THEORETICAL DETERMINATION OF THE ELASTIC CONSTANTS OF A SISAL/POLYESTER LAMINA COMPOSITE

Iréna Arivony RAKOTOMANANA[1], Michaël Godin RAKOTOMANANA[2], Charles Rodin RAKOTOMANANA[3], Josoa Albert RAMORASATA[4]

[1] PhD student, GMTI, INGE, Antananarivo, Madagascar
[2] PhD student, GMTI, INGE, Antananarivo, Madagascar
[3] Lecturer, GMTI, INGE, Antananarivo, Madagascar
[4] Professor, GMTI, INGE, Antananarivo, Madagascar

ABSTRACT

To know the elastic characteristics of a Sisal / Polyester laminate, it is important to determine which of the lamina that represent the basic element. The rule of mixture and the models of Halphin-Tsai and Tsai-Hahn are used to obtain the variations of the elastic constants of the lamina as a function of the quantity of fibers in the composite. According to the results obtained, the value of the longitudinal Young's modulus of the Sisal / Polyester increases with the quantity of fibers in the latter, unlike those of the other elastic constants. For the transverse Young's modulus and the minor Poisson's ratio of the studied lamina, the curves given by the rule of mixture are between those determined by the models of Halphin-Tsai and Tsai-Hahn. The curves also show a small value of the transverse Young's modulus and the in-plane shear modulus, and a minor Poisson's ratio very sensitive to the amount of fibers in the lamina, especially for a fiber mass fraction of less than 40%.

Keywords: *Composite, Lamina, Fiber, Sisal, Polyester, Elastic constant*

1. INTRODUCTION

The use of composite materials dates back centuries. But in recent years, thanks to advances in technology and scientific research on these materials, they are now used in many sectors because of their lightness and performance.

In addition, concern for environmental pollution encourages manufacturers to move towards biosourced materials. Vegetable fibers such as those of sisal are then the subject of several investigations because of their low density, low cost, and their biodegradability.

Sisal fibers are increasingly used in composite materials for the automotive, furniture and construction industries. They have good mechanical properties [1] and on the other hand, sisal is a plant with excellent environmental properties. Indeed, it absorbs for example more CO_2 than it produces.

To contribute to the mechanical characterization of sisal fibers reinforced composites, the elastic behavior of the lamina, the basis of a laminated structure, is studied in this paper. Its elastic properties are illustrated through the values of its elastic constants.

And since long-fiber composites exhibit better mechanical behavior than short fiber or particle composites, at least in the fiber-reinforced directions [2], the considered lamina is unidirectional long fibers.

2. CONSTITUENTS OF THE STUDIED LAMINA

The lamina studied has for reinforcement the sisal fibers and for matrix the unsaturated polyester resin.

2.1. Sisal fibers

The sisal fibers are extracted from sisal, whose botanical name is "Agave sisalana". These are anisotropic fibers [3]. Table 1 shows the mechanical characteristics of the sisal fibers needed for the study.

Table 1 : Some mechanical characteristics of sisal fibers [4]

	Sisal fiber
Density (g/cm^3)	1.5
Longitudinal Young’s modulus (GPa)	26.6
Transversal Young’s modulus (GPa)	1.4
Shear modulus (GPa)	0.82

2.2. Unsaturated polyester

Unsaturated polyester is the thermosetting resin most used in the manufacture of composite parts.

Table 2 : Some mechanical characteristics of the unsaturated polyester resin [5]

	Unsaturated polyester resin
Density (g/cm^3)	1.2
Young’s modulus (GPa)	4
Shear modulus (GPa)	1.4
Poisson’s Ratio	0.4

3. ADOPTED APPROACHES

Let be a rectangular axes system (1,2). The direction of the axis 1 is that of the fibers. And for the axis 2, it is directly perpendicular to the axis 1.

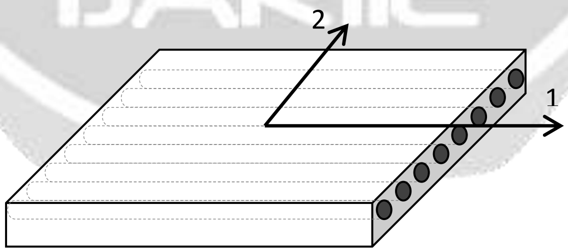

Fig 1 : Unidirectionnal lamina

The elastic behavior of a lamina depends on the values of its elastic constants, which are: longitudinal Young’s modulus E_1, transverse Young’s modulus E_2, in-plane shear modulus G_{12}, and Poisson’s ratios ν_{12} and ν_{21}.

