# THE EFFECTIVENESS OF MANIPULATIVES IN EDUCATIONAL SKILLS AMONG GRADE 2 PUPILS

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## ABSTRACT

*This study aimed to determine the effectiveness of manipulatives in educational skills among grade 2 pupils at Cateel Central Elementary School. It also attempted to determine the level of pre-test and post-test scores between the two groups, the significant difference between pre-test and post-test scores, and the significant difference between the learners' pre-test and post-test scores. This study employed a quantitative method, especially a quasi-experimental design. A researcher-made questionnaire was utilized to collect data. Results of the study revealed that the experimental group, who were taught proper fractions using manipulatives, improved significantly in their learning performance compared to the control group, who were taught using the traditional teaching and learning methods. Students' interest increased after the intervention, and the teaching and learning process became more interactive and engaging. The study found that using manipulatives stimulates learners' interest, increases their mathematical skills, and develops concentration and perseverance skills while learning about cause and effect and creatively analyzing and solving problems*.

## 1. INTRODUCTION

Solving mathematical problems, especially fractions, are deemed to be hard for learners. Learning fractional concepts is one of primary school students' first experiences with a mathematics concept beyond the fundamental four basic operators. However, students struggle to learn fractions, which significantly inhibits their intuitive knowledge (Hoon, 2021). This study's main aim is to examine why they face problems making sense of fractions and what can be done about it since learning fractions is foundational to understanding many more advanced areas of mathematics and science (Singh et al., 2020).

Using manipulatives on fractions when teaching 2nd graders helps students grasp abstract concepts because it gives them a concrete idea of the concept (Jao, 2013). While teachers use manipulatives frequently in the classroom, there is a scarcity of research on the actual effectiveness of manipulatives when teaching fractions. Manipulatives are objects an individual can handle in a sensory manner during which conscious and unconscious mathematical thinking will be fostered (Swann & Marshall, 2010). Manipulative materials can be touched and moved around and appeal to several senses, including the learners' socio-cultural needs (Yurof, 2013).

Learning fractions can be especially challenging for students with mathematics learning difficulties. Namkung, et. al. (2018) found that students with severe mathematics learning difficulties, as indexed by their whole-number competence below the 10th percentile, were 32 times more likely than students with intact whole-number knowledge to experience difficulty with fractions. Students with less severe mathematics learning difficulties (between the 10th and 25th percentile) were five times more likely to experience difficulty with fractions than students with intact whole number knowledge. Likewise, Resnick et al. (2016) found that students with inaccurate whole-number line estimation performance were twice as likely to show low-growth in fraction magnitude understanding compared to those with accurate whole-number line estimation skills.

As a result, the study explored the effectiveness of manipulatives in teaching fractions to second graders. The researcher in this study divided a second-grade class into two groups; one group was taught with manipulatives, and the other group was taught without manipulatives. Data was collected in the forms of pretests and posttests and was analyzed to find the effectiveness of manipulatives. Hopefully, this study could offer better strategies that would make teaching and learning fractions easier and more enjoyable.

## 2. REVIEW OF RELATED LITERATURE

This chapter presents a review of related literature on the topic of this action research. The discussion includes the importance of mathematical skills, problems on mathematical skills, strategies for enhancing mathematical skills, and the relevance of cognitive development theory in enhancing mathematical skills. Since this is a review of related literature, the researcher includes in the discussion the topics from various reliable sources like online articles, books, journals, and other researchers.

### 2.1 Importance of Mathematical Skills

The significance of mathematics in any field of discipline cannot be denied. Eisen et al. (2005) argued that the necessity of acquiring mathematical knowledge and skills plays a crucial in any occupational field as cited in (McCormick & Lucas, 2011). Mathematical skills are one of the fundamental skills that employers are looking for (Rosenberg et al., 2012). Likewise, the level of mathematical thinking of students in this information-based society is vital in developing process skills such as innovative ways to find a solution to a problem (Walshaw et al., 2009).

