Ultrametric hierarchical resource utilization model using *p*-adic numbers

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ABSTRACT

We present a new model of resource utilization which we call the ultrametric model. It is based on a modeling of hierarchical resource utilization by p-adic numbers. This model can be used on a system with several resources which can be classified in order of importance. The state of such a system is a p-adic number. We define an elementary resource utilization as this state of the system, and a representation of the uses in the form of balls in the set of p-adic numbers has been advanced. From these models, and using state transitions at each change in resource utilization whether it is the arrival of a new customer, or a departure, we can demonstrate that the probability of resource utilization is governed by a Cauchy problem: a fractional differential equation, and more particularly, with the Vladimirov operator.

Keyword: - *hierarchical, p-adic, resource, utilization, ultrametric*

1. INTRODUCTION

Theories on *p*-adic dynamical systems have been intensely advanced [1]. Flow dynamics, like algebraic geometry, have been proposed by Herman and Yoccoz [2] in the small divisor problem in non-Archimedean fields. It seems to be the first publication talking about dynamics in a non-Archimedean domain. Future developments were observed according to Silverman [3]. Several fields of applications of p-adic numbers in various fields have been explored by Khrennikov et al.: differential analysis [4], quantum mechanics [5], biology [6]–[8], cognitive sciences [9][10], imagery [11], system dynamics [12], psychology and finance [13], sociology and medicine [14] and many other fields [15], ...

For our case, we will propose an analysis of a dynamical system in a non-Archimedean domain. A system of hierarchical resources whose state changes over time according to the utilization of these resources. We have shown that the occupations of these resources are governed by the ultrametric diffusion equation whose resolution uses a Cauchy problem with Vladimirov's fractional derivative operator.

2. ULTRAMETRIC MODEL OF HIERARCHICAL MODEL

2.1 *p*-adic valuation

Let p be a prime number. For any rational number $r = p^k \frac{m}{n} \in \mathbb{Q}$, where m and n are coprime with p, we denote $v_p(r) = k$ or else for $r \neq 0$, and by convention $v_p(0) = +\infty$. We define the p-adic absolute value in $\mathbb{Q}$ as the map from $\mathbb{Q}$ to $\mathbb{R}_+ = [0, +\infty[$, denoted $|\,.\,|_p$ by :

$$|r|_p = p^{-v_p(r)} \qquad (1)$$

The map v_p from $\mathbb{Q}$ to $\mathbb{Z}$ is called p-adic valuation.
Note some properties of this p-adic absolute value in Proposition 1.

Proposition 1.

- For any rational number r_1 and r_2 : $|r_1.r_2|_p = |r_1|_p.|r_2|_p$
- For any rational number r_1 and r_2 : $|r_1+r_2|_p \le \max\{|r_1|_p, |r_2|_p\}$

Proof :

The rational numbers r_1 and r_2 can be written respectively like $r_1 = p^{k_1}\frac{m_1}{n_1}$ and $r_2 = p^{k_2}\frac{m_2}{n_2}$ where m_1, m_2, n_1 and n_2 are coprime with p. So we have $|r_1|_p = p^{-k_1}$ and $|r_2|_p = p^{-k_2}$. As for the product $r_1.r_2$ we have $r_1.r_2 = p^{k_1+k_2}\frac{m_1m_2}{n_1n_2}$. Knowing that m_1m_2 and n_1n_2 are coprime with p, we can say that the p-adic absolute value of the product $r_1.r_2$ is equal to $|r_1.r_2|_p = p^{-k_1-k_2} = |r_1|_p.|r_2|_p$.

For the sum, we have:

$$r_1+r_2 = p^{k_1}\frac{m_1}{n_1} + p^{k_2}\frac{m_2}{n_2} = \frac{p^{k_1}m_1n_2 + p^{k_2}m_2n_1}{n_1n_2} = \begin{cases} p^{k_1}.\dfrac{m_1n_2 + p^{k_2-k_1}m_2n_1}{n_1n_2} & \text{si } k_1 \le k_2 \\ p^{k_2}.\dfrac{p^{k_1-k_2}m_1n_2 + m_2n_1}{n_1n_2} & \text{si } k_1 > k_2 \end{cases}$$

$$= p^{\min(k_1,k_2)}.\frac{u}{v}$$

where we can prove that v and p are also coprime.

