USING GEOMETER'S SKETCHPAD SOFTWARE TO DESIGN LECTURES ON FINDING OPTIMAL SOLUTIONS FOR LINEAR OPTIMIZATION PROBLEMS TO ENHANCE THE APPREAL OF ADVANCED MATHEMATICS TEACHING AT UNVERSITIES

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ABSTRACT

In the scope of this paper, the author will present the method for finding the optimal solution to two optimization problems in economics: the problem of maximizing total income and the problem of minimizing total cost using the Geometer's sketchpad software. Additionally, a lesson plan will be designed for teaching how to find the optimal solution to the linear optimization model using the Geometer's sketchpad software.

Keywords: *maximizing total income, minimizing total cost, linear optimization model, optimal solution, Geometer's sketchpad.*

1. INTRODUTION

The reality of production and business activities is highly dynamic and diverse, especially in the fiercely competitive environment of the market economy. Producers, business owners, and enterprise managers must constantly weigh factors such as capital resources, the company's capabilities, market conditions, local potential, and natural and social circumstances against their goals of profitability and business growth. Their decisions on selecting solutions and courses of action must be carefully considered to achieve the best possible outcomes. Investment decisions made by business managers are always tied to specific objectives, representing an optimal choice based on predetermined goals.[1]

If all factors related to resources, objectives, and investment decision-making have a linear relationship, we can fully utilize a linear optimization model to describe, analyze, and determine the optimal choice for managers.

Currently, there are many software programs that support finding optimal solutions for linear optimization models, such as Excel, Geogebra, and Geometer's Sketchpad… Among them, Geometer's Sketchpad (GSP) is a dynamic geometry software with several outstanding advantages, including simple installation, ease of use, open-source availability, and smooth, precise drawing capabilities. Additionally, it allows for step-by-step lecture design in a convenient and fast manner, enabling direct teaching presentations without the need for editing or transferring to PowerPoint.

Within the scope of this paper, the author will demonstrate how to find optimal solutions for two economic optimization problems: the problem of maximizing total revenue and the problem of minimizing total cost using GSP software. Furthermore, the paper will present a lecture design for teaching optimal solution methods in linear optimization models with the help of GSP.

2. CONTENT

2.1 Introduction to GSP Software

Geometer's Sketchpad (commonly referred to as Sketchpad or GSP) is a commercial software designed for exploring Euclidean Geometry, Algebra, Calculus, and other branches of Mathematics. GSP includes classical geometric drawing tools, such as a ruler and compass, which serve as the foundation for constructing fundamental geometric elements. These include finding the midpoint of a segment, drawing a line through a point perpendicular (or parallel) to another line, constructing an angle equal to a given angle, and drawing the angle bisector. Although GSP is primarily designed for Geometry, it also provides several tools for Algebra, such as drawing number lines, plotting function graphs, graphing functions with variable coefficients, and plotting parametric functions. Additionally, it supports Calculus with features like calculating function limits at a specific point. By incorporating these capabilities, GSP enhances student engagement, comprehension, and learning outcomes. With GSP, Mathematics becomes less of a challenge for learners, making the subject more accessible and interactive.[4]

GSP is also designed for presentations and demonstrations. By allowing users to create multiple pages within a file, add text, insert external images, and apply direct interactive effects, GSP effectively enhances the teacher's presentation capabilities, making lessons much more engaging and dynamic. It can be said that Sketchpad is truly an essential teaching tool for users who want to explore and study geometric operations, from basic to complex concepts.[4]

The GSP software runs stably on devices using Windows XP and Windows 7 operating systems. To ensure smooth performance, users should install and use Sketchpad on devices with these supported operating system versions. The installation process and download links for GSP can be easily found on the Internet.

Figure 1: GSP Software Interface

2.2 The Model of the Maximum Total Revenue Problem and the Minimum Total Cost Problem

Based on real-life situations, by analyzing the resources that directly or indirectly influence the problem and the defined objectives, we can fully model these real-world issues in the form of mathematical models and solve them using mathematical tools.

