# TFSF boundary implementation in the FDTD Algorithm for the Study of the Propagation of Electromagnetic Waves in Vacuum

RANDRIAMAROSON Rivo Mahandrisoa [1], RAKOTOMIRAHO Soloniaina[2], Herinantenaina Edmond Fils[3]

[1] *PhD, SE-I-MSDE, ED-STII, Antananarivo, Madagascar*
[2] *Laboratory Manager, SE-I-MSDE, ED-STII, Antananarivo, Madagascar*
[3] *PhD, SE-I-MSDE, ED-STII, Antananarivo, Madagascar*

**ABSTRACT**

*The implementation of the Yee algorithm allows a numerical resolution of the propagation problem of EM (electromagnetic) waves propagating in a vacuum. In order to simulate an EM wave propagating in a defined direction, a TFSF boundary must be introduced. In the work presented here, we will present what the TFSF boundary is, and how the TFSF boundary could be introduced in the FDTD method. We will use three types of sources to illustrate the implementation of the TFSF boundary. We will consider a Gaussian source, a harmonic source, and a Ricker wavelet source. This work will result in the presentation of the FDTD algorithm with TFSF implementation, as well as the presentation of the results of the algorithm.*

**Keyword**: *FDTD, TFSF Boundary, electromagnetic wave, vacuum propagation, numerical implementation*

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## 1. INTRODUCTION

The wiring of the source on the origin of a FDTD grid has the disadvantage that no energy can pass through the source node. This problem can be corrected by using an additive source. However, the introduction of the source on a node other than the origin leads to the propagation of the field on both sides of the source while halving the amplitude of the source [1]. In order to remedy this problem, the introduction of additive source is done by implementing TFSF (Total Field / Scattered Field) boundary.

In order to be able to correctly implement this TFSF limit, the functions defining the source will have to be defined as well. In these works, three types of sources will be considered, a Gaussian source, a harmonic source and a Ricker wavelet source.

## 2. TFSF BOUNDARY

### 2.1. 1D solution of the wave equation

The wave equation, which governs the electric or magnetic field in one dimension region without a source, can be written as in Eq.1 [2].

$$\frac{\partial^2 f(x,t)}{\partial x^2} - \mu\varepsilon \frac{\partial^2 f(x,t)}{\partial t^2} = 0 \qquad (1)$$

Any function $f(\xi)$that is twice differentiable is a solution to the wave equation. In one dimension, it suffices that the argument $\xi$ is replaced $t \pm x/c$ with $c = 1/\sqrt{\mu\varepsilon}$ [2]. The goal is to build a source such that the excitement propagates only in one direction. To do this, the limit known as the Total-Field / Scattered-Field (TFSF) limit will be used.

### 2.2. Implementation of the TFSF boundary

In a TFSF formulation, the computational domain is divided into two regions: (1) the total field region that contains the incident field plus any scattered field, and (2) the scattered field region that contains only scattered fields. The incident field is introduced on a fictitious boundary, between the regions of the total field and the scattered field. The location of this boundary is somewhat arbitrary, but it is usually placed so that the broadcasters are contained in the total field region [3].

When updating fields, the update equations must be consistent. This means that only scattered fields should be used to update a node in the scattered field region and only the total fields should be used to update a node in the total field region. Fig.1 illustrates a one-dimensional grid in which the TFSF boundary is assumed to exist between the nodes $H_y\left(49+\frac{1}{2}\right)$) and $E_z(50)$ (in Fig.1, the nodes are represented in the form of a table with whole indices). The node $H_y\left(49+\frac{1}{2}\right)$is equivalent to$H_y\left(50-\frac{1}{2}\right)$. No matter where the boundary is placed, there will only be two nodes adjacent to the boundary, an electric field node and a magnetic field node. Defining the scattered field region as being to the left of the boundary and the total field region as being to the right, we see that $hy(49)$ is the last node of the scattered field region while $ez(50)$ is the first node in the total field region.