The need for continuous improvement of mathematical thinking in mathematics education is highly important for the reason that one of the concerns of today's mathematics education is to produce intelligent learners who will eventually become better citizens of the future (Cordova & Tan, 2018). In almost future jobs, college-level mathematical skills will be one of the most required skills (Huebner & Corbett, 2008). Laursen (2009) defined eight mathematics competencies that students must possess: mathematical thinking, problem handling, modeling, reasoning, representation, symbols and formalism, communications, and aids and tools as cited in Zeidmane (2012). Similarly, Cordova and Tan (2018) broke down mathematical proficiency into five strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Further, according to Corbeshly and Traxsaw (2010), the mathematical skills that can be acquired by the students upon entering the university can be categorized into four: subject knowledge, number sense, measurement and data, reasoning, and generalization.

These mathematical competencies can be grouped into a) competencies concerning the ability to ask and answer questions about utilizing mathematics (mathematical thinking, problem handling, reasoning, and modeling) and b) competencies that concern the students' ability to engage with mathematical language and tools (representation, symbols and formalism, communications, and aid and tools). The above categorization of mathematics skills were the demands skills nowadays. According to Achieve (2005), in today's demands, employers want to have individuals with the ability and skills to be effective in oral and written communications, independent researchers, and complex problem solvers as cited in (McCormick & Lucas, 2011).

### 2.2 Problems on Mathematical Skills

Mathematics has always been given special attention in school as the nature of the subject is related to many other fields and disciplines. Moreover, students' mathematics achievement has often been the focus and is seen as a critical global issue in many countries. Besides being perceived as a tough subject, problems in mathematics learning has also been related to the lack of regulation skills among students in learning math. Self-regulation is a broad construct that covers before, during, and after phases of learning. Self-regulation in learning is related to the 21st-century of learning competency (Wolters, 2010); thus, students who are not regulated will face difficulty in overcoming the obstructions or challenges they face while learning.

The rapid changes in the education system and delivery methods significantly impact students. This situation requires students to learn effectively and in a more self-directed manner (Winters, Greene & Costich, 2008). To achieve this, students must be trained to enhance their skills to choose the most appropriate learning strategy (Azevedo & Cornley, 2004). Failure of doing so will affect students' motivation to learn and eventually might diminish their interest to learn. Motivation is a very important element in the learning process as it is an inducer and propeller for one to do a task successfully. Therefore, motivation is essential for an individual to successfully face challenges in academic setting. Moreover, motivation will be used by students as the attribution or determinant to their behavior in learning and performance. Behaviors that are related to academic motivation such as the desire to do difficult tasks and stay longer in difficult situations will be the determinant for students' ability in facing daily school life challenges (Masaali, 2007). Based on the above mentioned statements, the current study was conducted to explore the daily challenges faced by students in the process of learning mathematics. This challenges also can be obstruction and difficulties experienced by students. This is significant due to the nature of advancement of current mathematics education, delivery system, and also the elements that have the potential to hinder students' mathematics learning progress, as discussed above.

### 2.3 Strategies on Enhancing Mathematical Skills

The movement of problem-solving expanded worldwide in the 1980s and problem-solving became the central focus in education (Fan & Zhu, 2007; Rosli et al., 2013) According to Novita et al. (2012), mathematical problem-solving refers to mathematical tasks that have the potential to provide intellectual challenges for enhancing learners' mathematical understanding and development. Since one of the objectives of mathematics education is to prepare learners for the workplace (Gravemeijer et al., 2017), the introduction of a competence-based curriculum in the Rwandan education system was meant to address this educational reform (Rwanda Education Board [REB], 2015). Learning to solve problems is the primary reason for studying mathematics (Novita et al., 2012). Likewise, mathematics does not only play a vital role in preparing learners for further education and future career development but also strengthens students' ability to solve real-world problems (Pandey, 1991). Though learners need skills such as critical thinking, effective communication, innovation, and the ability to solve problems collaboratively, existing pedagogical practices do not seem to have dealt with these aspects adequately (Scott, 2015).