Hypotheses:

- Fibers and matrix follow Hooke's law
- There are no voids in the lamina
- The adhesion between the fibers and the matrix is perfect
- The fibers are continuous and parallel
- The diameters and the space between the fibers are uniform

3.1. Rule of mixture [6]

For a unidirectional lamina, the method used to determine the expressions of the elastic constants is to consider a volume element representing a fiber in a matrix. These will then be presented as rectangular blocks of the same width.

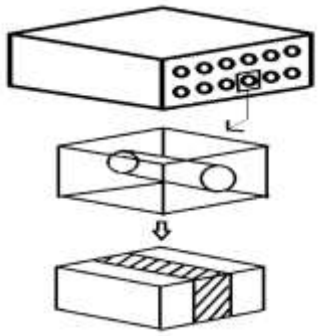

Fig 2 : Representation of the volume element of a lamina

Taking into account the distribution of forces, stresses and deformations in the material following each main direction, the following relations are obtained:

$$E_1 = V_m E_m + V_f E_{f1} \quad (1)$$

$$1/E_2 = V_m/E_m + V_f/E_{f2} \quad (2)$$

$$1/G_{12} = V_m/G_m + V_f/G_{f12} \quad (3)$$

$$\nu_{12} = V_m \nu_m + V_f \nu_{f12} \quad (4)$$

$$\nu_{21} = \nu_{12}\, E_2/E_1 \quad (5)$$

- E_m, G_m, V_m are respectively the Young's modulus, the in-plane shear modulus and the volume fraction of the matrix.
- E_{f1}, E_{f2}, G_{f12}, V_f are the longitudinal Young's modulus, the transversal Young's modulus, the in-plane shear modulus and the volume fraction of the fibers.
- ν_m et ν_{f12} are the Poisson's ratios of the matrix and fibers

3.2 Semi-empirical models

In the majority of cases of non-plant fiber composites, the values of the transverse Young's modulus and the in-plane shear modulus obtained with relations (2) and (3) seem to be far from the values obtained experimentally [6]. Therefore, new modeling techniques are needed. Among the semi-empirical models relating to elastic properties are those of Halphin and Tsai [7], and Tsai-Hahn [8].

3.2.1 Halphin-Tsai model

The following relations are those of Halphin-Tsai for transversal Young's modulus and in-plane shear modulus.

$$\frac{E_2}{E_m} = \frac{1 + \xi\eta V_f}{1 - \eta V_f} \quad (6)$$

with

$$\eta = \frac{(E_f/E_m) - 1}{(E_f/E_m) + \xi} \quad (7)$$

Where $\xi = 2$ in relations (6) and (7) concerning the transverse Young's modulus if the fibers are assumed to have circular sections and a square arrangement.

$$\frac{G_{12}}{G_m} = \frac{1 + \xi\eta V_f}{1 - \eta V_f} \quad (8)$$

with

$$\eta = \frac{(G_f/G_m) - 1}{(G_f/G_m) + \xi} \quad (9)$$

here $\xi = 1$ in relations (8) and (9) concerning the in-plane shear modulus if the fibers have the same shape and arrangement as before.

Moreover, according to Hewitt and Malherbe [9], one can also determine the value of the coefficient ξ , in order to estimate G_{12}, by the relation :

$$\xi = 1 + 40{V_f}^{10} \quad (10)$$

3.2.2 Tsai-Hahn Model

According to Tsai and Hahn, the value of E_2 can be obtained by the following expression :

$$E_2 = (V_f + \beta V_m) \times \frac{1}{\frac{V_f}{E_{2f}} + \beta\frac{V_m}{E_m}} \quad (11)$$

β is the stress distribution factor

For natural fibers, $\beta = 0{,}5$

4. RESULTS AND COMMENTS

The mechanical characteristics of a composite material depend on the amount of fibers in the latter. In practice, the volume fraction of the fibers should be between 25% and 75%. This corresponds respectively to the mass fractions of sisal fibers of 30% and 78%.