For the case of u, if it is prime with p, we have $|r_1+r_2|_p = p^{-\min(k_1,k_2)}$, therefore :

$$|r_1+r_2|_p = p^{-\min(k_1,k_2)} = \max\left(p^{-k_1}, p^{-k_2}\right) = \max\left(|r_1|_p, |r_2|_p\right).$$

If it is not prime with p, we can find a nonzero natural integer k such that $u = p^k u'$ and u' are prime with p. The p-adic absolute value of the sum is therefore :

$$|r_1+r_2|_p = p^{-(k+\min(k_1,k_2))} = \max\left(p^{-k-k_1}, p^{-k-k_2}\right) < \max\left(p^{-k_1}, p^{-k_2}\right) = \max\left(|r_1|_p, |r_2|_p\right).$$

So, $|r_1+r_2|_p \le \max\{|r_1|_p, |r_2|_p\}$.

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Like $v_p(0) = +\infty$, $|r|_p = 0$ if and only if $r = 0$. And these three properties show that $|\,.\,|_p$ defines a norm on the set of rational numbers $\mathbb{Q}$. The second property is called the strong triangle inequality.

A norm verifying the strong inequality is called a non-Archimedean norm or an ultrametric norm. A norm satisfying the usual triangle inequality is called the Archimedean norm.

2.2 Hierarchical resource system

We will consider a system composed of $m+1$ of T_i ($i = 0,...,m$) types of resources. In the following, we can say resource of type T_i or simply resource T_i. Each resource T_i is assumed to be discrete with a maximum quantity equal to $p-1$, i.e. the number of resources T_i used only takes values integers between 0 and $p-1$. If we denote by r_i the quantity of resources T_i used, we have $r_i \in \{0,1,..., p-1\}$ for all $i \in \{0,...,m\}$. In a real system, it may be that the maximum quantity $p-1$ of resources does not necessarily give a prime number *p*. We can use the first prime number that follows this maximum quantity.

This system is assimilated to a system of hierarchical resources. Resource T_0 is the most important, resource T_i is more important than resource T_{i+1}, and so on. This hierarchy is observed in most real cases. For example, the central processor resource is considered more important, and so on.

We call state of the system the quantities of resources used defined by the vector $r=(r_0,...,r_m)$. We recall that $r_i \in \{0,1,...,p-1\}$ and for a purely mathematical reason, we will fix $p>1$ and prime number. We can extend the state *r* of the system to a vector $r=(r_{-n},...,r_{-1},r_0,r_1,...,r_m)$ for types T_i of resources indexed by negative and positive integers $i=-n,...,0,...,m$. It is also necessary not to fix the numbers of coordinates *n* and *m* used to make possible the future addition of type of resources.

Such a vector space can be represented by a rational number of the form:

$$r = r_{-n}p^{-n} + ... + r_{-1}p^{-1} + r_0 + r_1 p + ... + r_m p^m, \quad r_j \in \{0,1,...,p-1\} \tag{2}$$

To use an ultrametric model, we will construct a complete metric space. The approach is to consider a vector of infinite coordinates of the form:

$$r = ... + r_{-n}p^{-n} + ... + r_{-1}p^{-1} + r_0 + r_1 p + ... + r_m p^m + ..., \quad r_j \in \{0,1,...,p-1\} \tag{3}$$

such that there exist integers *n* and *m* such that $r_{-j}=0$ for all $j>n$ and $r_j=0$ for all $j>m$. The space of these infinities of coordinates will be denoted Q_p in which we will define a metric as follows. Given two states $x=(x_j)$ and $y=(y_j)$, the distance between the two states x and y noted $d_p(x,y)=|x-y|_p$ is equal to:

$$d_p(x,y) = |x-y|_p = p^{-\nu} \tag{4}$$

such that ν is the natural integer defined by: $x_j = y_j$ for all $j<\nu$ and $x_\nu \neq y_\nu$.

