TRANSPORTATION PROBLEM

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ABSTRACT

The most important and successful applications in the optimization refers to Transportation Problem (TP), that is a special class of the Linear Programming (LP) in the Operation Research (OR). Transportation problem is considered a vitally important aspect that has been studied in a wide range of operations including research domains. As such, it has been used in simulation of several real life problems. The main objective of transportation problem solution methods is to minimize the cost orthe time of transportation.

An Initial Basic Feasible Solution (IBFS) for the transportation problem can be obtained by using the North-West corner rule, Minimum Cost Method and Vogel's Approximation Method. In this paper the best optimality condition has been checked.

Thus, optimizing transportation problem of variableshas remarkably been significant to various disciplines.

Key words: Transportation Problem, Linear Programming (LP).

INTRODUCTION

The transportation problem is a special class of linear programming problems, which deals with the transportation of a single homogeneous product from several sources (production or supply centres) to several sinks (destinations or warehouses). When addressing a Transportation problem, the practitioner usually has a given capacity at each supply point and a given requirement at each demand point. Many decision problems, such as production inventory, job scheduling, production distribution and investment analysis can be formulated as Transportation Problems. Good financial decisions concerning facility location also attempt to minimize total transportation and production costs for the entire system. The transportation problem can be represented as a single objective transportation problem or as a multi-objective transportation problem.

Since the first statement of the transportation problem, in its continuous form due to Monge (1781), it has been the subject of numerous mathematical publications. The problem is known under several terms, among which are the general, classical or Hitchcock transportation problem as well as the distribution problem. We will consider the problem in its discrete, dense and incapacitated form, without so called 'inadmissible' cells, and refer to this form, see (2.1), as the transportation problem for the remainder of this paper.

Studies of this problem date back to the 1930s and the work of A.N. Tolstoi, see Schrijver (2002) and were followed by the well-known publications of Hitchcock (1941), Kantorovich (1942) (continuous formulation) and Koopmans (1948). In the early 1950s Dantzig's specifically tailored version of his famous Simplex Method for the transportation problem was published (see Dantzig, 1951), which we refer to as (Primal) Transportation Simplex (TPS)[1], which until today remains the basis of several competitive algorithms for the problem.

MAIN OBJECTIVES

The main objective of Transportation Problem is to determine the amounts transported from each source to each sink to minimize the total transportation cost while satisfying the supply and demand restrictions.

BASIC STEPS

The basic steps for obtaining an optimum solution to a Transportation Problem are:

Step 1: Mathematical formulation of the transportation problem;
Step 2: Determining an initial basic feasible solution;
Step 3: To test whether the solution is an optimal one. If not, to improve it further till the optimality is achieved.

MATHEMATICAL FORMULATION

The transportation problem can be stated as an allocation problem in which there are m sources (suppliers) and n destinations (customers). Each of the m sources can allocate to any of the n destinations at a per unit carrying cost c_{ij}

(unit transportation cost from source i to destination j). Each sources has a supply of S_i units, $1 \le i \le m$ and each destination has a demand of d_j units, $1 \le j \le n$. The objective is to determine which routes are to be opened and the size of the shipment on those routes, so that the total transportation cost of meeting demand, given the supply constraints, is minimized.

THE DECISION VARIABLES

A transportation problem is a complete specification of how many units of the product should be transported from each source to each destination. Therefore, the decision variables are:

$x_{ij} =$ The amount of the shipment from source i to destination j,

where $i = 1, 2, \dots\dots, m$ and $j = 1, 2, \dots\dots, n$

THE OBJECTIVE FUNCTION

Consider the shipment from warehouse i to destination j. For any i and any j, the transportation cost per unit is c_{ij}; and the size of the shipment is x_{ij}. Since we assume the cost function is linear, the total cost of this shipment is given by $c_{ij} x_{ij}$. Summing over all i and all j now yields the overall transportation cost for all source-destination combinations. That is, our objective function is:

$$\text{Minimize:} \qquad z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

THE CONSTRAINTS

Considering warehouse i. The total outgoing shipment from this warehouse is the sum $x_{i1} + x_{i2} + \dots\dots + x_{in}$. In summation notation, this is written as $\sum_{j=1}^{n} x_{ij}$. Since the total supply from warehouse i is S_i, the total outgoing shipment cannot exceed S_i. That is, we must require

$$\sum_{j=1}^{n} x_{ij} \le S_i \quad ; \qquad i = 1, 2, \dots\dots, m$$

Again, considering destination j. The total incoming shipment at this outlet is the sum $x_{1j} + x_{2j} + \dots\dots + x_{mj}$. In summation notation, this is written as $\sum_{i=1}^{m} x_{ij}$. Since the demand at outlet j is d_j, the total incoming shipment should not be less than d_j. That is, we must require

$$\sum_{i=1}^{m} x_{ij} \ge d_j \quad ; \qquad j = 1, 2, \dots\dots, n$$

Of course, as physical shipments, the x_{ij} 's should be non-negative i.e. $x_{ij} \ge 0$.