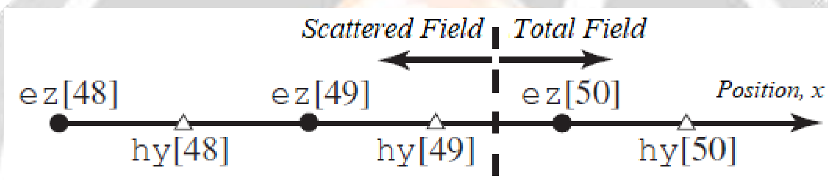


**Fig.1** : Part of one-dimensional matrices in the vicinity of a TFSF boundary.

When updating nodes adjacent to the boundary, there is an inconsistency. A neighbor on one side is not the same type of field as the field being updated. This means that a total field node will depend on a dispersed field node and, conversely, a dispersed field node will depend on a total field node. The solution to this problem is how the fields are entered into the grid using the TFSF boundary.

### 2.3. Electric Field update

Consider the usual updating equation of the electric field at the location m = 50 which is given in Eq.2 and illustrated in Fig.2.

$$\overbrace{E_z^{q+1}(50)}^{total} = \overbrace{E_z^{q}(50)}^{total} + \frac{\Delta_t}{\varepsilon\Delta_x}\left(\overbrace{H_y^{q+\frac{1}{2}}\left(50+\frac{1}{2}\right)}^{total} - \overbrace{H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)}^{scattered}\right) \tag{2}$$

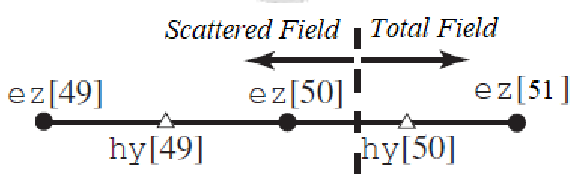


**Fig.2** : TFSF boundary between$E_z^{q+1}(50)$ and $H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)$

The TFSF boundary is between $E_z^{q+1}(50)$ and $H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)$ in Eq.2. Labels above the individual components indicate whether the field is in the total field region or in the scattered field region. $E_z^{q+1}(50)$ *and* $H_y^{q+\frac{1}{2}}\left(50+\frac{1}{2}\right)$ are total field nodes, whereas $H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)$ is a scattered field node. There only the incident field is missing. The incident field is added to $H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)$ in Eq.2. This added field must match the existing magnetic field at the location $50-\frac{1}{2}$ and at the time step $q+\frac{1}{2}$. Thus, a coherent update equation for $E_z^{q+1}(50)$ is given in Eq.3.

$$\overbrace{E_z^{q+1}(50)}^{total} = \overbrace{E_z^{q}(50)}^{total} + \frac{\Delta_t}{\varepsilon\Delta_x}\left(\overbrace{H_y^{q+\frac{1}{2}}\left(50+\frac{1}{2}\right)}^{total} - \overbrace{\left[\overbrace{H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)}^{scattered} + \overbrace{\left(-\frac{1}{\eta}E_z^{inc}\left(50-\frac{1}{2}, q+\frac{1}{2}\right)\right)}^{incident}\right]}^{total}\right) \quad (3)$$

The sum of the terms between the hook gives the total magnetic field for $H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)$. The incident field is here supposed to be known. It can be calculated analytically or, when the TFSF boundary involves multiple points, it can also be calculated with auxiliary FDTD simulation.

Instead of changing the update equation, it is best to keep the standard update equation and then apply a fix in a separate step. In this way, the field at node $H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)$ is updated during a two-step process (Eq.4 then Eq.5).

$$E_z^{q+1}(50) = E_z^{q+1}(50) + \frac{\Delta_t}{\varepsilon\Delta_x}\left(H_y^{q+\frac{1}{2}}\left(50+\frac{1}{2}\right) - H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)\right) \quad (4)$$

$$E_z^{q+1}(50) = E_z^{q+1}(50) + \frac{\Delta_t}{\varepsilon\Delta_x}\frac{1}{\eta}E_z^{inc}\left(50-\frac{1}{2}, q+\frac{1}{2}\right) \quad (5)$$