The figures which follow show the variations of the elastic constants of the Sisal / Polyester lamina as a function of the mass fraction M_f of the fibers.

4.1 Longitudinal Young's modulus

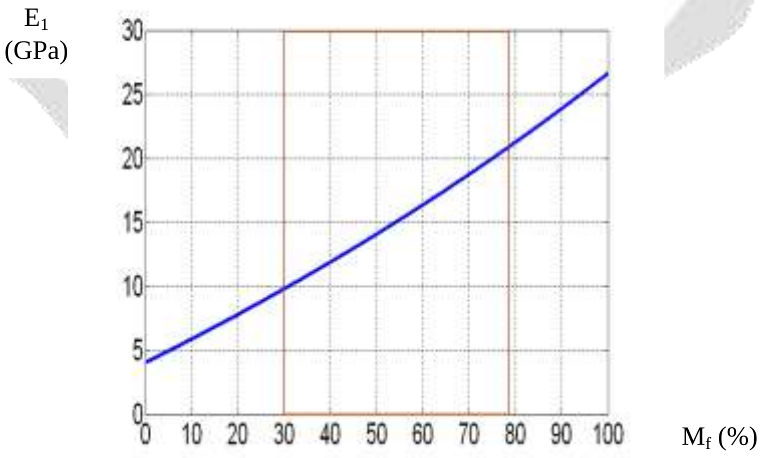

Fig 3 : Variation of longitudinal Young's modulus E_1 according to M_f

Fig.3 highlights the fact that the higher the quantity of fibers in the composite, the greater the value of the longitudinal Young's modulus of the latter increases and approaches that of the longitudinal Young's modulus of the fibers: 26.6 GPa. So there will be less deformation in the direction of the fibers under the action of a load.

And the smaller M_f is, the more the value E_1 approaches that of the longitudinal Young's modulus of the matrix: 4GPa.

4.2 Transverse Young's modulus

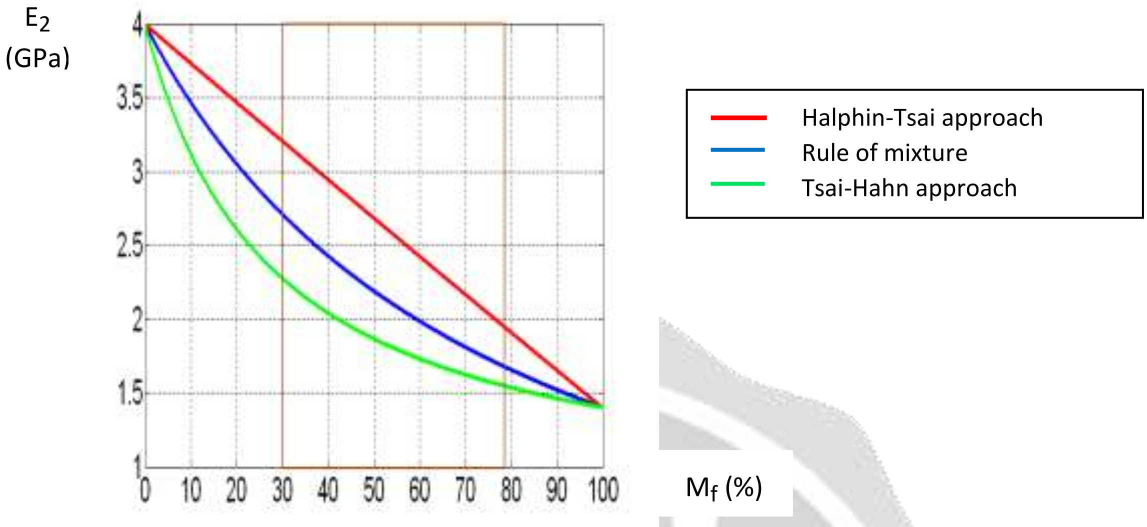

Fig 4 : Variation of the transverse Young's modulus E_2 according to M_f

Fig.4 shows a decrease in the value of the transverse Young's modulus of the lamina with the increase in the amount of fibers.
The maximum deviation of the E_2 values obtained by the Halphin-Tsai model and that of Tsai-Hahn is 0,9 GPa for a sisal fibers mass fraction of 40%.
Moreover, E_2 is low therefore, the fold could have a large deformation under the action of a transverse load.

