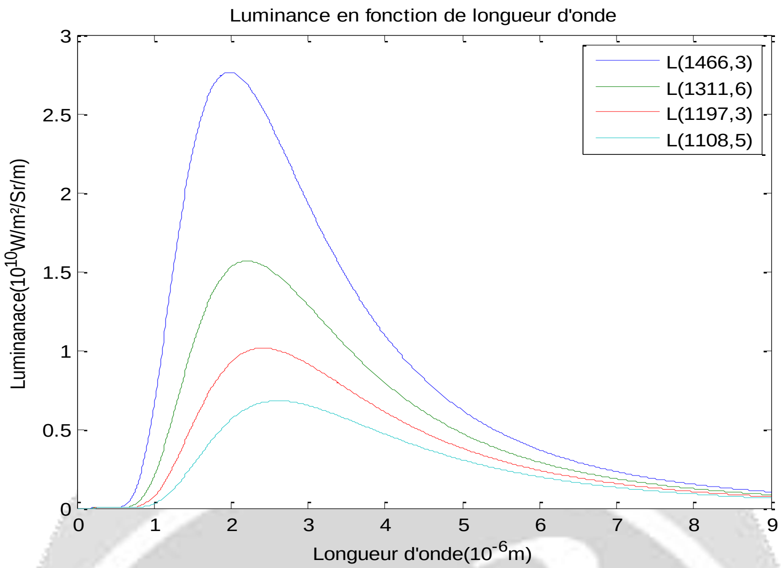

Fig. 8: representative curve of the luminance of a circular black body according to the wavelength for the height of 0.5m.

- **Without attenuation:**

At various temperatures and according to the heights of the paraboloid we have the following maximum wavelength values:

- H = 0,1m:

Table 2: Value of the maximal wave length given out by the circular black body to various temperature for H=0,1m (case without attenuation)

H(m)	0,1			
T(K)	881,76	778,57	720,01	666,6
$L_\lambda(T)_{max}(10^9 w/m^2/Sr/m)$	2,19	1,23	0,79	0,54
$\lambda_{max}(\mu m)$	3,3	3,7	4	4,3

We find that:

$\lambda_{max} \geq 3{,}3\mu m$.

- H= 0,5m:

Table 3: Value of the maximal wave length given out by the circular black body to various temperature for H=0,5m (case without attenuation)

H(m)	0,5			
T(K)	1612,4	1442	1336,6	1218,9
$L_\lambda(T)_{max}(10^{10} w/m^2/Sr/m)$	4,44	2,54	1,74	1,09
$\lambda_{max}(\mu m)$	1,8	2	2,2	2,4

We find that: $\lambda_{max} \geq 1{,}8\mu m$.

- **With attenuation:**

At various temperatures and according to the heights of the paraboloid we have the following maximum wavelength values:

- H = 0,1m:

Table 4: Value of the maximal wave length given out by the circular black body to various temperature for H=0,1m (case with attenuation)

H(m)	0,1			
T(K)	801,89	717,26	654,77	606,19
$L_\lambda(T)_{max}(10^{9} w/m^2/Sr/m)$	1,34	0,79	0,49	0,32
$\lambda_{max}(\mu m)$	3,6	4	4,4	4,8

We find that: $\lambda_{max} \geq 3{,}6\mu m$.

- H = 0,5m:

Table 5: Value of the maximal wave length given out by the circular black body to various temperature for H=0,5m (case with attenuation)

H(m)	0,5			
T(K)	1466,3	1311,6	1197,3	1108,5
$L_\lambda(T)_{max}(10^{10} w/m^2/Sr/m)$	2,76	1,57	1,01	0,68
$\lambda_{max}(\mu m)$	2	2,2	2,4	2,6

We find that: $\lambda_{max} \geq 2\mu m$.

In general, the maximum lengths that characterize the light emitted by our heated black body, at a temperature T, using our solar paraboloid concentrator, are between 0.67µm and 4.8µm. That is, included in the frequency band such that: Knowing that the frequency electromagnetic waves included in the band: are designated by "infrared". As the name suggests, this frequency band is immediately below the visible red light. Our result allows us to say that our black body heated using our dish concentrator, emits infrared.

4. Conclusion:

Our study allows us to conclude that the reunited solar energy at the focus of the paraboloid concentrator increases according to its height on which its surface depends. And that the maximum temperature that the black body placed at this meeting point can reach is proportional to the depth of the paraboloid raised to the power three out of eight. And according to Planck's law, this body can radiate similarly with respect to its absorption of black electromagnetic waves placed at this point, much smaller than the opening of the concentrator, heats up by absorbing all the reflected solar radiation by this system. And at a given equilibrium temperature, this body emits radiation to another body. This program, which follows Planck's law, allows researchers to determine the color of the heated body by studying the luminance according to this equilibrium temperature. For this study, we set the dimensions of the solar concentrator (height, focus), the black body and that of gray, taking into account that the latter is smaller in order to neglect radiation losses. We find that the black body heated in our study emits Infrared.

5. ACKNOWLEDGEMENT

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