Proposition 3.

The distance d_p between the two states *x* and *y* defined in equation (4) is ultrametric.

Proof :

For any three states r_1, r_2 and r_3, this distance d_p verifies the following properties:

- $d_p(r_1,r_2)=0$ if and only if $r_1=r_2$
- $d_p(r_1,r_2)=d_p(r_2,r_1)$
- $d_p(r_1,r_3) \le \max\{d_p(r_1,r_2), d_p(r_2,r_3)\}$

The first two properties follow from the definition of the *p*-adic distance from the *p*-adic absolute value.
For the third property, called strong triangle inequality, we use the second property of Proposition 1:

$$\begin{aligned} d_p(r_1,r_3) = |r_1-r_3|_p = |(r_1-r_2)+(r_2-r_3)|_p &\le \max\{|r_1-r_2|_p, |r_2-r_3|_p\} \\ &\le \max\{d_p(r_1,r_2), d_p(r_2,r_3)\} \end{aligned}$$

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2.3. Ultrametric resource model

Starting from the representation of the state by a rational number, we are interested in its representation in the completeness Q_p of the set of rational numbers equipped with the ultrametric distance d_p.

Each ball can be identified as a ball of radius $R = p^{\nu}$, $\nu \in \mathbf{Z}$. The ball $B_1(0)$ with center 0 and radius 1 will be denoted $\mathbf{Z}_p$. In this ultrametric space, any ball can be represented as a disjoint union of small balls. For example: $\mathbf{Z}_p = B_1(0) = \bigcup_{j=0}^{p-1} B_{1/p}(a^j) = \bigcup_{j_0 \cdots j_{\nu-1}=0}^{p-1} B_{1/p^{\nu}}(a^{j_0 \cdots j_{\nu-1}})$ where $a^j, a^{j_0 \cdots j_{\nu-1}} \in \mathbf{Z}_p$ such that $x_0 = j$ for a^j and $x_0 = j_0, ..., x_{n-1} = j_{n-1}$ for $a^{j_0 \cdots j_{\nu-1}}$. It is the famous property that, in an ultrametric space, any point of a ball can be considered as its center.

2.4 Resource utilization

2.4.1 Mathematical formulation

In our model, a *p*-adic ball represents a set of resource usage states of a system by fixing the usage of some resource.

For example:

- The resource utilization $R_j = B_{1/p}(a^j)$, which is the ball with center a^j and radius $1/p$, $R_j = B_{1/p}(a^j) = \{x \in \mathbf{Z}_p : x_0 = j\}$, is equal to the uses of all the resources while fixing *j* the utilization of the resource T_0.
- Resource usage $R_{ji} = B_{1/p}(a^{ji})$, which is the ball with center a^{ji} and radius $1/p$, $R_{ji} = B_{1/p}(a^{ji}) = \{x \in \mathbf{Z}_p : x_0 = j, x_1 = i\}$, is equal to the usages of all resources while setting *j* the usage of resource T_0 and *i* the usage of resource T_1.

A resource utilization, which is a state of the system, is a point in space Q_p. It is a ball of radius zero. The partition of a ball into disjoint balls of smaller radii corresponds to a partition of a resource use into disjoint sub-uses at a deeper level in the resource hierarchy.

An elementary use is a point of Q_p. But for more practical cases, balls of finite radius are used.

The examples below define balls of finite radius:

- The utilization of resource T_0 is greater than 90%.
- The utilization of resource T_0 is greater than 90% and that of resource T_1 is greater than 60%.
- Thresholds are useful for dimensioning the quantities of resources necessary for a system, and for decisions on the expansion of such resources: Probability of using resource T_0 is greater than p_0, that of T_1 is greater than p_1, and so on.

2.4.2 Probability of resource utilization governed by a diffusion equation

Now, let $q(x,t)$ be the probability that the system is in state *x* at any time *t*, and $q(B_{\nu},t)$ the probability that the state of the system at time *t* belongs to a ball $B_{\nu} = B_{p^{\nu}}(0)$ with center 0 and radius p^{ν}, $\nu \in \mathbf{Z}$. This probability can be represented by an integral as a function of a measure μ defined on the space Q_p:

$$q(B_{\nu},t) = \int_{B_{\nu}} q(x,t)\mu(dx) \tag{5}$$

Proposition 4.