Linear optimization models are a mathematical approach to solving optimization problems. They are used to determine the optimal value of a linear objective function while satisfying a set of linear constraints. This model is widely applied in various fields, including economics, production management, engineering, science, and technology.

In economics, producers and business managers must make investment decisions that align with their company's resources, as well as natural and social conditions, while still achieving their main objective—maximizing profit while minimizing costs.

Ultimately, most economic problems faced by businesses or production facilities fall into one of two categories: the **maximum total revenue problem** or the **minimum total cost problem**.

* **Maximum Total Revenue Model [1]**

Let profit function $f = c_1x_1 + c_2x_2 + \cdots + c_nx_n$, with constraint system:

$$\text{(I)} \quad \begin{cases} a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \le b_i;\, i = \overline{1,m} \\ x_j \ge 0;\, j = \overline{1,n}. \end{cases}$$

a_{ij} : The amount of resource type i required to produce one unit of product type j

b_i ($i = \overline{1,m}$): The amount of resources available to the enterprise

c_j :The cost of one unit of product type j, $j = \overline{1,n}$.

x_j :The number of units of product type j to be produced.

* *Mathematical Model*: Finding the solution $x^0 = (x_1, x_2, \dots, x_n)$ that satisfies the constraint system (I) such that the function *f* is maximized.

The solution x^0 is called the optimal solution, and the function f is called the objective function.

Example 1: Optimal Model for Maximizing Profit

A cooperative plans to produce wheat and potatoes on 21 hectares of land, with 27,000 m^3 of water and 120 labor units available. The resource requirements and profits for each crop are as follows:

- Wheat:
  - Requires 3 hectares of land, 3,000 m^3 of water, and 5 labor units per ton.
  - Generates 6 million VND in profit per ton.
- Potatoes:
  - Requires 1 hectare of land, 3,000 m^3 of water, and 20 labor units per ton.
  - Generates 8 million VND in profit per ton.

The cooperative aims to maximize total profit while ensuring that resource constraints are met.

Formulating the Linear Optimization Model

Let:

- *x* be the number of tons of wheat produced.
- *y* be the number of tons of potatoes produced.

Objective Function (Profit Maximization): Maximize *f(x,y) = 6x + 8 y*

Constraints

1. Land Constraint: $3x + y \leq 21$
2. Water Constraint: $3x + 3y \leq 27$
3. Labor Constraint: $5x + 20y \leq 120$
4. Non-Negativity Constraints: $x \geq 0; y \geq 0.$

* *Mathematical Model*: Finding the solution $c^0 = (x, y)$ that satisfies the constraint system

$$\begin{cases} 3x + y \leq 21 \\ 3x + 3y \leq 27 \\ 5x + 20y \leq 120 \\ x \geq 0, y \geq 0 \end{cases}$$

such that the profit function $f = 6x + 8y \to max.$

*** Minimum Total Cost Model**

Let the cost function $f\ = c_1x_1 + c_2x_2 + \cdots + c_nx_n$, with constraint system:

$$(II) \begin{cases} a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \geq b_i; i = \overline{1,m} \\ x_j \geq 0; j = \overline{1,n}. \end{cases}$$

a_{ij} : The amount of resource type i required in one unit of product type j;

b_i $(i = \overline{1,m}$) : The minimum amount of resources that the enterprise needs to meet.

c_j : The cost of one unit of product type j ($j = \overline{1,n}$) that needs to be purchased.

x_j : The number of units of product type j that need to be purchased.

* *Mathematical Model*: Finding the solution $x^0 = (x_1, x_2, \dots, x_n)$ that satisfies the constraint system (II) such that the function *f* is minimumed.