4.3 In-plane Shear modulus

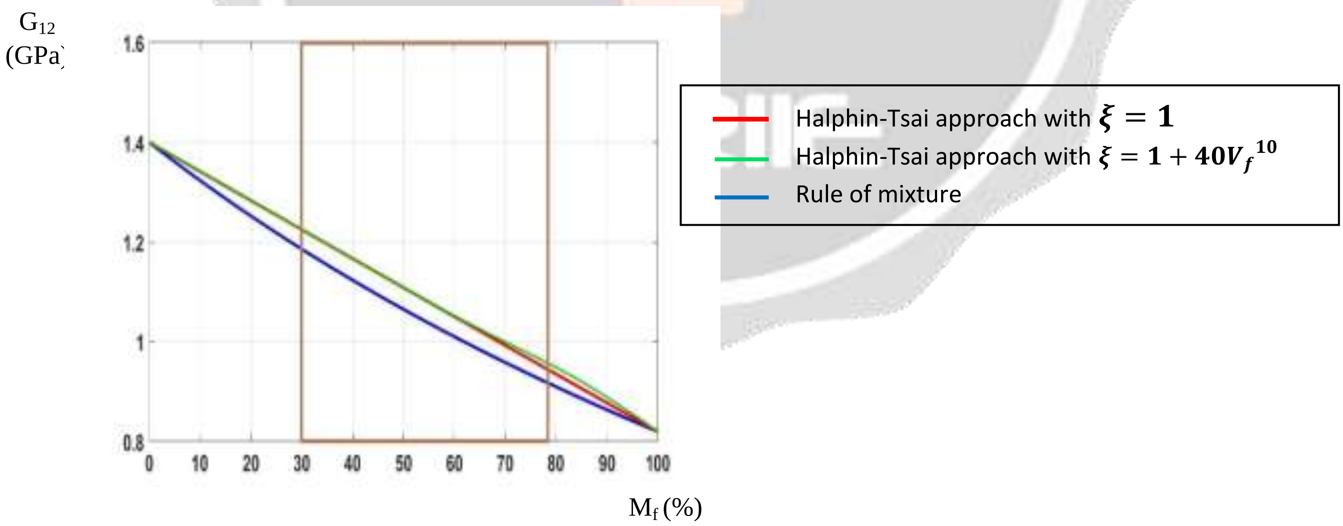

Fig 5 : Variation of the in-plane shear modulus G_{12} according to M_f

Fig.5 shows that the value of the in-plane shear modulus of the lamina varies very little and remains low.
For a fiber fraction of 60%, the maximum deviation of the G_{12} values is 0.04.

4.4 Major Poisson's ratio ν_{12}

A value of 0,25 was adopted for the major Poisson's ratio of sisal fiber since for the majority of plant fibers this coefficient is between 0,2 and 0,3.

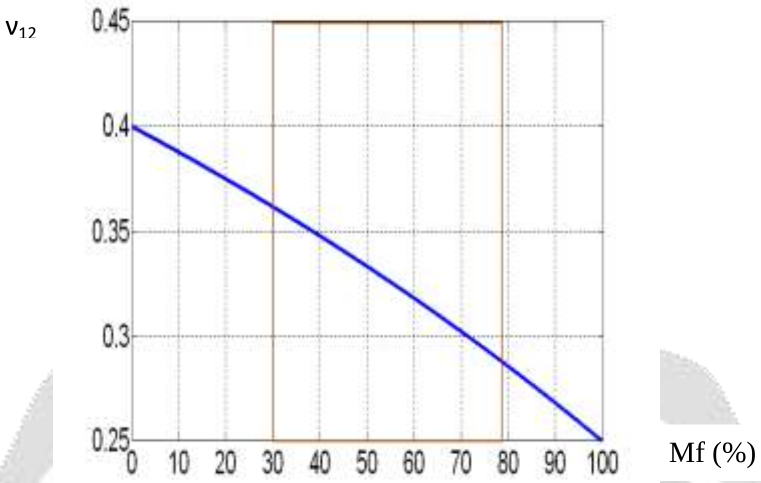

Fig 6 : Variation of the major Poisson's ratio ν_{12} according to M_f

The value of major Poisson's ration ν_{12} decreases as the fiber fraction M_f increases.