The probability of using resources $q(x,t)$ can be expressed by:

$$\frac{\partial q(x,t)}{\partial t}=\int_{Q_p}\left[p(x\mid y;t)q(y,t)-p(y\mid x;t)q(x,t)\right]\mu(dy) \tag{6}$$

Proof :

To better understand the proof, let us first take the case of any state e_i belonging to a set of real numbers. The probability of being in this state e_i at time $t+\varepsilon$, denoted $q(e_i,t+\varepsilon)$, is equal to:

$$q(e_i,t+\varepsilon)=q(e_i,t)+\sum_{j\neq i}\left(p(e_i\mid e_j,t)q(e_j,t)-p(e_j\mid e_i,t)q(e_i,t)\right)\varepsilon$$

The quantity $q(e_i,t)$ indicates the probability that the system was already in state e_i at time *t*. Another possibility added to this is that the system was in state e_j at time *t* and becomes in state e_i at time $t+\varepsilon$. The probability $p(e_i\mid e_j,t)$ defines this transition from state e_j to state e_i. This is why we have the term $p(e_i\mid e_j,t)q(e_j,t)$ with the duration ε for all the values $j\neq i$. A last possibility that we must subtract from the existence of the state e_i at the instant *t* is that the system has become at another state at time $t+\varepsilon$. This is why we have the term $p(e_j\mid e_i,t)q(e_i,t)$ with the duration ε for all values $j\neq i$.

And we have the differential notation for ε tending to 0:

$$\frac{q(e_i,t+\varepsilon)-q(e_i,t)}{\varepsilon}=\sum_{j\neq i}\left(p(e_i\mid e_j;t)q(e_j,t)-p(e_j\mid e_i;t)q(e_i,t)\right)$$

$$\frac{dq(e_i,t)}{dt}=\sum_{j\neq i}\left(p(e_i\mid e_j;t)q(e_j,t)-p(e_j\mid e_i;t)q(e_i,t)\right)$$

Now, in the space Q_p, we can proceed to the same step of the demonstration. We have by analogy:

$$\frac{\partial q(x,t)}{\partial t}=\int_{Q_p}\left[p(x\mid y;t)q(y,t)-p(y\mid x;t)q(x,t)\right]\mu(dy)$$

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In the case where the transitions are homogeneous in time, that is to say that $p(e_i\mid e_j;t)$ are independent of time, $p(e_i\mid e_j;t)=p(e_i\mid e_j)$, we have:

$$\frac{\partial q(x,t)}{\partial t}=\int_{Q_p}\left[p(x\mid y)q(y,t)-p(y\mid x)q(x,t)\right]\mu(dy) \tag{7}$$

In the case where the transition is symmetric, i.e. $p(x\mid y)=p(y\mid x)$, we have:

$$\frac{\partial q(x,t)}{\partial t}=\int_{Q_p}p(x\mid y)\left[q(y,t)-q(x,t)\right]\mu(dy) \tag{8}$$

when the distance between them increases, and vice versa. We can then set $p(x\mid y)$ as defined in equation (9) for any number:

$$p(x\mid y)=\frac{C_\alpha}{\left|x-y\right|_p^{1+\alpha}} \tag{9}$$

where C_α is a normalizing constant. The choice of α depends on the characterization of the transition according to user behaviors: arrival processes, and service demand laws.

Proposition 5.