Example 2: Optimal Model for Minimizing Transportation Cost

A livestock farm needs to rent trucks to transport 140 pigs and 9 tons of animal feed. The rental company offers two types of trucks:

- Large trucks:
  - 10 trucks available
  - Can carry 20 pigs and 0,6 tons of feed per truck
  - Rental cost: 4.000.000 VND per truck
- Small trucks:
  - 9 trucks available
  - Can carry 10 pigs and 1,5 tons of feed per truck
  - Rental cost: 3.000.000 VND per truck

The farm must choose the number of each type of truck to rent so that all pigs and feed are transported while minimizing the total rental cost.

Formulating the Linear Optimization Model

Let:

- x be the number of large trucks rented.
- y be the number of small trucks rented.

Objective Function (Cost Minimization): Minimize f(x, y) = 4.x + 3.y (million VND)

Constraints

1. Pig Transportation Constraint: (Total pigs transported must be at least 140)

$$20x + 10y \geq 140$$

2. Feed Transportation Constraint: (Total feed transported must be at least 9 tons)

$$0{,}6x + 1{,}5y \geq 90$$

3. Truck Availability Constraints: $x \leq 10; y \leq 9$
4. Non-Negativity Constraints: $x \geq 0; y \geq 0$.

* *Mathematical Model*: Finding the solution $c^0 = (x, y)$ that satisfies the constraint system

$$\begin{cases} 20x + 10y \geq 140 \\ x \leq 10 \\ y \leq 9 \\ 0{,}6x + 1{,}5y \geq 9 \\ x \geq 0, y \geq 0 \end{cases}$$

such that the cost function $f = 4x + 3y \to min$.

2.3 Design a lecture on finding optimal solutions for two problems of maximizing total income and minimizing total cost using Geometer' Sketchpad software.

An optimal solution for a linear optimization model is a set of values for the decision variables that maximize or minimize the objective function while satisfying all constraints.

This model represents a linear programming problem (LPP) that can be solved using optimization techniques or software like Excel Solver, Geogebra, or Geometer's Sketchpad (GSP)… The GSP software not only helps solve problems but also serves as a teaching tool with visually appealing, easy-to-understand, and dynamic presentations.

The following are insights on designing a lesson to find the optimal solution for a linear optimization problem using GSP software through two specific examples:

Lesson Design: Finding the Optimal Production Plan for Wheat and Potatoes to Maximize Cooperative Profit (Solving Example Problem 1)

* *Mathematical Model*: Finding the solution $c^0 = (x, y)$ that satisfies the constraint system

$$\begin{cases} 3x + y \le 21 \\ 3x + 3y \le 27 \\ 5x + 20y \le 120 \\ x \ge 0, y \ge 0 \end{cases}$$

such that the profit function $f = 6x + 8y \to max$.

Step 1: Copy the model and objective of the problem, and create control buttons to prepare for the presentation.

Note: Select control buttons for each object by clicking on the object, then choosing *Edit Control Button → Hide/Show*. It is recommended to name each presentation step to avoid confusion during execution.

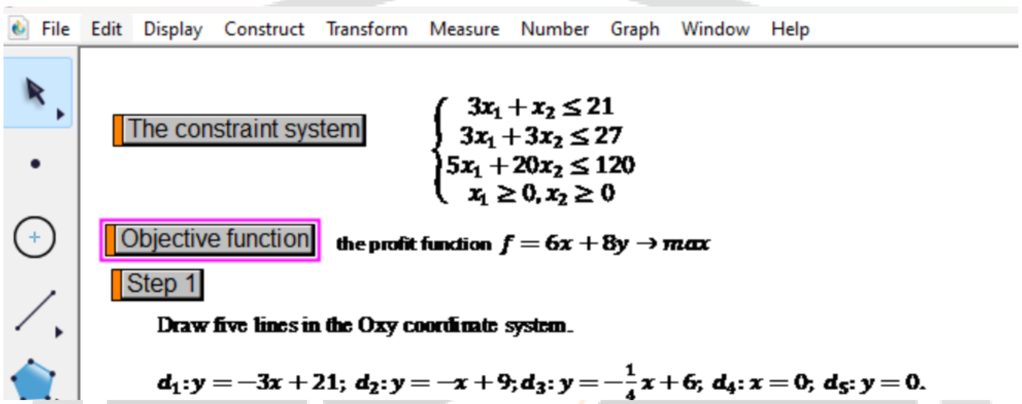

Figure 2: Control nodes for the object.