The impedance $\eta$ can be written in the form $\sqrt{\mu_r\mu_0/\varepsilon_r\varepsilon_0} = \eta_0\sqrt{\mu_r/\varepsilon_r}$. The coefficient $\Delta_t/\varepsilon\Delta_x$ can be expressed by $\eta_0 S_c/\varepsilon_r$ [3]. By combining these terms, with a number of Courant equal to unit and free space (where $\varepsilon_r = \mu_r = 1$), the correction equation Eq.5 became Eq.6.

$$E_z^{q+1}(50) = E_z^{q+1}(50) + E_z^{inc}\left(50-\frac{1}{2}, q+\frac{1}{2}\right) \quad (6)$$

Equation 6 shows that the incident field that existed half a time step in the past and a left half-step left of $E_z^{q+1}(50)$ is added to this node. A field moving to the right requires half a time step to cover half of a spatial step.

**2.4. Magnetic Field update**

Consider the update equation for $H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)$ that is given by Eq.7.

$$\overbrace{H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)}^{scattered} = \overbrace{H_y^{q-\frac{1}{2}}\left(50-\frac{1}{2}\right)}^{scattered} + \frac{\Delta_t}{\mu\Delta_x}\left(\overbrace{E_z^{q}(50)}^{total} - \overbrace{E_z^{q}(49)}^{scattered}\right) \quad (7)$$

As was the case for updating the electric field adjacent to the TFSF boundary, it is not a coherent equation since the terms are scattered field quantities except for $E_z^{q}(50)$ which is in the region of the total field. To correct this, the incident field could be subtracted from $E_z^q(50)$. Rather than modifying Eq.7, the necessary correction would be made as a separate equation (Eq.8).

$$H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)=H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)-\frac{\Delta_t}{\mu\Delta_x}E_z^{inc}(50,q) \tag{8}$$

With a unit Courant and the free space ($\eta_0$, being the characteristic impedance of the vacuum), the Eq.9 is obtained.

$$H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)=H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)-\frac{1}{\eta_0}E_z^{inc}(50,q) \tag{9}$$

### 2.5. Field correction equations

Nothing needs to affect the origin of the source to a particular node of the grid. There is no reason to associate the location *x = 0* (origin of the source) to the left end of the grid. In the TFSF formulation, it is usually more convenient to set the origin with respect to the TFSF limit itself. That the origin *x = 0* corresponds to the node $E_z\ (50)$. Such a displacement requires 50 to be subtracted from previously given spatial indices for the incident field. Then the correction equations become Eq.10 for the magnetic field and Eq.11 for the electric field.

$$H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)=H_y^{q+\frac{1}{2}}\left(50-\frac{1}{2}\right)-\frac{1}{\eta_0}E_z^{inc}(0,q) \tag{10}$$

$$E_z^{q+1}(50)=E_z^{q+1}(50)+E_z^{inc}\left(-\frac{1}{2},q+\frac{1}{2}\right) \tag{11}$$

### 2.6. Algorithm FDTD_1D with TFSF Boundary

To set up a TFSF boundary, just translate the Eq.10 and Eq.11 into the necessary declarations. A logic diagram that implements a TFSF boundary between *hy(49)* and *ez(50)* is presented in Fig.3. The algorithm is similar to the FDTD_1D calculation algorithm (see [1]). The only differences are the suppression of the additive source and the addition of the two correction equations to blocks 4 and 6. In these two blocks, the source is defined by the function $ez_{inc}$, having as argument the spatial step and the not temporal. In block 6, the half step forward in time is obtained with q + 0.5. The half step back in space is obtained with the value - 0.5 for the spatial argument.