The probability of using resources $q(x,t)$ is governed by the differential equation (10):

$$\frac{\partial q(x,t)}{\partial t} = D^{\alpha} q(x,t), \quad \text{with } q(x,0) = q_0(x) \tag{10}$$

where D^{α} is the Vladimirov operator for a function of complex-valued p-adic variable:

$$D^{\alpha} f(x) = \frac{1}{\Gamma_p(-\alpha)} \int_{\mathbb{Q}_p} \frac{f(x)-f(y)}{|x-y|_p^{1+\alpha}} \mu(dy) \tag{11}$$

And $\Gamma_p(-\alpha) = \frac{p^{\alpha}-1}{1-p^{-1-\alpha}}$

Proof :

Starting from equation (8) and using the transition probability that we posed in (9), we obtain:

$$\begin{aligned} \frac{\partial q(x,t)}{\partial t} &= \int_{\mathbb{Q}_p} \frac{C_{\alpha}}{|x-y|_p^{1+\alpha}} [q(y,t)-q(x,t)] \mu(dy) \\ &= C_{\alpha} \int_{\mathbb{Q}_p} \frac{q(y,t)-q(x,t)}{|x-y|_p^{1+\alpha}} \mu(dy) \end{aligned}$$

The integral on the right side of this equality is called Vladimirov's fractional derivative operator D^{α}. As well as the evolution of the probability of using resources is described by:

$$\frac{\partial q(x,t)}{\partial t} = D^{\alpha} q(x,t)$$

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This equation (10) which defines the resource utilization is called the p-adic diffusion equation. The resolution of such an equation uses a Cauchy problem like the one we solved in [16], or as other authors have developed in [17][18].

3. APPLICATIONS

We will take an example of the application of this new model.

3.1 System definition

Consider a system composed of 3 types of resources: T1, T2, T3. T1 has 3 resources, T2 and T3 has 2 resources each. The resource T1 is the most important of the considered system, then T2 and finally T3.

The resources utilization on this system can be represented by the set $\mathbb{Q}_5$, set of 5-adic numbers (p-adic whose p = 5), since 5 is the most prime number greater than the maximum quantity of its resources.

3.2 System Resource Usage Status

The state of the system is represented by a 5-adic number.

Example:

- The utilization of 1 T1 resource, 1 T2 resource, and 1 T3 resource is represented by $r = 1+1p+1p^2 = 1+1\times 5+1\times 5^2 = 31$. Here, 31 represents a 5-adic number and we will denote it $(31)_5$.
- The utilization of 2 T1 resources, 1 T2 resource, and 0 T3 resource is represented by $r = 2+1p+0p^2 = 2+1\times 5+0\times 5^2 = (7)_5$.
- And so on.

Table-1 represents the possible states of this system.

Table-1: Ultrametric model system states

T1	T2	T3	*r* (expression of Hensel)	State of the system
0	0	0	$0+0p+0p^2$	$(0)_5$
0	0	1	$0+0p+1p^2$	$(25)_5$
0	0	2	$0+0p+2p^2$	$(50)_5$
0	1	0	$0+1p+0p^2$	$(5)_5$
0	1	1	$0+1p+1p^2$	$(30)_5$
0	1	2	$0+1p+2p^2$	$(55)_5$
0	2	0	$0+2p+0p^2$	$(10)_5$
0	2	1	$0+2p+1p^2$	$(35)_5$
0	2	2	$0+2p+2p^2$	$(60)_5$
1	0	0	$1+0p+0p^2$	$(1)_5$
1	0	1	$1+0p+1p^2$	$(26)_5$
1	0	2	$1+0p+2p^2$	$(51)_5$
1	1	0	$1+1p+0p^2$	$(6)_5$
1	1	1	$1+1p+1p^2$	$(31)_5$
1	1	2	$1+1p+2p^2$	$(56)_5$
1	2	0	$1+2p+0p^2$	$(11)_5$
1	2	1	$1+2p+1p^2$	$(36)_5$
1	2	2	$1+2p+2p^2$	$(61)_5$
2	0	0	$2+0p+0p^2$	$(2)_5$
2	0	1	$2+0p+1p^2$	$(27)_5$
2	0	2	$2+0p+2p^2$	$(52)_5$
2	1	0	$2+1p+0p^2$	$(7)_5$
2	1	1	$2+1p+1p^2$	$(32)_5$
2	1	2	$2+1p+2p^2$	$(57)_5$
2	2	0	$2+2p+0p^2$	$(12)_5$
2	2	1	$2+2p+1p^2$	$(37)_5$
2	2	2	$2+2p+2p^2$	$(62)_5$
3	0	0	$3+0p+0p^2$	$(3)_5$
3	0	1	$3+0p+1p^2$	$(28)_5$
3	0	2	$3+0p+2p^2$	$(53)_5$
3	1	0	$3+1p+0p^2$	$(8)_5$
3	1	1	$3+1p+1p^2$	$(33)_5$