Step 2: On the GSP software interface, select the arrow tool from the left toolbar. Then, choose *Graph Functions → Compact Oxy Axes System*. This will create the Oxy coordinate plane.

Step 3: Draw five lines in the Oxy coordinate system.

$d_1: y = -3x + 21;\ d_2: y = -x + 9; d_3: y = -\frac{1}{4}x + 6;\ d_4: x = 0;\ d_5: y = 0.$

By accessing the toolbar, select *Graph → Draw New Function Graph*. A dialog box will then appear for entering the equations.

Note that after drawing the lines, it is necessary to adjust them to fit within the defined Oxy plane. This can be done by moving the cursor over the line, right-clicking, selecting *Properties → Graphing*, and choosing an appropriate range for x.

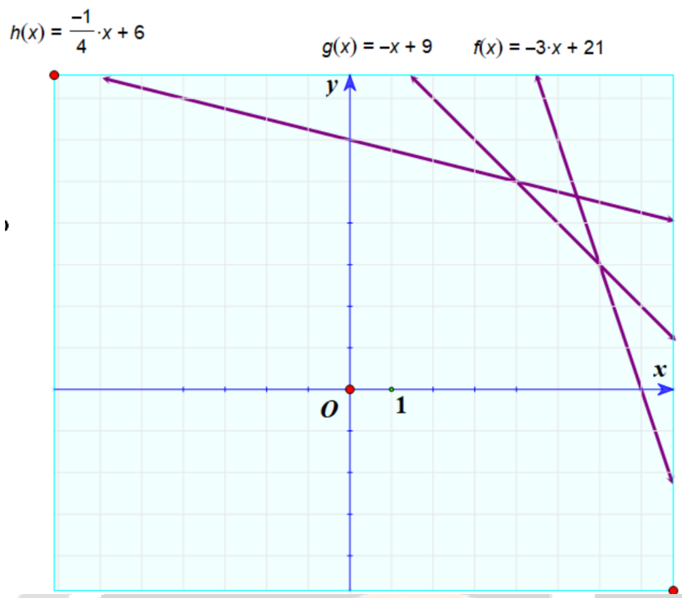

Figure 3: The lines in the constraint system.

Step 4: Since the origin O(0,0) satisfies the inequalities in the system, the solution region will include the origin. We eliminate the half-planes that do not contain O.

To represent the solution region, use the mouse to click on special points on the plane (usually points at the corners of the plane) that belong to the half-plane not containing point O. Then, press the **Ctrl + P** key combination. This will shade out the non-solution region.

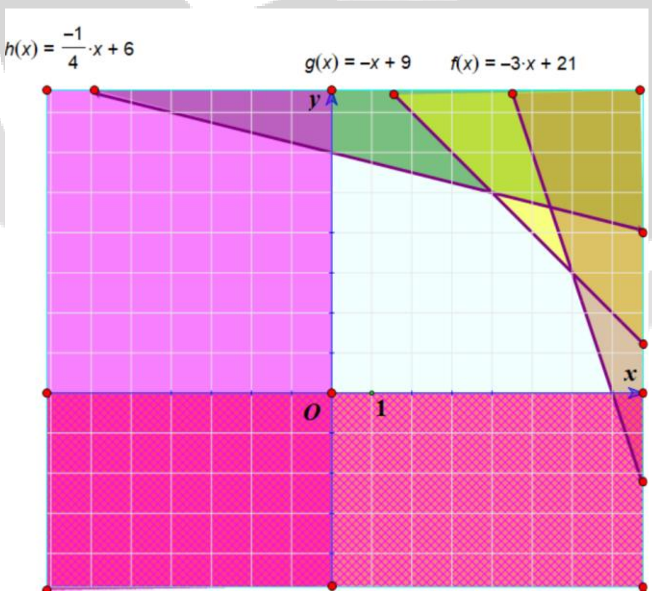

Figure 4: Represent the solution region.