The FDTD algorithm with a TFSF implementation can be summarized as follows:

1. Updated magnetic field

2. Correction of the Magnetic Field Value at the TFSF Limit

3. Updated electric field

4. Correction of the electric field value at the TFSF limit

5. Repeat the previous four steps until the fields are obtained for the desired duration.

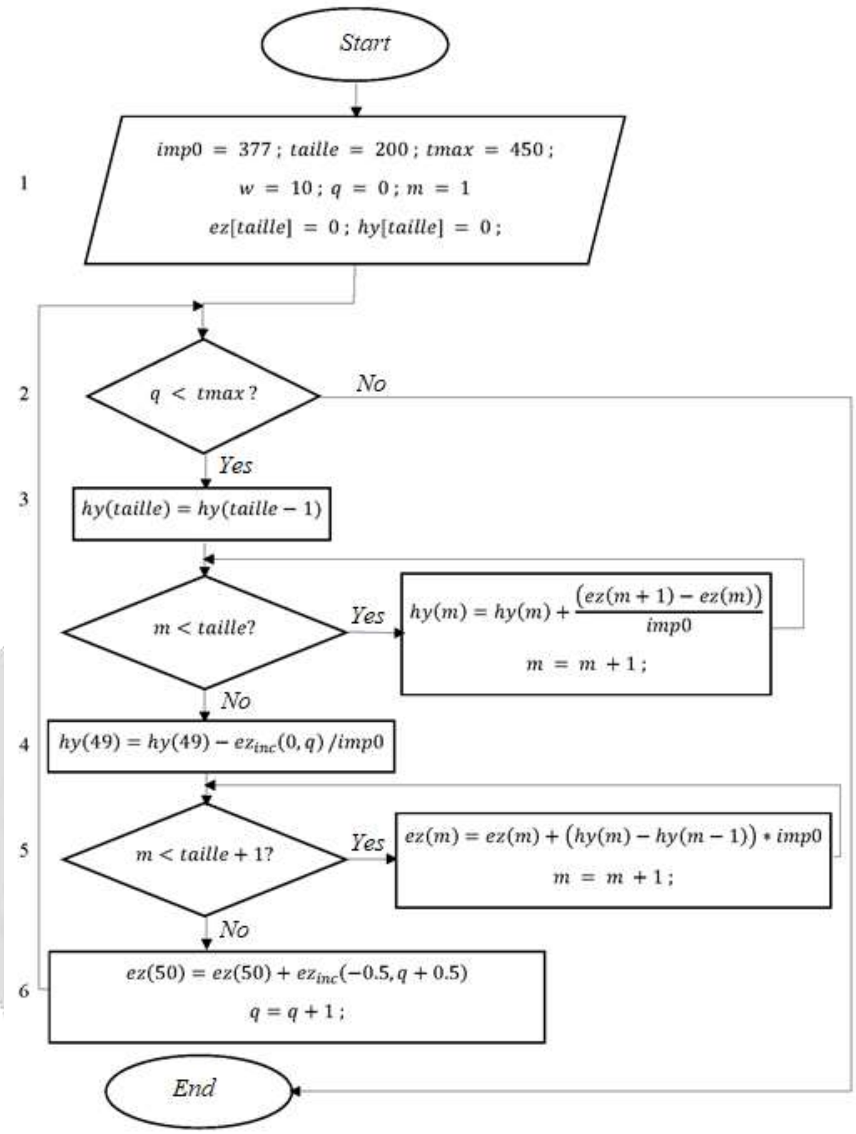


**Fig.3** : Flowchart of the calculation algorithm FDTD_1D with TFSF between *hy(49)* and *ez(50)*.

## 3. SOURCE FUNCTIONS

In order to implement a source function ($ez_{inc}$), for the additive source, in an FDTD grid, the incident field must be specified as a function of space and time.

### 3.1. Gaussian pulse

3.1.1. Discretization of a Gaussian

In the continuous world, a Gaussian can be expressed as in Eq.12 [4].

$$f_g(t) = e^{-\left(\frac{t-d_g}{w_g}\right)^2} \tag{12}$$

where $d_g$ is the time delay and $w_g$ is the pulse width. The Gaussian has its maximum value at $t = d_g$ and has a value of $e^{-1}$ when $t = d_g \pm w_g$.