Moreover, the gap between science, technology, engineering, and mathematics (STEM) education as well as the required workplace skills in the industry, academia, and the government is still quite appalling (Jang, 2016). Hence, there is a need for understanding the kind of mathematical skills needed in today's world and the shifts that should be made in both content and pedagogy for preparing students to acquire relevant 21st-century skills including mathematical reasoning, critical thinking, and mathematical problem-solving (Turiman et al., 2012). These skills can be developed by employing teaching strategies that actively involve students in the learning process and using a kind of assessment that involves students' intellectual challenges rather than memory retrieving (Ebiendele, 2012). This could be attributed to the fact that assessment drives students' learning (Kanjee, 2009) and plays a great role in the implementation of the curriculum in which students are expected to acquire different skills and competencies (Drijvers et al., 2019).

Some scholars believe that mathematics teachers should show students how to solve problems (Lester, 2013), others think that it is better when students themselves manage to find out how to solve problems (Schoen et al., 2019), while others have argued that students can solve the real-world problem after memorizing mathematical facts (Ebiendele, 2012; Wolf et al., 1991). Nevertheless, Stacey (2005) argues that problem-solving is a teaching method rather than a goal in itself. Since problem-solving is conceived at a very early age, prekindergarten, kindergarten, and primary children can discover problems while playing (Kelly 2006). In that regard, this review focuses on assessment strategies for assessing mathematical problem-solving skills from kindergarten through to upper secondary levels of education.

The study conducted by Nizeyimana and Muthukrishna (2003) revealed that teachers employ unsound practices while implementing the curriculum. By focusing on increasing the number of students who may pass the national examinations, some teachers spend much of their time on past paper examination explorations. This is why there is a need to orient both pre-service and in-service teachers on appropriate instructional and assessment approaches that support the development of students' mathematical problem-solving skills. Similarly, integrating an authentic assessment with teaching strategies that ensure that students have got learning opportunities that can enable them to seek solutions consistently is very critical (Hopkins 2000). Effective use of assessment towards mathematical problem-solving can have a positive impact on education and society through the advancement and measurement of competencies that are needed to create productive workers and citizens (Griffin et al., 2012). Within the current economy's standards and requirements, we need to develop students' full potentials (Griffin et al., 2012), whereby teachers teach and assess students' mathematical knowledge in a way that allows them to perform what they undoubtedly understand (Kelly, 2006). Since assessment is a critical and important educational component in supporting learning and qualification (Moss, 2013; Taras, 2008), it is important to point out that "assessment tasks should be constructed to measure the particular aspects of mathematical competencies to align assessment with curriculum" (Pettersen, 2017). Hence, there is a call for national reform by changing how students are being assessed in mathematics (Berenson & Carter, 2018).

### 2.4 Relevance of Cognitive Development Theory in Enhancing Mathematical Skills

According to Burns & Silbey (2000), hands-on experiences and multiple ways of representing a mathematical solution can be ways of fostering the development of this cognitive stage. The importance of hands-on activities cannot be overemphasized at this stage. These activities provide students an avenue to make abstract ideas concrete, allowing them to get their hands on mathematical ideas and concepts as useful tools for solving problems. Because concrete experiences are needed, teachers might use manipulatives with their students to explore concepts such as place value and arithmetical operations. Existing manipulative materials include: pattern blocks, Cuisenaire rods, algebra tiles, algebra cubes, geoboards, tangrams, counters, dice, and spinners. However, teachers are not limited to commercial materials, they can also use convenient materials in activities such as paper folding and cutting. As students use the materials, they acquire experiences that help lay the foundation for more advanced mathematical thinking. Furthermore, students' use of materials helps to build their mathematical confidence by giving them a way to test and confirm their reasoning.