Step 5: Identify the intersection points of the corresponding lines and name them. You can also create hide/show control buttons to display these intersection points during the presentation.

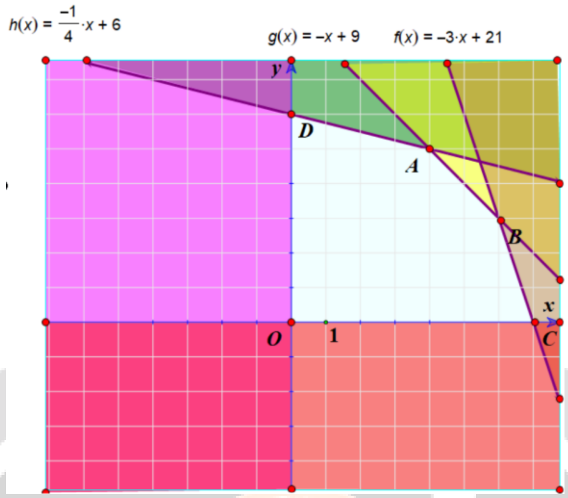

Figure 5: Solution polygon.

We observe that the solution region (the unshaded area) is the polygon ABCOD. Here A(4; 5); B(6;3); C(7; 0); O(0; 0) and D(0; 6).

Step 6: Draw the level line 6x + 8y = 24 or $y = \frac{-3}{4}x + 3$ to represent the objective function graph.

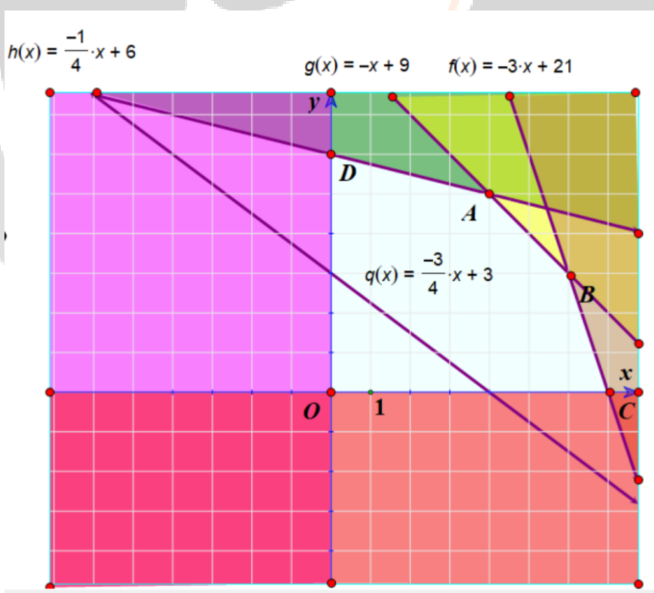

Figure 6: Contour graph.

Move the level line parallel to the direction of the normal vector, sweeping through the solution region from bottom to top. Stop when further movement causes the level line to no longer intersect the solution region. We observe that the last point in the solution region is vertex A(4,5), which is where the objective function reaches its maximum.$f_{max} = 64$.

Step 7: After designing all the presentation steps, click on the control buttons to hide the objects. When starting the lesson, you can press each control button to sequentially display the presentation steps along with dynamic visuals.