Since Eq.12 is only a function of time, that is a function of $q\Delta_t$ in the discretized world, the temporal step $\Delta_t$ must be explicitly indicated. However, if the delay and the pulse width are specified in terms of time steps, the term $\Delta_t$ appears both in the numerator and the denominator of the exponent. For example, for $d_g = 30\Delta_t$ and $w_g = 10\Delta_t$ the source function can be written as in Eq.13, where $\Delta_t$ does not appear in the expression of the function. The discretized version of a function $f(q\Delta_t)$ will be denoted by $f[q]$.

$$f_g(q\Delta_t) = f_g[q] = e^{-\left(\frac{q-30}{10}\right)^2} \tag{13}$$

3.1.2. Gaussian moving in the direction of x positive

For the incident field in propagation, $t$ in Eq.13 is replaced by $t - x/\,c$. In discretized space-time, this argument is given by Eq.14 [2] [4].

$$t - \frac{x}{c} = q\Delta_t - \frac{m\Delta_x}{c} = \left(q - \frac{m\Delta_x}{c\Delta_t}\right)\Delta_t = (q-m)\Delta_t \tag{14}$$

where the number of Courant $\frac{c\Delta_t}{\Delta_x} = 1$ was used to write the last equality. This expression can be used for the Gaussian source function argument to obtain a propagation wave denoted $E_z^{inc}$ (Eq.15).

$$E_z^{inc}[m,q] = exp\left[-\left(\frac{(q-m)\Delta_t - 30\Delta_t}{w\Delta_t}\right)^2\right] = exp\left[-\left(\frac{(q-m)-30}{w}\right)^2\right] \tag{15}$$

Equation 15 can be used as a source of a Gaussian in a FDTD implementation when implementing TFSF boundary.

**3.2. Harmonic sources**

3.2.1. Discretization of a harmonic source function

A harmonic source can be defined as in Eq.16 [4].

$$f_h(t) = cos(\omega t) \tag{16}$$

In Eq.16, there is no explicit numerator or denominator in the argument. By replacing $t$ with $q\Delta_t$, the time step had to be explicitly indicated. However, for a plane wave propagating in the free space, the wavelength $\lambda$ and the frequency $f$ are linked by Eq.17. Thus, the argument $\omega t$ (that is, $2\pi f t$) can be written as in Eq.18.

$$f\lambda = c \implies f = \frac{c}{\lambda} \tag{17}$$

$$\omega t = \frac{2\pi c}{\lambda} t \tag{18}$$

For a given frequency, the wavelength is a fixed length. For a length, it can be expressed in terms of spatial pitch (Eq.19).

$$\lambda = N_\lambda \Delta_x \tag{19}$$

where $N_\lambda$ is the number of points per wavelength. By linking the frequency to the wavelength and the wavelength to $N_\lambda$, the discrete version of the harmonic function can be defined in Eq.20 and Eq.21.

$$f_h(q\Delta_t) = cos\left(\frac{2\pi c}{N_\lambda \Delta_x} q\Delta_t\right) = cos\left(\frac{2\pi}{N_\lambda}\frac{c\Delta_t}{\Delta_x} q\right) \tag{20}$$

$$f_h[q] = cos\left(\frac{2\pi S_c}{N_\lambda} q\right) \tag{21}$$

The expression of Eq.21 contains the Courant number and the parameter $N_\lambda$. In this form, it is not necessary to indicate an explicit value for the time step. Instead, the number of Courant $S_c$ and the number of spatial steps per wavelength $N_\lambda$ are specified.

3.2.2. Harmonic source function moving in positive x direction

A harmonic wave moving in the positive x direction is given by Eq.22 [2][4].

$$f_h(x,t) = cos(\omega t - kx) = cos\left(\omega\left(t - \frac{k}{\omega}x\right)\right) \tag{22}$$

with $k = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r} = \frac{\omega\sqrt{\mu_r\varepsilon_r}}{c}$, the argument can be written as in Eq.23. The expression of all quantities in terms of discrete values belonging to the FDTD network gives Eq.24. Therefore, the discretized form of Eq.22 is given by Eq.25.