T1	T2	T3	*r* (expression of Hensel)	State of the system
3	1	2	$3+1p+2p^2$	$(58)_5$
3	2	0	$3+2p+0p^2$	$(13)_5$
3	2	1	$3+2p+1p^2$	$(38)_5$
3	2	2	$3+2p+2p^2$	$(63)_5$

The system therefore has 36 different states.

3.3 Analyzes of distances between resource utilization states

To calculate the distance between these states, we will take some examples:

- State 1: 2 T1 resources, 1 T2 resource and 0 T3 resource (equivalent to $r_1=(7)_5$)
  State 2: 1 T1 resource, 0 T2 resource and 0 T3 resource (equivalent to $r_2=(1)_5$)
  The distance between these two states is equal to $d=\left|r_1-r_2\right|_5=\left|7-1\right|_5=\left|6\right|_5=\left|2^1 3^1 5^0\right|_5=5^{-0}=1$
- State 1: 3 T1 resources, 2 T2 resources and 0 T3 resources (equivalent to $r_1=(13)_5$)
  State 2: 3 T1 resources, 1 T2 resource and 0 T3 resources (equivalent to $r_2=(8)_5$)
  The distance between these two states is equal to $d=\left|r_1-r_2\right|_5=\left|13-8\right|_5=\left|5\right|_5=\left|5^1\right|_5=5^{-1}=0,2$
- …

The calculations allow us to obtain that these distances can only take one of the following 4 values: $5^{-\infty}$, 5^0 , 5^{-1}, 5^{-2} , that is 0, 1, 0.2 and 0.04.

Chart-1 and Chart-2 represent the distances between two states of the system. They are called initial state and final state.