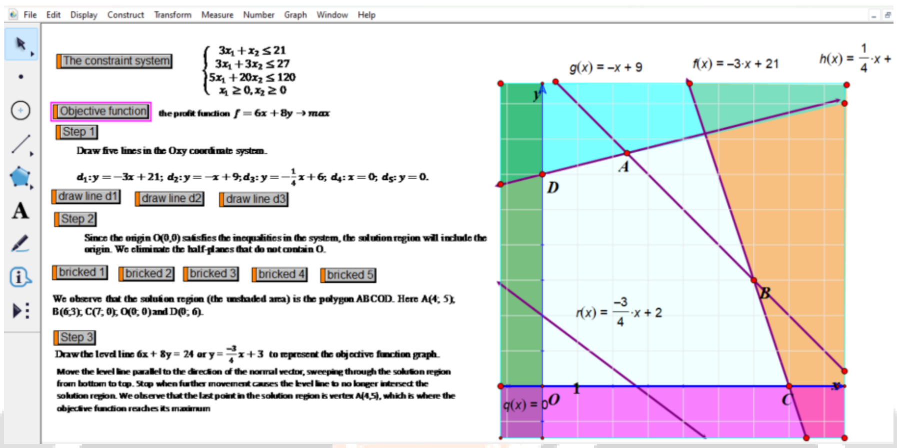

Figure 7: Description of the lecture process.

Lesson Design: Finding the Optimal Rental Plan for Two Types of Vehicles to Minimize Costs (Solving Example Problem 2)

* *Mathematical Model*: Finding the solution $c^0 = (x, y)$ that satisfies the constraint system

$$\begin{cases} 20x + 10y \geq 140 \\ x \leq 10 \\ y \leq 9 \\ 0{,}6x + 1{,}5y \geq 9 \\ x \geq 0, y \geq 0 \end{cases}$$

such that the cost function $f = 4x + 3y \to min$.

We follow the same steps as in the previous example. As a result, we will design the presentation as follows:

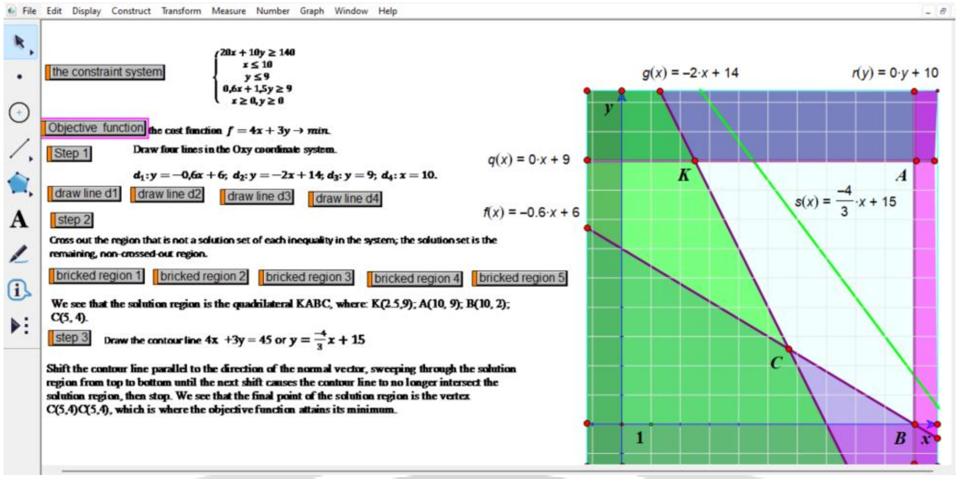

Figure 8: Description of the lecture process.

Before starting the presentation, hide the objects that will be displayed. Then, press the control buttons to gradually reveal the objects according to the lesson's progression.

3. CONCLUSION

The problem of maximizing total income and the problem of minimizing total costs are two important optimization problems in optimization theory. Besides formulating the mathematical model from real-life situations, solving these problems to find the optimal solution is crucial in helping producers and businesses make important strategic decisions.

By following the designed steps, we can effectively visualize and solve the optimization problem using Geometer's Sketchpad. The step-by-step presentation helps illustrate the problem clearly, making it easier to understand and analyze the optimal solution. This approach can be applied to various optimization problems in different fields.

Using GSP software to design lessons and find optimal solutions for linear optimization problems not only helps instructors convey the lesson's ideas and objectives but also enhances student engagement. The dynamic visuals, scientific and easy-to-understand design, and innovative teaching methods stimulate learners' interest, making the learning process more effective.

4. REFERENCES

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