$$\omega\left(t - \frac{k}{\omega}x\right) = \omega\left(t - \frac{\sqrt{\mu_r\varepsilon_r}}{c}x\right) \tag{23}$$

$$\omega\left(t - \frac{\sqrt{\mu_r\varepsilon_r}}{c}x\right) = \frac{2\pi c}{N_\lambda\Delta_x}\left(q\Delta_t - \frac{\sqrt{\mu_r\varepsilon_r}}{c}m\Delta_x\right) = \frac{2\pi}{N_\lambda}(S_c q - \sqrt{\mu_r\varepsilon_r}m) \tag{24}$$

$$f_h[m,q] = cos\left(\frac{2\pi}{N_\lambda}(S_c q - \sqrt{\mu_r\varepsilon_r}m)\right) \tag{25}$$

Equation 25 can be used as a source in an FDTD implementation. However, when temporal and spatial indices are zero, this source takes the unit value. If this is the initial activation value of the source, this may cause unwanted artifacts. It is usually better to gradually increase the source by using a sine function instead of a cosine, since sine is initially zero.

### 3.3. Ricker Wavelet

It is generally advantageous to use pulsed sources, which can introduce a broad spectrum of frequencies, rather than a harmonic source. The Gaussian pulse is potentially an acceptable source, except that it contains a DC component. DC sources have the ability to introduce artifacts that are not physical. Therefore, a different pulsed source, the Ricker wavelet, which has no DC component and which can have its most energetic frequency set to the desired frequency will be considered.

3.3.1. Discretization of a Ricker wavelet

The Ricker wavelet is equivalent to the second derivative of a Gaussian. It has no DC component, and its spectral content is fixed by a single parameter. The Ricker wavelet is defined in Eq.26 [5].

$$f_r(t) = \left(1 - 2\{\pi f_p[t - d_r]\}^2 exp\left(-\{\pi f_p[t - d_r]\}^2\right)\right) \tag{26}$$

where $f_p$ is the "peak frequency" and $d_r$ is the time delay. The peak frequency is the frequency with the largest spectral content. The delay can be set to any desired quantity, but it is convenient to express it as a multiple of $1/f_p$, defined in Eq.27.

$$d_r = M_d\frac{1}{f_p} \tag{27}$$

where $M_d$ is the multiple of the delay (which is not necessarily an integer).

The peak frequency $f_P$ has a corresponding wavelength $\lambda_P$. This wavelength can be expressed in terms of spatial step such that $\lambda_P = N_p\Delta_x$, where $N_p$ (number of points per wavelength at the peak frequency) does not need to be an integer (Eq. 28). The Courant number $S_C$ is $c\Delta_t/\Delta_x$ so that the spatial step can be expressed by $\Delta_x = c\Delta_t/S_C$. Using this in Eq.28, Eq.29 is obtained. The delay can therefore be expressed in Eq.30.

$$f_p = \frac{c}{\lambda_p} = \frac{c}{N_p \Delta_x} \tag{28}$$

$$f_p = \frac{S_c}{N_p \Delta_t} \tag{29}$$

$$d_r = M_d \frac{1}{f_p} = M_d \frac{N_p \Delta_t}{S_c} \tag{30}$$

Taking the time $t$ being q$\Delta_t$ and expressing $f_P$ and $d_r$ as in Eq.29 and Eq.30, the discrete form of Eq.26 can be written as in Eq.31.

$$f_r[q] = \left(1 - 2\pi^2 \left[\frac{S_c q}{N_p} - M_d\right]^2\right) exp\left(-\pi^2 \left[\frac{S_c q}{N_p} - M_d\right]^2\right) \tag{31}$$

The parameters specifying $f_r[q]$ are the number of Courant $S_C$, $N_P$ the number of points per wavelength at the peak frequency, and the multiple delay $M_d$. There is no $\Delta_t$ in Eq.31. This function is independent of temporal and spatial steps, but it depends on their ratio via the Courant number.