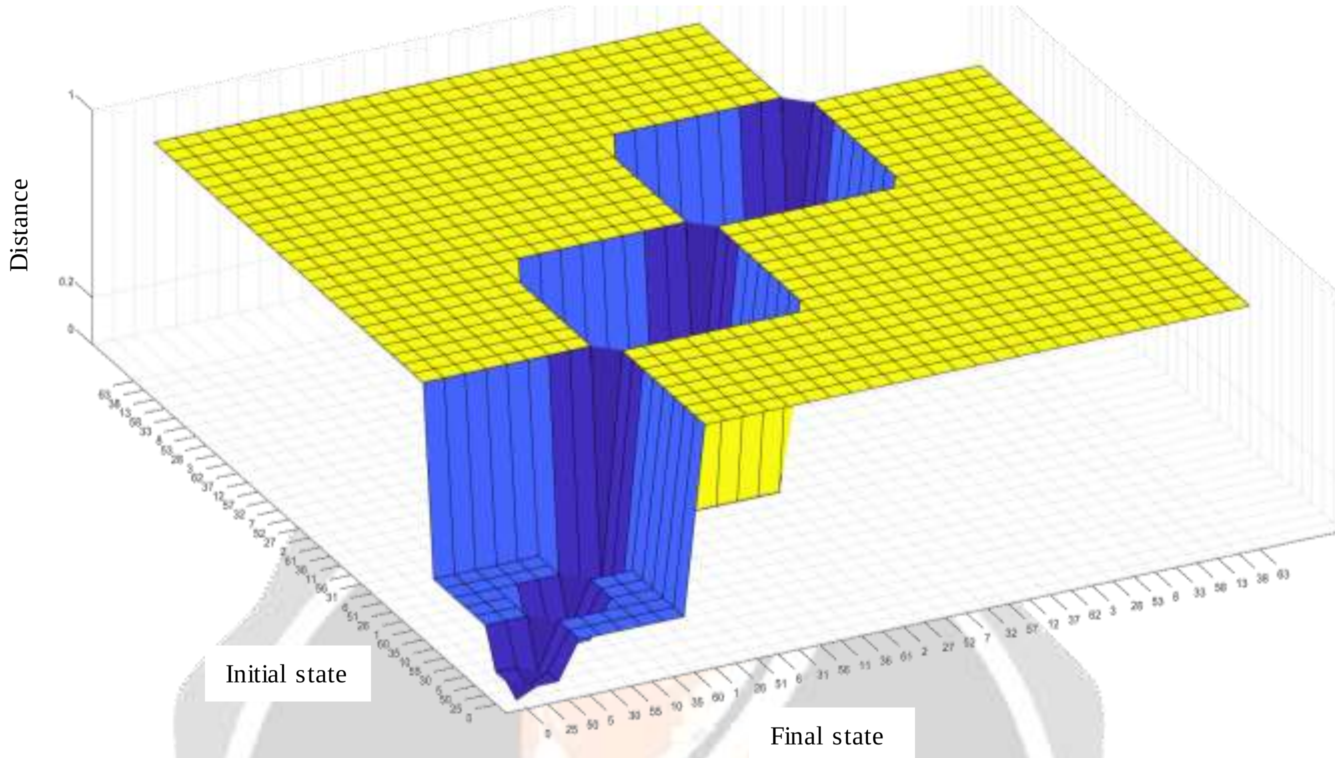

Chart-1: Distance between system states in 3D form

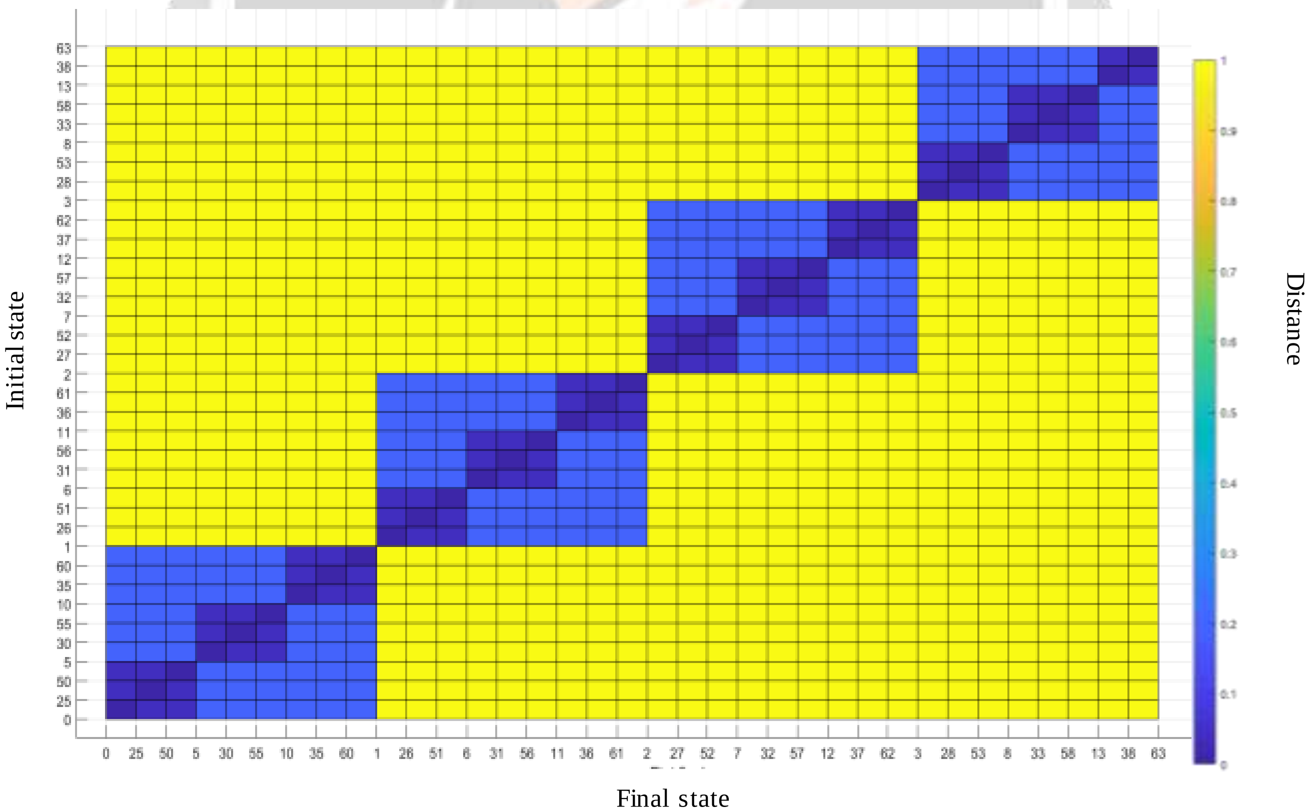

Chart-2: Distance between system states as a plane coloring

It can be seen that the more the change affects the number of lower priority resources (T3 in this example), the distance between the two states is small. This is explained by the blue shift in Chart-1 and Chart-2. And the more the change is on the number of higher priority resources (T1 in this example), the distance between the two states is great. This is explained by the yellow color in Chart-1 and Chart-2.

Example: We will start from state $(7)_5$ which is equal to T1 = 2, T2 = 1 and T3 = 0 resources used.

- For a final state equal to $(32)_5$: T1 = 2, T2 = 1 and T3 = 1 resources used. The change concerns the resources of T3. The distance between these states is 0.04 (blue in Chart-1 and Chart-2).
- For a final state equal to $(57)_5$: T1 = 2, T2 = 1 and T3 = 2 resources used. The change concerns the resources of T3. The distance between these states is still 0.04 (blue in Chart-1 and Chart-2).
- For a final state equal to $(12)_5$: T1 = 2, T2 = 2 and T3 = 0 resources used. The change concerns the resources of T2 which have a higher priority than those of T3. The distance between these states is 0.2.
- For a final state equal to $(6)_5$: T1 = 1, T2 = 1 and T3 = 0 resources used. The change concerns the resources of T1 which have a higher priority than those of T2 and T3. The distance between these states is 1 (yellow in in Chart-1 and Chart-2).

Hence the importance of using our model in the case of hierarchical resources.

3.4 The probability law of the quantities of resources used