4.5 Minor Poisson's ratio ν_{21}

Taking into account the values E_2 given by the different models, three curves of ν_{21} according to M_f were obtained.

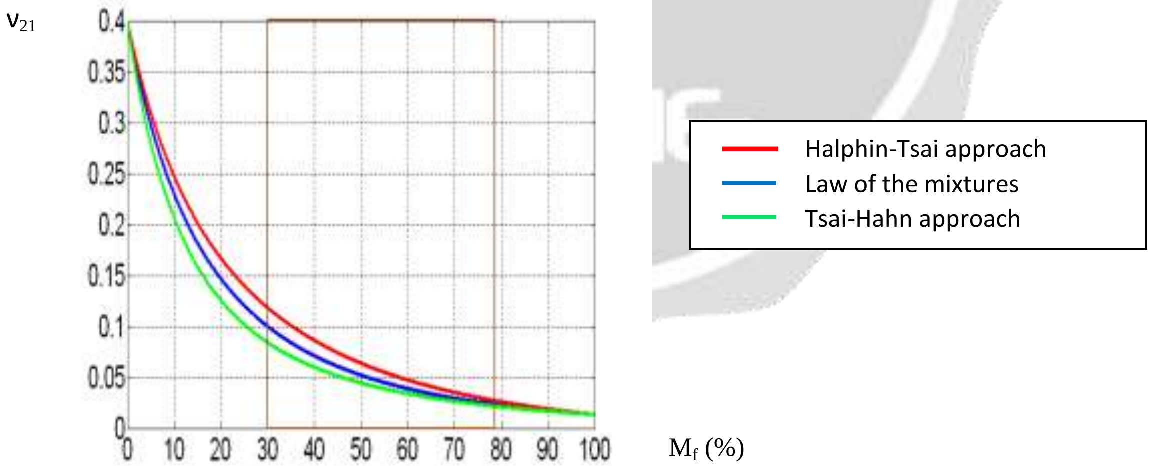

Fig 7 : Variation of minor Poisson's ratio ν_{21} according to M_f

The maximum difference between the values of ν_{21}using the Halphin-Tsai and Tsai-Hahn approach is 0,04 for a fiber fraction of about 20%. Moreover, for M_f lower than 40%, the decrease in the value of $\nu_{21}$is more marked.

5. CONCLUSION

The different models used, namely those of Haphin-Tsai, Tsai-Hahn and the rule of mixture, made it possible to determine the variations of the elastic constants of the Sisal / Polyester lamina according to the quantity of fibers in the composite.

According to the results, for the lamina studied, the values of the transversal Young's modulus E_2 and the minor Poisson's ratio ν_{21} obtained by the rule of mixture represent approximately the averages of the values found by the two semi-empirical models.

This study made it possible to obtain a theoretical estimate of the elastic behavior of the sisal / polyester lamina in order to better understand the mechanical properties of composite structures with sisal fiber reinforcement.

6. REFERENCES

[1] Characterization of Sisal Fibers for use as Reinforcement in Polymer Composites, International journal of Engineering and Innovative Technology (IJEIT), Volume 4, Issue 8, February 2015
[2] Lionel GENDRE, Matériaux composites et structures composites, Paris-Saclay University, May 2011
[3] Richard Ntenga, Modélisation multi-échelle et caractérisation de l'anisotropie élastique des fibres végétales pour le renforcement de matériaux composites, July 2012
[4] Gentles F., J. Anderson, L. Thomason, Characterisation of the transverse thermoelastic properties of natural fibres used in composites
[5] Centre d'Animation Régional en Matériaux Avancés, Glossaire des matériaux composites, 2006
[6] Hashin, Z., Theorie of fiber reinforced materials, NASA tech. rep. Contract no. NAS18818, November 1970
[7] Halphin, J.C. and Tsai, S.W., Effect of environment factors on composite materials, Air Force tech. rep. AFML-TR-67-423, June 1969.
[8] Tsai, S.W. and Hahn H.T., Introduction to composite materials, 1980
[9] Hewitt, R.L. and Malherbe, M.D.De, An approximation for the longitudinal shear modulus of continuous fiber composites, J. Composite Mater., 4, 280,1970