One of the important challenges in mathematics teaching is to help students make connections between the mathematics concepts and the activity. Children may not automatically make connections between the work they do with manipulative materials and the corresponding abstract mathematics: “children tend to think that the manipulations they do with models are one method for finding a solution and pencil-and-paper math is entirely separate” (Burns & Silbey, 2000). For example, it may be difficult for children to conceptualize how a four by six inch rectangle built with wooden tiles relates to four multiplied by six, or four groups of six. Teachers could help students make connections by showing how the rectangles can be separated into four rows of six tiles each and by demonstrating how the rectangle is another representation of four groups of six. Providing various mathematical representations acknowledges the uniqueness of students and provides multiple paths for making ideas meaningful. Engendering opportunities for students to present mathematical solutions in multiple ways (e.g., symbols, graphs, tables, and words) is one tool for cognitive development in this stage. Eggen & Kauchak (2000) noted that while a specific way of representing an idea is meaningful to some students, a different representation might be more meaningful to others.

## 3. METHODOLOGY

### 3.1 Research Design

The study was done using the one-group pretest-post-test experimental design (Koenig et al., 2012; Flasch et al., 2017). The effectiveness of the manipulatives and question series were measured by looking at whether there was an improvement, significant changes, or differences between the controlled and experimental groups' learning results.

### 3.2 Research Instrument

The research instrument used was a researcher-made questionnaire that underwent validity and reliability testing. The validity test was established through factor analysis with a KMO of 0.748 and Bartlett's Test of Sphericity of 0.000, which both suggest that there is enough sample and correlation among data for the test. Scree plot and rotated component matrix showed that Item 3 of the pre-tested questionnaire must be removed to make it valid. On the other hand, the reliability test was accomplished using Cronbach's Alpha with a coefficient of 0.772, which showed that the questionnaire was highly reliable upon removing Items 6, 8, and 11.

The topic and objective are from the 3rd Quarter lesson as reflected in the Math curriculum guide of the Department of Education (DepEd). Furthermore, during the Math class schedule, Mondays through Thursdays, the researcher conducted the action research for 100 minutes per day: 50 minutes in the morning and 50 minutes in the afternoon. The activities used by the researcher in a week's sessions were based on the topic and phases of fraction circle or pizza fractions.

The researcher-made instrument was pre-tested on 26 pupils who were the intended population but were not included in the final study. Once their responses had been tallied, the researcher sent them to a statistician for factor analysis.

Moreover, the preliminary study did not only improve the manipulatives accuracy but also aided in its development by recommending it to a specific master teacher to ensure the content of the questionnaire was aligned with the researcher's chosen competency based on the Math curriculum guide – Visualizes, represents and identifies proper fractions with denominators of 10 and below..

### 3.3 Respondents of the Study

The researcher utilized the complete enumeration method in choosing the 26 Grade 2 pupils of Cateel Central Elementary School. The respondents were grouped into the control group (13 respondents) and the experimental group (13 respondents). Their responses were gathered through a Pre-Test and a Post Test. Since they have two sessions, morning and afternoon, through a toss coin, the morning session was picked as the experimental group, which has 13 respondents, and the afternoon session automatically, the control group with 13 respondents resulting in 26 research respondents.

## 4. RESULTS AND DISCUSSION

This chapter presents the results and discussion of the study. The discussion is arranged in order of the statements of the problem: the level of pre-test scores between the experimental and control group; the level of post-test scores between the experimental and control group; the comparison between pre-test scores between of experimental and control group; comparison between post-test scores of the experimental and control group.

### 4.1 Level of Pre-Test Scores

According to Sherman and Richardson (2013), the two main reasons for teachers not using manipulatives in their mathematics classrooms were that teachers were uncertain of how to use them, and they felt that manipulative instruction was inappropriate for students above the fourth grade.

Table 1 showed the level of pre-test scores between the control and experimental group.