3.3.2. Ricker wavelet moving in the direction of x positive

Equation 31 gives the Ricker wavelet as only a function of time. It is necessary to set an incident field in time and space. Obtaining a plane wave solution to the wave equation is done by modifying the argument of any function that is twice differentiable. Given the Ricker wavelet $f_r(t)$, $f_r\left(t \pm \frac{x}{c}\right)$is a solution to the wave equation where *c* is the propagation velocity. The plus sign corresponds to a wave traveling in the negative direction x and the minus sign corresponds to a wave traveling in the positive direction x. Therefore, a roving Ricker wavelet can be constructed by replacing the *t* argument in Eq.26 by $t - \frac{x}{c}$. The value of the function now depends on time and location, that is, it depends on two variables (Eq.32) [2] [5].

$$f_r\left(t - \frac{x}{c}\right) = f_r(x,t) = \left(1 - 2\pi^2 f_p^2 \left[t - \frac{x}{c} - d_r\right]^2\right) exp\left(-\pi^2 f_p^2 \left[t - \frac{x}{c} - d_r\right]^2\right) \tag{32}$$

As before, Eq.29 and Eq.30 can be used to rewrite $d_r$ and $f_P$ in terms of Courant number, points per wavelength at peak frequency, time step and multiple of delay. . By replacing *t* with $q\Delta_t$, $x$ by $m\Delta_x$ and using the identity $\frac{x}{c} = \frac{m\Delta_x}{c} = \frac{m\Delta_t}{S_c}$, the Eq.33 is obtained.

$$f_r[m,q] = \left(1 - 2\pi^2 \left[\frac{S_c(q-m)}{N_p} - M_d\right]^2\right) exp\left(-\pi^2 \left[\frac{S_c(q-m)}{N_p} - M_d\right]^2\right) \tag{33}$$

Equation 33 gives the value of Ricker's wavelet to the time index *q* and the space index *m*. When *m* equals zero, Eq.33 is reduced to Eq.31.

## 4. SIMULATION OF ADDITIVE SOURCES

In order to implement an additive source in the grid, instead of setting the value of *ez (1)* to the value of the function, the source function is added to *ez (50)*. The source is introduced after the updating equations that are unchanged, with the subtraction of the incident field for the magnetic field and the addition of the incident field for the electric field. The TFSF boundary is defined between *hy (49)* and *ez (50)*.

Snapshots of *Ez* taken at time steps 20, 40, 80 and 120 are illustrated in Fig.4, Fig.5 and Fig.6. The incident field is from node 50 and propagates only to the right of the TFSF boundary. Fig.4 corresponds to a Gaussian source function, while Fig.5 and Fig.6 respectively correspond to a harmonic source function and a Ricker's wavelet.

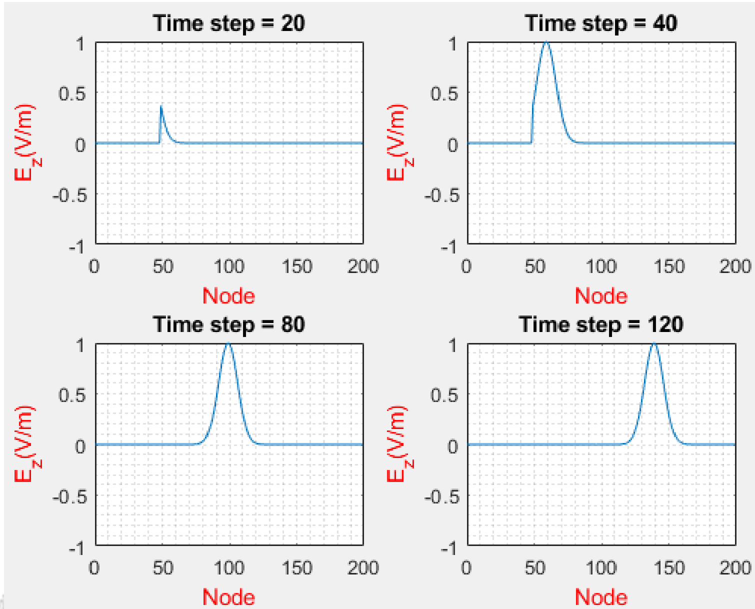


**Fig.4** : Snapshots of the *Ez* field generated by the FDTD_1D calculation algorithm with TFSF limit with a Gaussian source