From these distances, we find that 35 pairs of states have a distance of 0, then 918 pairs of states with a distance of 1, then 204 pairs of a distance of 0.2 and 68 pairs of a distance of 0.04.

If we take the state transition probability equal to $p(x,y)=\dfrac{C}{|x-y|_5}$, we have the normalization constant $1=918\times\dfrac{C}{1}+204\times\dfrac{C}{0,2}+68\times\dfrac{C}{0,04}$. We will find $C=\dfrac{1}{3638}$.

The probability of transition from a state x to a state y is therefore equal to $p(x,y)=\dfrac{1}{3638|x-y|_5}$ where $|x-y|_5$ designates the ultrametric distance between these two states which are 0 if $x = y$, 1 if $x - y$ is not a multiple of 5, 0.2 if $x - y$ is a multiple of 5, and 0.04 if $x - y$ is a multiple of 25.

According to equation (10), the probability of using resources is governed by the equation $\dfrac{\partial q(x,t)}{\partial t}=D^0 q(x,t),\quad \text{with } q(x,0)=q_0(x)$.

Where $D^0 q(x,t)=\left(1-\dfrac{1}{5}\right)\int_{\mathbb{Q}_5}\dfrac{q(x,t)-q(y,t)}{|x-y|_5}\mu(dy)$.

The advantage highlighted in this model is the combined study of the uses of the 3 types of resources. The separate study can lead to errors by not considering other resources.

4. CONCLUSION

p-adic numbers can be used to model hierarchical resource utilization. The importance of this type of modeling has been highlighted. We proved that the probability of resource utilization is governed by a Cauchy problem: a fractional differential equation, and particularly, with the Vladimirov operator. There are still many aspects to study: the existence of a stationary solution, in order to determine the asymptotic behavior of our model.

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