**Table 1:** Level of pre-test scores between the control and experimental groups

| Group | Total Score | Standard Deviation | Mean | Grade Percentage | Remarks |
|---|---|---|---|---|---|
| Control | 16 | 2.95 | 10.769 | 83.65 | Satisfactory |
| Experimental | 16 | 1.59 | 10.769 | 83.65 | Satisfactory |

As observed in the table, the result showed that both the control and experimental group obtained satisfactory scores in their pre-test. This implied that learners already recognize various representations of the same fractions, select and use benchmark fractions, manipulate fractional units, understand fraction magnitude, and exercise flexibility in fractional operational strategies. This result aligned to the findings of Schneider and Stern (2010), which contest that pupils already learn how to read and represent the value of a fraction. They start placing fractions on a graduated number line. They learn how to simplify fractions (i.e., introduction to equivalent fractions). They learn how to add and subtract fractions with small and common denominators.

### 4.2 Difference on the Pretest Scores

The pre-test scores between the control and experimental group were compared by utilizing the data treatment tool, mean, to get their mean comparison. Table 2 presents the mean comparison between the pre-test scores of the control and experimental group. The results show that the pre-test score between the two groups did not differ significantly.

**Table 2:** Mean comparison between pre-test scores of control and experimental group

| Group | Mean | Standard Deviation | t-value | p-value | Interpretation |
|---|---|---|---|---|---|
| Control | 10.769 | 2.95 | 0.000 | 1.000 | Pre-test scores between the two groups do not differ significantly. |
| Experimental | 10.769 | 1.59 | | | |

The result implies that both control and experimental groups did not have varying knowledge about converting visual representation to fractions. This also implies that both have the same level of understanding of fractions.

Similarly, Allen (2007) conducted a three-day study among second-graders in a Mathematics class. The group was given a pre-test and a post-test. The units contained hands-on, manipulative games, partner activities, and everyday Mathematics tools. The result showed understanding and a positive attitude toward learning concepts that were previously difficult.

According to Burns & Silbey (2000), hands-on experiences and multiple ways of representing a mathematical solution can be ways of fostering the development of this cognitive stage. The importance of hands-on activities cannot be overemphasized at this stage. These activities provide students an avenue to make abstract ideas concrete, allowing them to get their hands on mathematical ideas and concepts as useful tools for solving problems.

### 4.3 Level of Post-Test Scores

Table 2 showed the level of post-test scores between the control and experimental group.

**Table 3:** Level of post-test scores between the control and experimental groups

| Group | Total Score | Standard Deviation | Mean | Grade Percentage | Remarks |
|---|---|---|---|---|---|
| Control | 16 | 0.38 | 11.85 | 87.02 | Very |

| | | | | | |
|---|---|---|---|---|---|
| | | | | | Satisfactory |
| Experimental | 16 | 1.04 | 14.85 | 95.69 | Outstanding |

As shown in table 2, the control group's post-test scores were very satisfactory, while the experimental group was outstanding. This implied that the manipulatives enhanced the learners' skills in recognizing fractions' representations. Although they already are satisfactory in learning fractions, as shown in the results, they clearly improved more. Also, through manipulatives, they were given the opportunity to be actively involved in learning. As asserted by Yusof (2013), pupils need to be actively involved in their learning and manipulate objects in their surroundings to generate a better understanding of mathematical concepts.

Thus, in teaching and learning fractions, pupils should be given opportunities to explore the concept of the fraction through hands-on experiences, share their ideas among themselves, and more importantly, learn fraction concepts with the help of appropriate manipulatives. Calma's (2012) findings are similar to those of the present study. Findings reveal that the post-test results of the control and experimental group respondents were significantly higher than their pre-test results. This suggests that students gained knowledge of the operations of fractions after being exposed to concrete manipulatives instruction.