Then the linear programming model representing the transportation problem is generally given as

$$\text{Minimize} \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}.$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \le S_i \quad ; \quad i = 1, 2, \dots\dots, m$$

$$\sum_{i=1}^{m} x_{ij} \ge d_j \quad ; \quad j = 1, 2, \dots\dots, n$$

$$x_{ij} \ge 0. \qquad \text{for all } i \text{ and } j.$$

In mathematical terms the above problem can be expressed as finding a set of x_{ij} 's, $i = 1, 2, \dots\dots, m$; $j = 1, 2, \dots\dots, n$ to

$$\text{Minimize} \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} .$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} = S_i \quad ; \quad i = 1, 2, \ldots \ldots, m$$

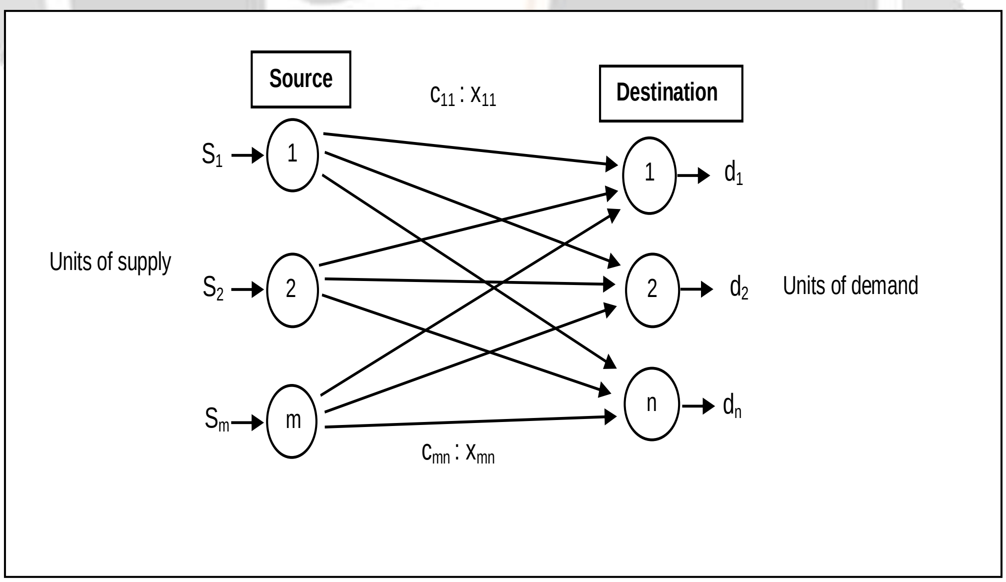
$$\sum_{i=1}^{m} x_{ij} = d_j \quad ; \quad j = 1, 2, \ldots \ldots, n$$

$$x_{ij} \geq 0 . \quad \text{for all } i \text{ and } j .$$

NETWORK REPRESENTATION

We represent each source and destination by a node, the amount of supply at source i by S_i, the amount of demand at destination j by d_j, the unit transportation cost between source i and destination j by c_{ij} and the amount transported from source i to destination j by x_{ij}. The arcs joining the source and destination represent the route through which the commodity is transported. Then the general transportation problem can be represented by the network as in the following figure.

Network Representation of Transportation Problem



BALANCED TRANSPORTATION PROBLEM

When the total supply amount is equal to the total demand then this transportation problem is called the Balanced Transportation problem. That is in the balance transportation problem

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} d_j$$

where $\sum_{i=1}^{m} S_i$ is the total supply amount and $\sum_{j=1}^{n} d_j$ is the total demand amount.

UNBALANCED TRANSPORTATION PROBLEM

An unbalanced transportation problem is one in which the total supply of the sources and the total demand at destination centre are not equal. This creates the following two situations:

1. When the total capacity of the origins exceeds the total requirement of destinations, a dummy destination is introduced in the transportation table which absorbs the excess capacity. The cost of shipping from each origin to this dummy destination is assumed to be zero. The insertion of a dummy destination establishes equality between the total sources capacities and total destination requirements.
2. When total capacity of origins is less than the total requirement of destinations, a dummy origin is introduced in the transportation table to meet the excess demand. The cost of shifting from the dummy origin to each destination is assumed to be zero. The introduction of a dummy origin in this case establishes the equality between the total capacity of sources and the total requirement of destinations.

MATRIX TERMINOLOGY

The matrix used in the transportation models consists of squares called “cells”, which when stacked from ‘columns’ vertically and rows ‘horizontally’. The cell allocated at the intersection of a row and a column is designated by its row and column heading. Thus the cell located at the intersection of row A and column 4 is called cell (A, 4). Unit transportation costs are positioned in each cell. The availabilities of each source are placed in the last column while the requirements are placed in the last row.

Matrix Representation of Transportation Problem

Sources	**Warehouses**					**Supply**
	1	**2**	**3**	**4**	**5**	15
A	2	5	8	1	9	25
B	5	6	8	7	10	20
C	1	2	5	8	2	10
Demand	16	10	18	12	14	

CONCLUSIONS

In today’s highly competitive market, various organizations want to deliver products to the customers in a cost effective way, so that the market becomes competitive. To meet this challenge, transportation model provides a powerful framework to determine the best ways to deliver goods to the customer. In this article, a new approach named allocation table method (ATM) for finding an initial basic feasible solution of transportation problems is proposed. Efficiency of allocation table method has also been tested by solving several number of cost minimizing transportation problems and it is found that the allocation table method yields comparatively a better result. Finally it can be claimed that the allocation table method may provides a remarkable Initial Basic Feasible Solution by ensuring minimum transportation cost. This will help to achieve the goal to those who want to maximize their profit by minimizing the transportation cost.

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