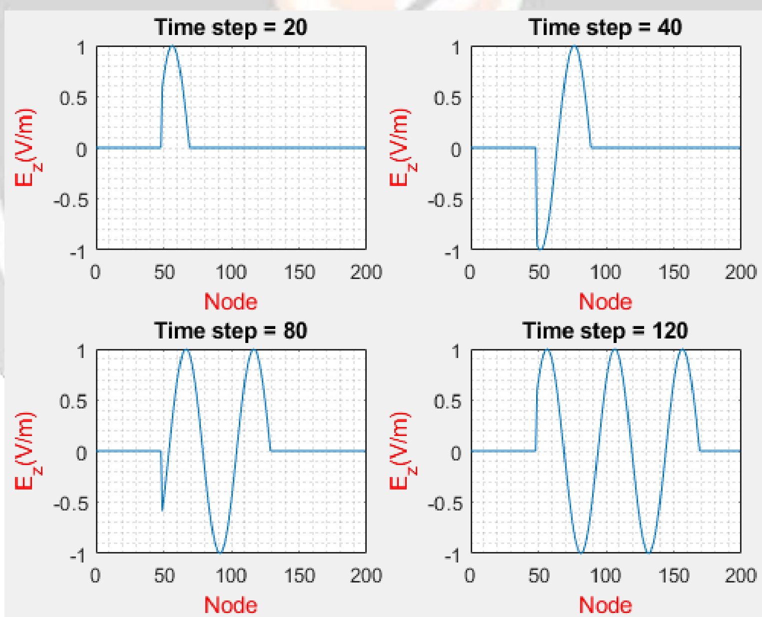


**Fig.5** : Snapshots of the *Ez* field generated by the FDTD_1D calculation algorithm with TFSF limit with sinusoidal harmonic as source

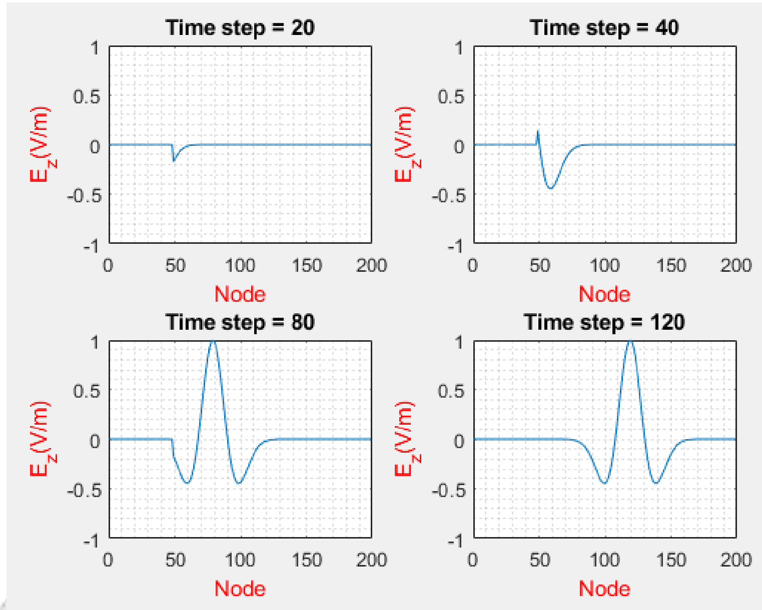


**Fig.6** : Snapshots of the *Ez* field generated by the FDTD_1D calculation algorithm with TFSF limit with a Ricker wavelet source.

## 5. CONCLUSION

This paper gives an overview of the TFSF boundary implementation in a FDTD simulation, for electromagnetic propagation propagating in a vacuum, modeled by a 1D space. The implementation of the TFSF boundary is defined by the subtraction of the incident wave from the magnetic field, and by the addition of the incident wave to the electric field, in the neighborhood of the considered source node. The introduction of the TFSF boundary makes it possible to introduce to any node of the grid, a wave propagating at the right of the source. The function defining the incident wave is a function of the spatial step and the temporal step for a Gaussian source. For a harmonic source, the source function is defined by the spatial step, the time step and the number of points per wavelength of the source. The use of a Ricker wavelet as a source defines the source function as a function of spatial step, time step and number of points per wavelength at the wavelet peak frequency.

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