Using manipulatives makes mathematics learning more meaningful to students (Stein & Bovalino, 2001), as they are materials designed to represent explicitly and concretely mathematical ideas that are abstract (Moyer, 2001). The meaning of concrete needs to be further defined to understand the role of concrete manipulatives and any concrete-to-abstract pedagogical sequence.

When students work with manipulatives and then are given a chance to reflect on their experiences, not only is mathematical learning enhanced, math anxiety is significantly reduced (Heuser, 2000). Exploring manipulatives, especially self-directed exploration, provides an exciting classroom environment and promotes in students a positive attitude toward learning (Moch, 2001).

**4.4 Difference on the Post-Test scores**

The post-test scores of the experimental and control groups were compared using the data treatment tool, mean, to get their mean comparison. Table 2 shows the mean post-test scores of the control and experimental groups. The results showed that the post-test score differs significantly between the two groups. The result prevailed that the learners from the experimental group excelled more compared to the learners from the control group.

**Table 4:** Mean comparison between post-test scores of control and experimental group

| Group | Mean | Standard Deviation | t-value | p-value | Interpretation |
|---|---|---|---|---|---|
| Control | 11.85 | 0.38 | –9.000 | 0.000 | Post test scores between the two groups differs significantly. |
| Experimental | 14.62 | 1.04 | | | |

Based on the results, a computed p-value of 0.000 shows a highly significant difference between the mean pre-test and post-test scores of the respondents in the experimental group. This group that used manipulatives showed greater improvement in the study. Yusof (2013) concluded that the intervention lessons in teaching operations on fractions through a cooperative approach while using manipulatives statistically significantly improved the pupils' overall attainment in fraction works. There was a highly significant improvement in pupils' overall performance in the post-test (after intervention lessons) compared to the pre-test before intervention lessons.

This finding indicates that manipulatives significantly increased the achievement scores of the experiment group. It can be said that manipulatives, as mentioned earlier, improved the learners' abilities to recognize fraction representations. Although they are already satisfactory in terms of learning fractions, the results clearly show that they have developed more. Manipulatives also allow them to participate actively in their learning. According to Hedens (2015), using manipulatives to teach fractions in cooperative learning is in line with the curriculum framework because this strategy is pupil-centered, allows for idea exploration, and allows students to manipulate objects from their surroundings.

**4.6 Implication to Mathematics Education**

This study aimed to determine the effect of manipulatives on improving student performance in mathematics. Education is constantly under the microscope. Within education, we are constantly trying to create the ideal school: one driven by a vision that involves an element of dreaming about the best that can happen.

Unfortunately, vision without action is merely a dream. Bringing the vision to reality requires the concerted effort of all education stakeholders, and this belief is certainly relevant as we consider mathematics pedagogy.

Understandably, math skills are crucial to functioning in today's world (Burns & Hamm, 2013). These skills are not just important in school and mathematical classes but also our daily lives. Golafshani (2013) stated that there is no doubt that everyone believes that mathematics is important. However, many students have poor math skills, indicating that changes are needed in teaching mathematics. Golafshani (2013) found a growing consensus around the use of manipulatives in instructional practices. One may inquire as to just what constitutes a manipulative. Johnson (1993) noted manipulatives as objects used for instruction that vary in shape, size, and color. Ultimately, manipulatives can be any objects that an innovative teacher wishes to use to improve a student's mathematical skills. Gauthier (2004) noted that manipulatives have a place not only at the elementary level but also in the higher grades.

## 5. CONCLUSION

Based on the findings, this paper concludes:

1. The pre-test scores between the two groups were satisfactory. This means that the learners already have background knowledge of fractions.

2. The post-test scores between the control and experimental groups differ significantly. This implies that both groups improved their skills in understanding fractions through manipulatives.

3. The pre-test scores between the two groups did not differ significantly, which implies that both have the same level of background knowledge in terms of understanding fractions.

4. The post-test scores differ significantly. This implies that although both groups improved more in understanding fractions, experimental group learners exceeded expectations and got outstanding. Hence, the scores differ significantly.

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